Curvature oscillations in modified gravitational baryogenesis and high energy cosmic rays

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Baryogenesis

Dominance of matter over antimatter in the universe

- The local universe is clearly matter dominated.
- On the other hand, matter and antimatter seem to have similar properties $\Longrightarrow$ we could expect a matter-antimatter symmetric universe.
- A satisfactory model of our universe should be able to explain the origin of the matter-antimatter asymmetry.

The term **baryogenesis** is used to indicate the generation of the asymmetry between baryons and antibaryons.

Sakharov Principles (1967):

1. Non-conservation of baryonic number
2. Breaking of symmetry between particles and antiparticles
3. Deviation from thermal equilibrium (Th.Eq.)

SBG and GBG do not demand an explicit C and CP violation and can proceed in Th.Eq.

Spontaneous Baryogenesis (SBG)

Cosmological baryon asymmetry can be created by SBG in thermal equilibrium:

- A. Cohen, D. Kaplan (1987, 1988), A.Cohen, D.Kaplan, A. Nelson (1991)

"spontaneous" $\Longrightarrow$ spontaneous breaking of underlying symmetry of the theory

- Unbroken phase: the theory is invariant with respect to the global $U(1)$-symmetry, which ensures conservation of total baryonic number.

Spontaneous symmetry breaking: the Lagrangian density acquires the term

$$\mathcal{L}_{SBG} = (\partial_\mu \theta) J^\mu_B \;\Longrightarrow\; \mathcal{L}_{SB} = \dot{\theta}\, n_B\,, \quad n_B \equiv J^0_B \;\; (\theta = \theta(t))$$

$\boldsymbol{\theta}$ is the (pseudo) Goldstone field, $\boldsymbol{J^\mu_B}$ is the baryonic current of matter fields, and n_B is the baryonic number density

Stimulated by SBG the idea of gravitational baryogenesis (GBG) was put forward:

- H. Davoudiasl et al. Phys. Rev. Lett. **93** (2004) 201301, hep-ph/0403019.

Gravitational Baryogenesis (GBG)

The scenario of SBG was modified by the introduction of the coupling of the baryonic current to the derivative of the curvature scalar ***R***:

$$\mathcal{L}_{GBG} = \frac{1}{M^2}(\partial_\mu R) J_B^\mu$$

- M is a constant parameter with the dimension of mass.

GBG mechanism can successfully explain the magnitude of the cosmological baryon asymmetry of the universe.

The addition of the curvature dependent term to the Hilbert-Einstein Lagrangian of GR leads to higher order gravitational equations of motion which are strongly unstable with respect to small perturbations.

- EA, A.D. Dolgov, "*Intrinsic problems of the gravitational baryogenesis*", Phys.Lett. B769 (2017) 171-175, arXiv:1612.06206. "*Instability of gravitational baryogenesis with fermions*", JCAP 1706 (2017) no.06, 001, arXiv:1702.07477.

GBG with scalars

The action of the scalar model:

$$S = -\int d^4x\,\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}R + \frac{1}{M^2}(\partial_\mu R)J^\mu - g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi^* + U(\phi,\phi^*)\right] + S_m$$

- where $m_{Pl} = 1.22\cdot 10^{19}$ GeV is the Planck mass, S_m is the matter action.

The baryonic number is carried by scalar field ϕ with potential $U(\phi,\phi^*)$.

If the potential $U(\phi)$ is not invariant w.r.t. the $U(1)$-rotation $\phi \to e^{i\beta}\phi \Longrightarrow$ the baryonic current defined in the usual way is not conserved.

$$J_\mu = iq(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)$$

- Here q is the baryonic number of ϕ

The corresponding equation of motion for the curvature scalar:

$$\frac{m_{Pl}^2}{16\pi}R + \frac{1}{M^2}\left[(R+3D^2)D_\alpha J^\alpha + J^\alpha D_\alpha R\right] - D_\alpha\phi\, D^\alpha\phi^* + 2U(\phi) = -\frac{1}{2}\,T^\mu_\mu$$

- D_μ is the covariant derivative, T^μ_μ is the trace of EM tensor of matter

EoM in FLRW background: $ds^2 = dt^2 - a^2(t)d\mathbf{r}^2$, $H = \dot{a}/a$

In the homogeneous case the equation for the curvature scalar:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{1}{M^2}\left[(R + 3\partial_t^2 + 9H\partial_t) D_\alpha J^\alpha + \dot{R}\, J^0\right] = -\frac{T^{(tot)}}{2}$$

- $T^{(tot)}$ is the trace of the energy-momentum tensor of matter including contribution from the ϕ-field.

The covariant divergence of the baryonic current in homogeneous case:

$$D_\alpha J^\alpha = \frac{2q^2}{M^2}\left[\dot{R}\left(\phi^* \dot{\phi} + \phi \dot{\phi}^*\right) + \left(\ddot{R} + 3H\dot{R}\right)\phi^* \phi\right] + iq\left(\phi \frac{\partial U}{\partial \phi} - \phi^* \frac{\partial U}{\partial \phi^*}\right)$$

The expectation values of the products of the quantum operators ϕ, ϕ^*, and their derivatives after the thermal averaging:

$$\langle \phi^* \phi \rangle = \frac{T^2}{12}, \quad \langle \phi^* \dot{\phi} + \dot{\phi}^* \phi \rangle = 0$$

- T is the plasma temperature

Exponential instability of GBG with scalars

Equation of motion for the classical field $\boldsymbol{R}$ in the cosmological plasma:

$$\frac{m_{Pl}^2}{16\pi} R + \frac{q^2}{6M^4}\left(R + 3\partial_t^2 + 9H\partial_t\right)\left[\left(\ddot{\boldsymbol{R}} + 3H\dot{R}\right) T^2\right] + \frac{1}{M^2}\dot{R}\langle J^0\rangle = -\frac{T^{(tot)}}{2}$$

- $\langle J^0 \rangle$ is the thermal average value of the baryonic number density of ϕ.
- This term can be neglected, since it is small initially and subdominant later.

Keeping only the linear in R terms we obtain the linear fourth order equation:

$$\frac{d^4 R}{dt^4} + \mu^4 R = -\frac{1}{2} T^{(tot)}, \quad \mu^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2}$$

The homogeneous part of this equation has exponential solutions:

$$R \sim e^{\lambda t}, \quad \lambda = |\mu| e^{i\pi/4 + i\pi n/2}, \quad n = 0, 1, 2, 3$$

- There are two solutions with positive real parts of $\boldsymbol{\lambda}$.

Curvature scalar is exponentially unstable w.r.t. small perturbations, so $\boldsymbol{R}$ should rise exponentially fast with time and quickly oscillate around this rising function.

GBG with fermions

We start from the action in the form:

$$S = -\int d^4x\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}R - \mathcal{L}[Q,L]\right] + S_{matt}$$

with

$$\begin{aligned}\mathcal{L}[Q,L] &= \frac{i}{2}(\bar{Q}\gamma^\mu\nabla_\mu Q - \nabla_\mu\bar{Q}\,\gamma^\mu Q) - m_Q\bar{Q}\,Q \\ &+ \frac{i}{2}(\bar{L}\gamma^\mu\nabla_\mu L - \nabla_\mu\bar{L}\gamma^\mu L) - m_L\bar{L}\,L \\ &+ \frac{g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L) + (\bar{Q}^cQ)(\bar{L}Q)\right] + \frac{d}{M^2}(\partial_\mu R)J^\mu + \mathcal{L}_{matt}\end{aligned}$$

- Q is the quark (or quark-like) field with non-zero baryonic number B_Q, L is another fermionic field (lepton)
- ∇_μ is the covariant derivative of Dirac fermion in tetrad formalism
- $J^\mu = B_Q\bar{Q}\gamma^\mu Q$ is the quark current with $\boldsymbol{\gamma^\mu}$ being the curved space gamma-matrices; $d = \pm 1$ is dimensionless coupling constant.
- The four-fermion interaction between quarks and leptons is introduced to ensure the necessary non-conservation of the baryonic number

Gravitational EoM for curvature scalar in fermionic case

Taking trace of gravitational EoM we obtain:

$$-\frac{m_{Pl}^2}{8\pi}R = m_Q\bar{Q}Q + m_L\bar{L}L + \frac{2g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L) + (\bar{Q}^c Q)(\bar{L}Q)\right] - \frac{2d}{M^2}(R + 3D^2)D_\alpha J^\alpha + T_{other}$$

- T_{other} is the trace of the energy momentum tensor of all other fields.
- At relativistic stage we can take $T_{other} = 0$.

To calculate the current divergence, we use kinetic equation, which leads to an explicit dependence of $D_\alpha J^\alpha$ on $\dot{R}$, if the current is not conserved. As a result we obtain high (fourth) order equation for $\boldsymbol{R}$.

We study solutions in homogeneous and isotropic FRW background:

$$ds^2 = dt^2 - a^2(t)d\mathbf{r}^2\,, \quad D_\alpha J^\alpha = (\partial_t + 3H)J^t \equiv (\partial_t + 3H)n_B$$

Curvature Instability

The contribution of thermal matter into EoM for R can be neglected

$$\frac{m_{Pl}^2}{8\pi} R = \frac{2d}{M^2}(R + 3D^2)(\partial_t + 3H)n_B$$

From the kinetic equation we find the baryonic number density:

$$n_B \sim \frac{9d}{10}\frac{g_s B_Q \dot{R}}{M^2 T}$$

Neglecting the H-factor in comparison with time derivatives of R, we arrive to:

$$\frac{d^4 R}{dt^4} + \lambda^4 R = 0, \qquad \lambda^4 = \frac{5 m_{Pl}^2 M^4}{36\pi g_s B_Q^2 T^2}$$

- g_s is the number of quark spin states.

This equation has extremely unstable solution with instability time by far shorter than the cosmological time. This instability would lead to an explosive rise of $\boldsymbol{R}$

Stabilization of GBG in R^2-modified gravity

- EA, A.D. Dolgov, K. Dutta, and R. Rangarajan, *"Gravitational Baryogenesis: Problems and Possible Resolution"*, Symmetry 15 (2023) 2, 404.

Possible stabilization mechanism might be achieved in R^2-modified gravity:

$$S_{Grav} = -\frac{m_{Pl}^2}{16\pi}\int d^4x\,\sqrt{-g}\left(R - \frac{R^2}{6M_R^2}\right)$$

R^2-term:

- In the early universe generates inflation (Starobinsky) and density perturbations.
- leads to excitation of the scalar degree of freedom: scalaron with the mass M_R.
- Amplitude of the observed density perturbations demands: $M_R = 3\cdot 10^{13}$GeV.

Bosonic case: baryonic number is carried by a complex scalar field ϕ:

$$S_{tot}[\phi] = -\int d^4x\,\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}\left(R - \frac{R^2}{6M_R^2}\right) + \frac{1}{M^2}(\partial_\mu R)J^\mu_{(\phi)}\right] - \int d^4x\,\sqrt{-g}\,[g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi^* - U(\phi,\phi^*)] + S_{matt}$$

Stabilization: Bosonic Case

The equation for the curvature evolution in spatially flat FLRW-metric:

$$\frac{m_{Pl}^2}{16\pi}\left[R+\frac{1}{M_R^2}(\partial_t^2+3H\partial_t)R\right]+\frac{1}{M^2}\left[(R+3\partial_t^2+9H\partial_t)D_\alpha J^\alpha_{(\phi)}+\dot{R}\,J^0_{(\phi)}\right]$$
$$+2U(\phi)-(D_\alpha\phi)(D^\alpha\phi^*)=-\frac{T^\mu_\mu(\phi)}{2}$$

With the divergence of the baryonic current:

$$D_\alpha J^\alpha_{(\phi)}=\frac{2q^2}{M^2}\left[\dot{R}\,(\phi^*\dot{\phi}+\phi\dot{\phi}^*)+(\ddot{R}+3H\dot{R})\,\phi^*\phi\right]+iq\left(\phi\frac{\partial U}{\partial\phi}-\phi^*\frac{\partial U}{\partial\phi^*}\right)$$

we obtain the 4th order differential equation:

$$\frac{m_{Pl}^2}{16\pi}\left(R+\frac{1}{M_R^2}D^2R\right)+\frac{q^2}{6M^4}\left(R+3\partial_t^2+9H\partial_t\right)\left[\left(\ddot{R}+3H\dot{R}\right)T^2\right]=-\frac{T^\mu_\mu(\phi)}{2}$$

Keeping only the dominant terms we simplify the above equation to:

$$\frac{d^4R}{dt^4}+\frac{\kappa^4}{M_R^2}\frac{d^2R}{dt^2}+\kappa^4R=-\frac{1}{2}\,T^\mu_\mu(\phi),\qquad \kappa^4=\frac{m_{Pl}^2M^4}{8\pi q^2T^2}$$

Stabilization: Fermionic Case

$$S_{tot}[Q,L] = -\int d^4x\sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}\left(R - \frac{R^2}{6M_R^2}\right) + \frac{d}{M^2}(\partial_\mu R)J_Q^\mu - \mathcal{L}[Q,L]\right]$$

The equation for the curvature evolution:

$$\begin{aligned} - \quad & \frac{m_{Pl}^2}{8\pi}\left(R + \frac{1}{M_R^2}D^2 R\right) = m_Q \bar{Q} Q + m_L \bar{L} L + \\ + \quad & \frac{2g}{m_X^2}\left[(\bar{Q}\,Q^c)(\bar{Q}L) + (\bar{Q}^c Q)(\bar{L}Q)\right] - \frac{2d}{M^2}(R + 3D^2)D_\alpha J_Q^\alpha + T_{matt} \end{aligned}$$

- In the early universe, when various species are relativistic $T_{matt} = 0$.
- Higher order equation for R originates after we substitute the current divergence $D_\alpha J^\alpha$ calculated from kinetic equation in external field R.

In complete analogy with the previous cases we obtain:

$$\frac{d^4 R}{dt^4} + \frac{\kappa_f^4}{M_R^2}\frac{d^2 R}{dt^2} + \kappa_f^4 R = 0, \quad \kappa_f^4 = \frac{5 m_{Pl}^2 M^4}{36\pi g_s B_Q^2 T^2}$$

Stability condition

The characteristic equation for the solution $R \sim \exp(\lambda t)$:

$$(*) \quad \lambda^4 + \frac{\kappa^4}{M_R^2}\lambda^2 + \kappa^4 = 0 \qquad \Longrightarrow \; \lambda^2 = -\frac{\kappa^4}{2M_R^2} \pm \kappa^2 \sqrt{\frac{\kappa^4}{4M_R^4} - 1}$$

No instability, if $\lambda^2 < 0$ and Eq. $(*)$ has only oscillating solutions.

Stability condition (boson case):

$$\kappa^4 = \frac{m_{Pl}^2 M^4}{8\pi q^2 T^2} > 4M_R^4 \; \Longrightarrow \; M > 3 \cdot 10^4 \text{ GeV} \left(\frac{q\,T}{\text{GeV}}\right)^{1/2}$$

The value of λ depends upon the relation between κ and M_R:

- $\kappa \sim M_R \Longrightarrow$ the frequency of oscillations is of the order of M_R, $|\lambda| \sim M_R$.
- $\kappa \gg M_R \Longrightarrow$ 2 possible solutions: $|\lambda| \sim M_R$ and $|\lambda| \sim M_R(\kappa/M_R)^2 \gg M_R$.

High frequency oscillations of R would lead to **efficient gravitational particle production** and, as a result, to damping of the oscillations.

Production of dark matter particles by oscillating curvature

Heavy DM particles have been created in the model of the Starobinsky inflation:

$$S(R^2) = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\,\left[R - \frac{R^2}{6M_R^2}\right]$$

The width of the scalaron decay into a pair of fermions with mass m_f:

$$\Gamma_f = \frac{m_f^2 M_R}{6M_{Pl}^2}$$

- This result is obtained for particles with masses $m_f \ll M_R$.
- X-particles created by the scalaron decay into heavier fermions could form DM, if $m_f \sim 10^7$ GeV, $M_X \sim 10^6$ GeV (EA, A. Dolgov, R.Singh, *R^2-cosmology and New Windows for Superheavy Dark Matter*, Symmetry 13 (2021) 5, 877)

We are interested in the case when the scalaron decays create particles with mass about 10^{21} eV, that is the energy of UHECR.

- EA, A.D. Dolgov, A.A. Nikitenko, *"Cosmic rays from heavy particle decays"*, e-Print: 2305.03313 [hep-ph]

Scalaron decay into extremely heavy DM

The width of the scalaron decay into superheavy leptons with mass $M_L \sim M_R/2$:

$$\Gamma_L = \frac{M_L^2 M_R}{6 M_{Pl}^2} \sqrt{1 - \frac{4M_L^2}{M_R^2}}$$

- The phase space factor $(1 - 4M_L^2/M_R^2)^{1/2}$ makes it possible to arrange the density of presumably DM particles L equal to the observed density of DM.

However, with the canonical energy scale of gravitational interaction with $M_{Pl} = 1.22 \cdot 10^{19}$ GeV, the life-time of such DM-particles turns out to be too long to allow for any observable consequences of their decays.

A possible way out could be opened by diminishing the fundamental gravity scale at small distances down to a lower value $M_* < M_{Pl}$. This could lead to a considerable increase of decay probability of DM-particles.

Decays through virtual Black Holes

- Usually dark matter particles are supposed to be absolutely stable.
- Zeldovich mechanism (1976): decay of any presumably stable particles is possible through creation of virtual black holes.
- The rate of the proton decay calculated in the canonical gravity, with the energy scale equal to M_{Pl}, is extremely tiny and the corresponding life-time is by far longer than the universe age.

However, the smaller scale of gravity and huge mass of DM particles both lead to a strong amplification of the Zeldovich effect.

Superheavy DM particles with $M_X \sim 10^{12}$ GeV may decay through the virtual BH with life-time only a few orders of magnitude longer than the universe age.

Decays of such particles could make essential contribution to the UHECR.

Multidimensional Modification of Gravity

Model: the observable universe with the SM particles is confined to a 4-dim brane embedded in a $(4+d)$-dim bulk, while gravity propagates throughout the bulk.

- N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **429**, 263 (1998); I.Antoniadis, N.Arkani-Hamed, S.Dimopoulos, G.Dvali, Phys.Lett.B**436**,257(1998).

The Planck mass M_{Pl} becomes a long-distance 4-dimensional parameter and the relation with the effective gravity scale at small distances, M_*, is given by:

$$M_{Pl}^2 \sim M_*^{2+d} R_*^d, \quad R_* \sim \frac{1}{M_*}\left(\frac{M_{Pl}}{M_*}\right)^{2/d},$$

- R_* is the size of the extra dimensions.

We choose $M_* \approx 3 \times 10^{17}$ GeV, so $R_* \sim 10^{(4/d)}/M_* > 1/M_*$.

Proton decay through virtual Black Hole

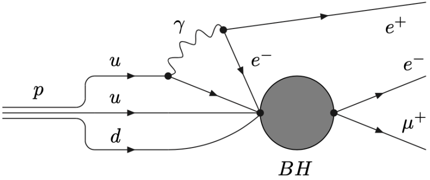

Figure: C. Bambi, A. D. Dolgov and K. Freese, Nucl. Phys. B **763** (2007), 91-114.

The width of the proton decay $p \to l^+ \bar{q} q$:

$$\Gamma_p = \frac{m_p\,\alpha^2}{2^{12}\,\pi^{13}} \left(\ln \frac{M_{Pl}^2}{m_q^2}\right)^2 \left(\frac{\Lambda}{M_{Pl}}\right)^6 \left(\frac{m_p}{M_{Pl}}\right)^{4+\frac{10}{d+1}} \int_0^{1/2} dx x^2 (1-2x)^{1+\frac{5}{d+1}}$$

- $m_p \approx 1$GeV, $m_q \sim 300$ MeV, $\Lambda \sim 300$ MeV is the QCD scale parameter, $\alpha = 1/137$, and d is the number of "small' extra dimensions.

Proton life-time: $\tau_p = 7.3 \times 10^{198}$ years $\gg t_U \approx 1.5 \times 10^{10}$ years.

X-particle decay $X \to L^+ \bar{q}_* q_*$: $m_p \Longrightarrow M_X \sim 10^{12} GeV$, $M_{Pl} \Longrightarrow M_*,\ m_{q_*} \sim M_X$.

Heavy proton type dark matter

The life-time of X-particles:

$$\tau_X \approx 10^{-24}\mathrm{s} \cdot \frac{2^{11}\pi^{13}}{3\alpha_*^2}\left(\frac{\mathrm{GeV}}{\mathrm{M_X}}\right)\left(\frac{\mathrm{M_*}}{\Lambda_*}\right)^6\left(\frac{\mathrm{M_*}}{\mathrm{M_X}}\right)^{4+10/(\mathrm{d}+1)}\left(\ln\frac{\mathrm{M_*}}{\mathrm{m_{q_*}}}\right)^{-2}\mathrm{I(d)}^{-1},$$

where we took $1/\mathrm{GeV} = (2/3)\times 10^{-24}$ s and

$$I(d) = \int_0^{1/2} dx x^2 (1-2x)^{1+\frac{5}{d+1}}, \quad I(7) \approx 0.0057.$$

Now all the parameters, except for Λ_*, are fixed:

- $M_* = 3\times 10^{17}$ GeV, $M_X = 10^{12}$ GeV, $m_{q_*} \sim M_X$, and $\alpha_* = 1/50$

The life-time of X-particles can be estimated as:

$$\tau_X \approx 7\times 10^{12}\ \mathrm{years}\left(10^{15}\,\mathrm{GeV}/\Lambda_*\right)^6 \quad vs \quad t_U \approx 1.5\times 10^{10}\ \mathrm{years}$$

A slight variation of Λ_* near 10^{15} GeV allows to fix the life-time of DM particles in the interesting range. They would be stable enough to behave as the cosmological DM and their decay could make considerable contribution into cosmic rays at ultra high energies.

Conclusions

- The derivative coupling of baryonic current to the curvature scalar in GBG scenarios leads to higher (4th) order equations for gravitational field.
- These equations are unstable with respect to small perturbations of the FRW background and such instability leads to an exponential rise of the curvature.
- For a large range of cosmological temperatures the development of the instability is much faster than the universe expansion rate.
- The problem of stability can be solved by adding to the Hilbert-Einstein action the R^2-term that leads to oscillations of curvature and to efficient particle production.
- In the model of high dimensional gravity modification may exist superheavy DM particles stable with respect to the conventional particle interactions. However, such DM particles should decay though the virtual BH formation.
- With a proper choice of the parameters the life-time of such quasi-stable particles may be larger than the universe age only by 3-4 orders of magnitude.
- This permits X-particles to make an essential contribution to the flux of high energy cosmic rays.
- The considered mechanism may lead to efficient creation of cosmic ray neutrinos of very high energies observed at IceCube and Baikal detectors.

THE END

THANK YOU FOR ATTENTION