Monodromy oscillons: an effective analytic description

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dedicated to the memory of Valery Rubakov

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also: D. G. Levkov, VM, E. Ya. Nugaev, A. G. Panin, JHEP12(2022)079 • arXiv:2208.04434

Oscillons: introduction

Scalar field theory

$$\partial_t^2 \varphi - \Delta\varphi = -V'(\varphi)$$

Generic lifetimes:

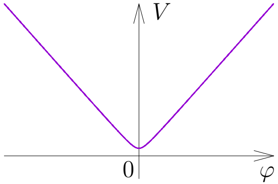
$\gtrsim 10^5$ periods

Example:

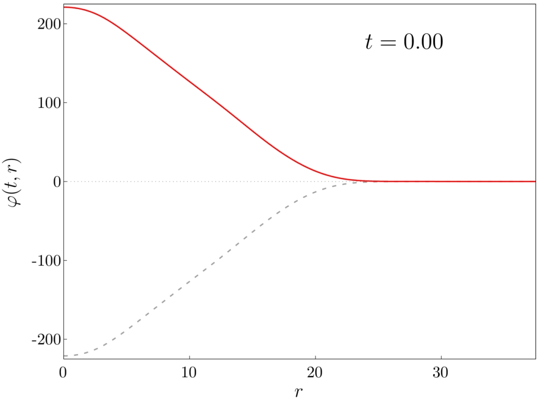
$$V(\varphi) = \sqrt{1+\varphi^2}$$

(axion–monodromy model)

McAllister, Silverstein, Westphal '10



$d = 3$



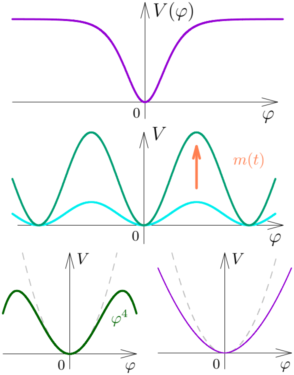
Scalar field theory

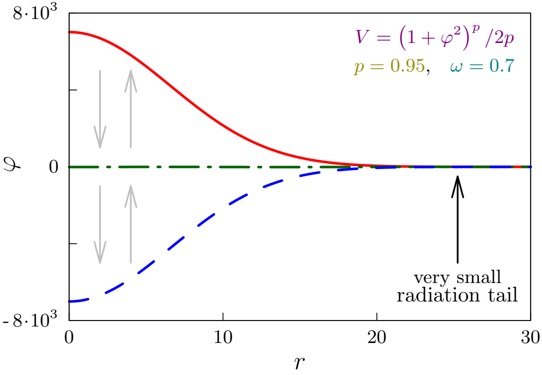
$$\partial_t^2 \varphi - \Delta\varphi = -V'(\varphi)$$

Generic lifetimes:

$\gtrsim 10^5$ periods

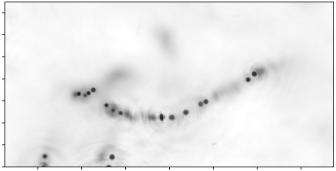
Plethora of theories:





Oscillons in cosmology

- nucleate during generation of axion or ultra-light DM



Kolb, Tkachev '94

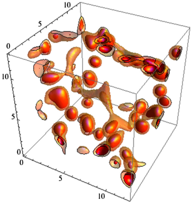
Vaquero, Redondo, Stadler '19

Buschmann, Foster, Safdi '20

- accompany cosmological phase transitions

Dymnikova, Kozel, Khlopov, Rubin '00
Gleiser, Graham, Stamatopoulos '10

- formed by inflaton field during preheating



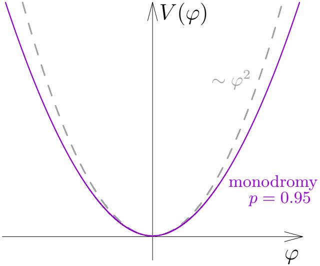
Amin, Easther, Finkel, Flauger, Herzberg' 12

Hong, Kawasaki, Yamazaki '18

Why are oscillons so long-lived? How to describe them?

Monodromy potentials

$$V(\varphi) = \frac{1}{2p}\left(1+\varphi^2\right)^p, \qquad p \lesssim 1$$



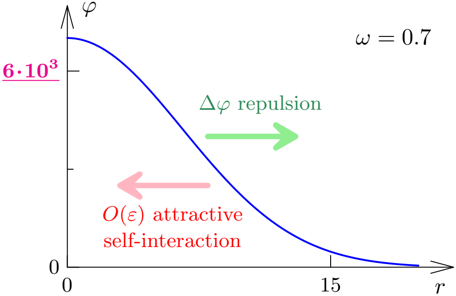
- Small attractive nonlinearity:

$$\boxed{\varepsilon \equiv 1 - p}$$

$$\Downarrow$$

- Large radius: $R^{-2} \sim O(\varepsilon)$.
- Lifetime: up to $\mathbf{10^{14}}$ periods!

  Ollé, Pujolàs, Rompineve '20
- Very strong fields: how to account for small nonlinearities**?**



Isolating small nonlinearity at strong fields

$$\partial_t^2\varphi - \Delta\varphi = -V'(\varphi)$$

- Zero-order approximation: still a **parabola**, but **not** expansion around the vacuum

$$-V'(\varphi) = -\mu^2\varphi - \delta V'(\varphi)$$

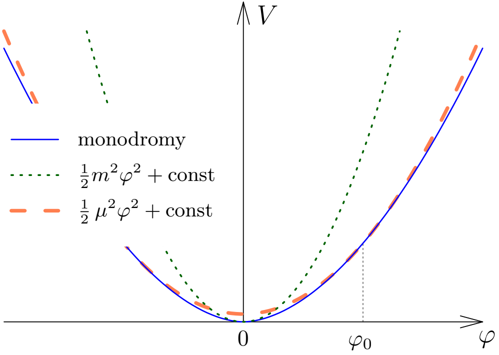
$$\delta V \equiv V - \mu^2\varphi^2/2$$

- Wise choice of $\mu \neq m$ to make $\delta V'$ small:

$$\mu^2 = V'(\varphi_0)/\varphi_0$$

for some scale $\varphi_0 \sim \varphi$

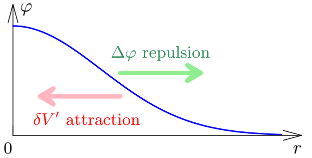
- In the end: scale φ_0 — tuned to the oscillon amplitude.



Example: monodromy potential

$$V'(\varphi) = \left(1+\varphi^2\right)^{-\varepsilon}\cdot\varphi = \underbrace{\left(1+\varphi_0^2\right)^{-\varepsilon}}_{\mu^2}\cdot\varphi + \delta V'$$

Effective Field Theory (EFT): slowly changing variables



- Oscillons: $\delta V' \sim \Delta\varphi \sim O(\varepsilon)$

$$\Downarrow$$

- Zero-order approximation:

$$\partial_t^2\varphi - \cancel{\Delta\varphi} = -\mu^2\varphi - \cancel{\delta V'}$$

linear oscillator →

Action-angle: $\varphi = \sqrt{2I/\mu}\,\cos\theta$

$$\pi_\varphi \equiv \partial_t\varphi = -\sqrt{2I\mu}\,\sin\theta$$

Solution: $I(t) = \text{const}, \quad \theta = \mu t.$

- Leading order: restore $\Delta\varphi$ and δV

$I(t, \boldsymbol{x}), \theta(t, \boldsymbol{x})$ now depend on $\boldsymbol{x}$ but **slowly**.

- Classical field action:

$$\mathcal{S} = \int dt\, d^3\boldsymbol{x} \left[\underbrace{\pi_\varphi\partial_t\varphi - \mu^2\varphi^2/2}_{I\partial_t\theta - \mu I} \underbrace{- (\partial_i\varphi)^2/2 - \delta V}_{subleading} \right]$$

Effective Field Theory (EFT): averaging perturbations

$$\mathcal{S} = \int dt\, d^3\boldsymbol{x} \left[\underbrace{\pi_\varphi \partial_t \varphi - \mu^2 \varphi^2/2}_{I\partial_t\theta - \mu I} \underbrace{- (\partial_i \varphi)^2/2 - \delta V}_{\textit{subleading}} \right]$$

Averaging over period : $t \longrightarrow \theta$

- $\partial_i I$, $\partial_i \theta$ — slow varying $\Longrightarrow$ moved ***out*** of the averages.

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle \overset{t\to\theta}{=} \int_0^{2\pi} \frac{d\theta}{2\pi}\,(\partial_i\varphi)^2 \approx \frac{(\partial_i I)^2}{4I\mu} + \frac{I}{\mu}(\partial_i\theta)^2 + \cancel{\langle\partial_I\Phi\,\partial_\theta\Phi\rangle\partial_i I\partial_i\theta}$$

$\varphi = \sqrt{2I/\mu}\cos\theta$

symmetry $\theta \to -\theta$

$$\delta V \longrightarrow \langle\delta V\rangle = \int_0^{2\pi}\frac{d\theta}{2\pi}\,\delta V(I,\theta) = \frac{1}{2p}\Big(\mathcal{A}_p(\varsigma) - p\mu I\Big)$$

$$\mathcal{A}_p(\varsigma) \equiv \left\langle\left(1+\varsigma\cos^2\theta\right)^p\right\rangle = (1+\varsigma)^{p/2} P_p\left(\frac{1+\varsigma/2}{\sqrt{1+\varsigma}}\right)$$

$\varsigma = 2I/\mu$

Monodromy oscillons: leading-order effective action

Effective action in the leading order

$$\mathcal{S}_{\text{eff}} = \int dt\, d^3\boldsymbol{x} \left[I\partial_t\theta - \mu I - \frac{(\partial_i I)^2}{8I\mu} - \frac{I(\partial_i\theta)^2}{2\mu} - \frac{\mathcal{A}_p(\varsigma)}{2p} + \frac{\mu I}{2} \right]$$

- Action depends on φ_0 as $O(\varepsilon^2)$
  After second-order corrections — $O(\varepsilon^3)$
- Final step: make "scale" φ_0 and "mass" μ **running**:

$\varphi = \sqrt{2I/\mu}\cos\theta \Longrightarrow \varphi_0^2 = 2I/\mu(\varphi_0^2) \Longrightarrow \mu = \mu(I)$ as planned

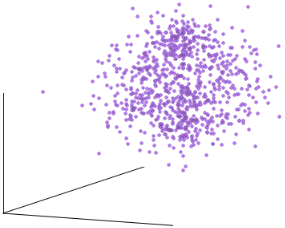
or simply $\varphi_0 = \sqrt{2I}$

- Global symmetry: $\theta \to \theta + \alpha$

$\Downarrow$

- Conserved charge: $N = \int d^3\boldsymbol{x}\; I(t, \boldsymbol{x})$

$+$ attraction $\Longrightarrow$ solitons!



Monodromy oscillons as nontopological solitons

- Stationary ansatz: $I(t, \boldsymbol{x}) = \psi^2(\boldsymbol{x}),\ \theta(t, \boldsymbol{x}) = \omega t$

  or **minimize** energy E at fixed charge N.

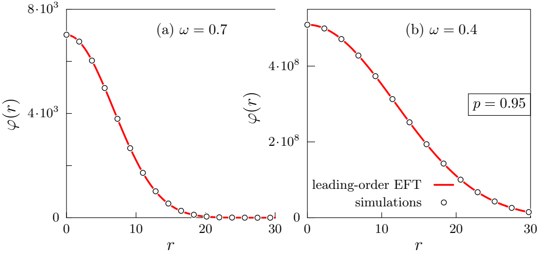
$$\frac{dE}{dN} = \omega$$

Monodromy oscillons profile equation

$$\omega\psi = \mu\psi - \frac{\Delta\psi}{2\mu} + \psi(\partial_i\psi)^2\,\frac{\partial_I\mu}{2\mu^2} + \left(\partial_\varsigma \mathcal{A}_p/\mu^2 p - 1/2\right)(\mu - \psi^2\partial_I\mu)\psi$$

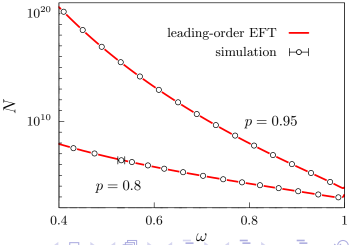
- Field values restored:

$$\varphi(t, \boldsymbol{x}) = \sqrt{2I/\mu(I)}\cos\omega t$$



- Exact adiabatic invariant:

$$N = \int d^3\boldsymbol{x} \int_t^{t+T} \frac{dt}{2\pi}\,(\partial_t\varphi)^2$$



Higher-order corrections

- Goal: Develop asymptotic expansion in $\varepsilon \sim R^{-2}$:

$$\mathcal{S}_{\rm eff} = \underbrace{\mathcal{S}^{(1)}_{\rm eff}}_{\varepsilon^0+\varepsilon^1} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\rm eff}}_{\varepsilon^2} + \underbrace{\mathcal{S}^{(3)}_{\rm eff}}_{\varepsilon^3} + \ldots}^{\text{corrections}}$$

- Field corrections:

$$I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\textit{fast}}, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\text{fast}}$$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Eqs. for δI, $\delta\theta$:

$$\partial_t\, \delta I = \partial_\theta \varphi(\Delta\varphi - \delta V')$$

$$\partial_t \delta\theta = -\partial_I \varphi(\Delta\varphi - \delta V') + \Big\langle \partial_I \varphi(\Delta\varphi - \delta V') \Big\rangle$$

- Solve order-by-order in δI, $\delta\theta \Longrightarrow$ plug $\delta I(\bar{I}, \bar{\theta})$, $\delta\theta(\bar{I}, \bar{\theta})$ into action

+ stationary ansatz: $\bar{I} = \psi^2(\boldsymbol{x})$, $\bar{\theta} = \omega t$

Higher-order corrections

Second-order effective action

$$\mathcal{S}_{\rm eff} = \mathcal{S}_{\rm eff}^{(1)} + \mathcal{S}_{\rm eff}^{(2)}$$

$O(\varepsilon^3)$ – sensitive to φ_0

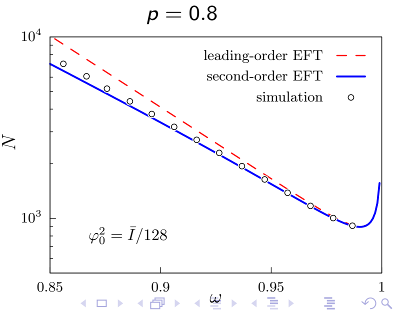
$$\mathcal{S}_{\rm eff}^{(2)} = \int dt\, d^3\boldsymbol{x} \left\{ \frac{1}{2\mu^2}(\Delta\psi + \mu^2\psi)^2 - \mathcal{C}_{1,p}\,(\Delta\psi + \mu^2\psi) + \mathcal{C}_{0,p} \right\}$$

Note. $\sim \varepsilon^2$ contribution, includes 4 spatial derivatives

$\mathcal{C}_{i,p}(\psi^2/\mu)$ — form factors

- $\bar\theta \to \bar\theta + \alpha$ — still a global symmetry
- Test: Detune the "scale" φ_0 to show 2nd order improvement:

$$\cancel{\varphi_0^2 = 2\bar{I}} \implies \varphi_0^2 = \bar{I}/128$$

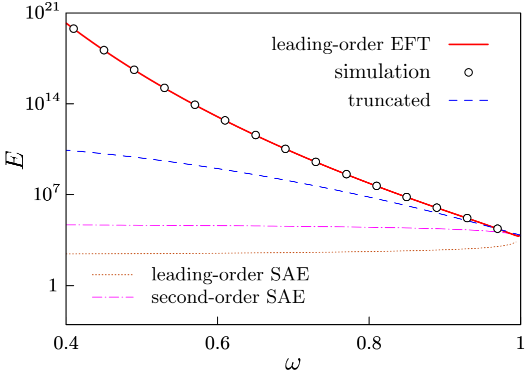


Monodromy: small-amplitude vs. EFT vs. $\varphi^2 \ln \varphi^2$

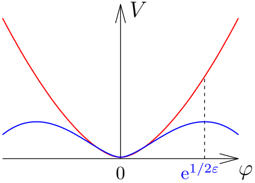
- Small-amplitude expansion: $|\varphi| \ll 1, \quad R \gg m^{-1}$
- Monodromy potential: expansion in ε at $|\varphi| \gg 1$

$$V = \underbrace{\frac{\varphi^2}{2}\left[1 + \varepsilon - \varepsilon \ln \varphi^2\right.}_{\text{admits exactly periodic solutions}} + O(\varphi^{-2}) + O(\varepsilon^2 \ln^2|\varphi|)\Big] .$$

$d = 3; \quad p = 0.95$

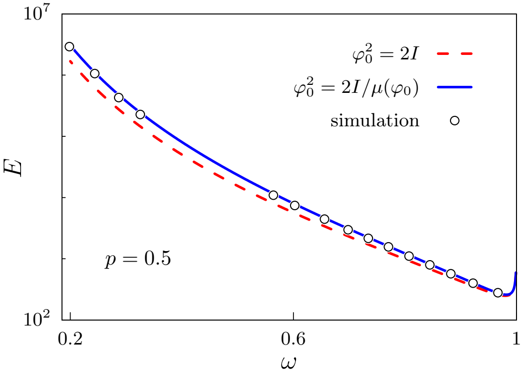


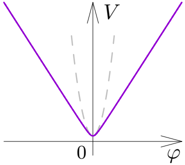
ε-expansion **breaks down** at $\varepsilon \ln|\varphi| \gtrsim 1$.



Axion-monodromy potential: $V(\varphi) = \sqrt{1+\varphi^2}$

- Significantly nonlinear: $p = 0.5$.
- How does that affect the EFT precision?





$\delta N/N \lesssim 0.4$

$\Downarrow$

$\delta N/N \lesssim 0.1$

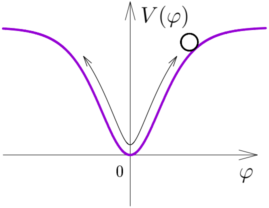
- Proper choice of φ_0 scale cures the method!
- Does not mean the EFT series converge well: $\varepsilon = 0.5$.

- No small nonlinearity, but still **consider large-sized oscillons**

  $\Downarrow$

  pursue gradient expansion

- Zero order approx.: $\partial_t^2 \varphi - \cancel{\Delta\varphi} = -V'(\varphi) \implies$ Nonlinear oscillator



- Action-angle variables in full nonlinearity

$$\varphi = \Phi(I, \theta), \quad \dot{\varphi} = \Pi(I, \theta)$$

- Hamiltonian: $h = \dot{\varphi}^2/2 + V(\varphi) \equiv h(I)$

- Classical solution: $I = \text{const},\quad \theta = \Omega t + \text{const},\quad \boxed{\Omega = \frac{\partial h}{\partial I}}$

- Single subleading term in the classical action:

$$\mathcal{S} = \int dt\, d^d\boldsymbol{x} \Bigg(\underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \Bigg)$$

- Averaging over period

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} (\partial_i\Phi(I,\theta))^2\, d\theta$$

Generalization to generic potentials

- Slow-varying $\partial_i I$, $\partial_i \theta$ are moved *out* of the average

$$\langle(\partial_i\varphi)^2\rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i\theta)^2}{\mu_\theta(I)} + \cancel{\langle\partial_I\Phi\,\partial_\theta\Phi\rangle\partial_i I\partial_i\theta}$$

$$\mu_I \equiv \left\langle(\partial_I\Phi)^2\right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle(\partial_\theta\Phi)^2\right\rangle^{-1}$$

Leading-order effective action for generic potential

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\boldsymbol{x} \left(I\partial_t\theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i\theta)^2}{2\mu_\theta(I)} \right)$$

- Oscillon profile equation

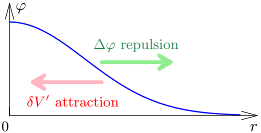
$$\Omega = \partial h/\partial I$$

$$-\frac{2\psi^2}{\mu_I}\Delta\psi - (\partial_i\psi)^2\frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

- Longevity & EFT applicability for large oscillons:

$$\left|\frac{d^2h}{dI^2}\right| = \left|\frac{d\Omega}{dI}\right| \ll \frac{\Omega}{I}$$

— potential is close to quadratic!



EFT.

- Large oscillons — held together by weak nonlinearity
- Parameter of the expansion: $(mR)^{-2} \sim O(\varepsilon)$
- Global $U(1)$ symmetry $\Longrightarrow$ oscillons
- Conditions for existence of long-lived oscillons:

$$V(\varphi) \begin{cases} \text{attractive} \\ \text{nearly quadratic potential} \end{cases}$$

- $\begin{cases} \text{"running mass" } \mu \\ \text{expansion in } \Delta\varphi \text{ and } \delta V' \end{cases}$ → great precision!

Perspective.

- Decay of oscillons — nonperturbative in EFT?

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THANK YOU FOR YOUR ATTENTION!

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