

NUMERICAL SIMULATION OF A LOW-ENERGY ELECTRON ELECTROSTATIC INTEGRAL SPECTROMETER WITH ADIABATIC MAGNETIC COLLIMATION

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Results are given of the numerical simulation of the electromagnetic field of a tritium beta-decay spectrometer as proposed previously in ref. [1].

It is shown that the use of an electrostatic analyzer in combination with a collimating magnetic field makes it possible to obtain a resolution of about 1 eV at the end point of the spectrum.

1. Introduction

In ref. [1] an experiment was proposed on the measurement of the electron antineutrino rest mass by measuring the tritium beta spectrum using an electrostatic integral spectrometer in combination with an adiabatically collimating magnetic field.

The magnetic field has an axially symmetric magnetic mirror trap configuration (a magnetic bottle) with the bottleneck ratio (the ratio of the field strength at a maximum H_n to the field strength at a minimum H_0) $\gamma \sim 10^3-10^4$.

The adiabatic invariant of particle motion in its magnetic moment $\mu = v_{\perp}^2 / 2H$ [2]), where $v_{\perp} = v \sin \beta$ is the component of the total velocity v perpendicular to a force line of the field, β is the angle of particle momentum with the force line, and H is the field strength along the force line of motion.

The tritium source is located (fig. 1) near the left bottleneck of the field. During the motion of electrons from the neck to the magnetic field minimum (the median plane of the magnetic bottle) the conservation of μ ensures adiabatic collimation of the momenta of electrons.

The field of an electrostatic analyzer is set up by a distributed potential applied to the axially symmetric

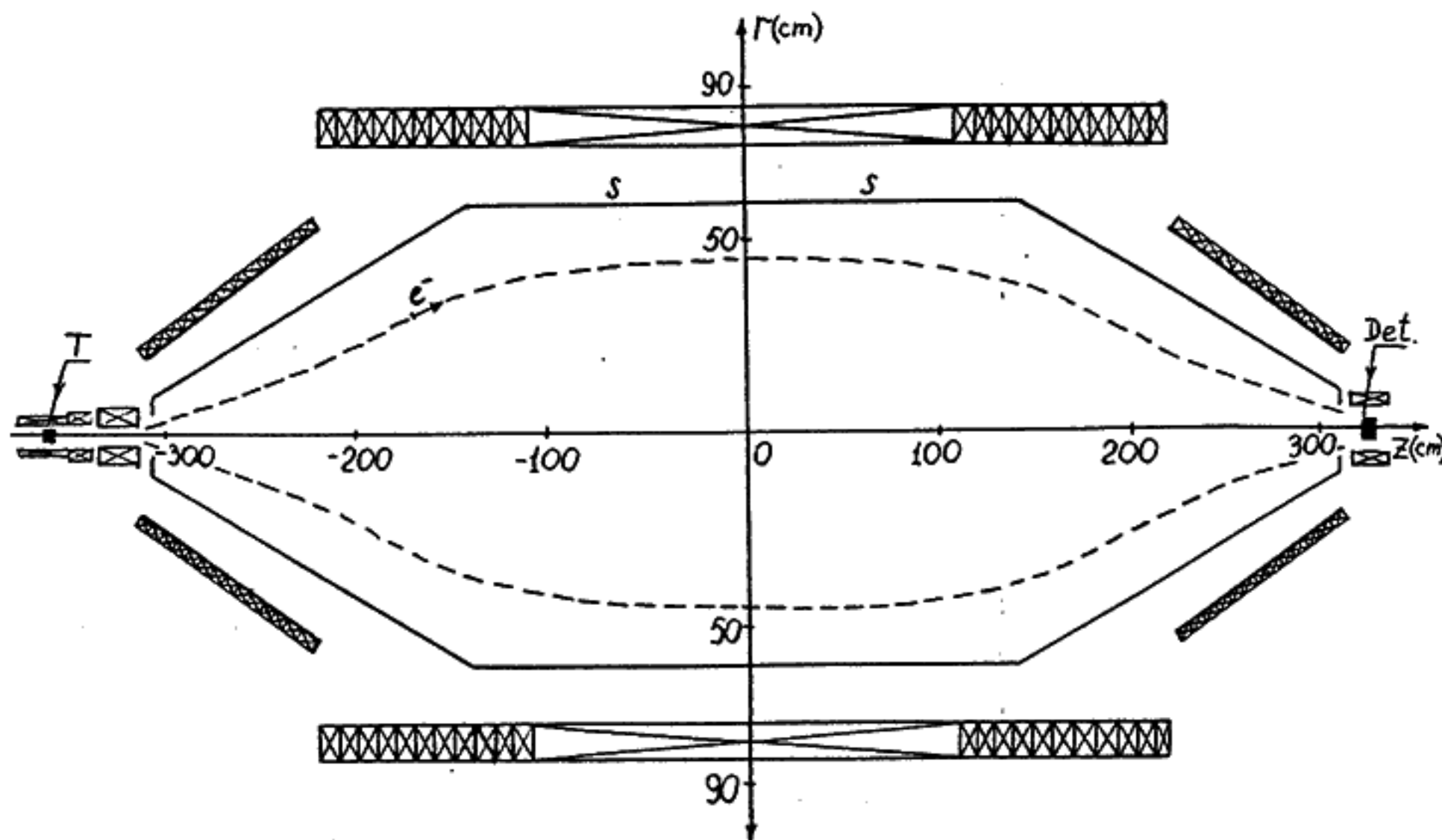


Fig. 1. Configuration of the solenoids and the potential carrying surface which sets up an electromagnetic field of the spectrometer. The dashes represent the boundary force line of the magnetic field at $H_0 = 60$ G.

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surface S (fig. 1). In this case along each force line of the magnetic field the potential maximum is reached in the median plane $z = 0$.

In the left neck of the magnetic field the angles of electron momenta with the force line β_n fall within the interval $[0, \pi/2]$ (forward hemisphere). If the electron energy E_0 is such that the longitudinal component of its total velocity is completely decelerated at the potential maximum then the residual energy of the transverse motion will not exceed $\Delta E = E_0/\gamma$, which follows from the conservation of the adiabatic invariant μ . Thus, under the conditions of ideal adiabaticity of electron motion, which was supposed in ref. [1], the spectrometer resolution $\Delta E/E_0 = 1/\gamma$.

As was already mentioned in ref. [1], the fulfillment of adiabaticity conditions for a spectrometer with a stopping electric field located only in the vicinity of the median plane requires an enormous length of the spectrometer magnetic field. The use of a stopping potential distributed over the whole spectrometer length allows one to make the spectrometer much shorter, however, some difficulties can arise with electrons passing with energies significantly higher than the stopping potential.

The main goal of the present work was to clarify to what extent adiabatic requirements and a 100% transmission efficiency can both be satisfied in a real spectrometer design.

In this paper we describe real configurations of the electric and magnetic fields of the tritium beta-decay spectrometer and determine the influence of various dynamic effects on its resolution including the degree of conservation of the adiabatic invariant μ .

2. Features of spectrometer fields

The spectrometer magnetic field is set up by a system of solenoids illustrated in fig. 1. The field in the left neck $H_n \sim 12$ T and the field at the minimum $H_0 \sim 60$ G (a variant with $H_0 \sim 10$ G has also been considered). The tritium source is located on the left of the neck in the region with field $H \sim 3$ T. The radius of the "window" for electrons in the left $r_n = 1$ cm so that for $H_0 = 60$ G the diameter of the spectrometer operating area is approximately 90 cm at the maximum.

The right (from the median plane) half of the spectrometer serves to accelerate electrons which have been decelerated in its left half and which have passed through the potential and also to collect electrons in the detector.

The carrying surface S and the distribution of the potential over it are symmetric with respect to the spectrometer median plane $z = 0$.

The detector of 3–5 cm diameter is located in the right neck of the magnetic field of around 2–3 T field strength.

The magnetic and electric fields of the spectrometer have to satisfy the following conditions:

(i) All the electrons that have escaped from the left neck with energies greater than the minimum stopping energy must reach the detector.

In reality for determining the neutrino mass it is sufficient to measure the tritium beta-decay spectrum above 16 keV, but the magnetic field obtained is an ideal conductor even at zero potential, that is when the adiabaticity conditions are not fulfilled. An analysis has shown that violations of the adiabatic invariance of the magnetic moment in this case have mainly a nature of phase oscillations whose amplitude is damped to zero in the region of the necks (fig. 2). So all the electrons penetrated through the left bottle neck will attend the detector.

(ii) For electrons with energies which are close to the given stopping energy E_0 the fulfillment of the adiabaticity conditions must be maximal, i.e. without disturbing the ideal resolution $\Delta E/E = 1/\gamma$.

(iii) Stopping of electrons must be the same over all the force lines of their motion – from the axis of symmetry of the spectrometer at the boundary force line determined by the equation $r^2 H(z) = r_n^2 H_n$, where $H(z)$ is the field along the axis of symmetry. To this end the non-uniformity of the electric potential in the median plane must be less than the resolution value $1/\gamma$.

(iv) The electric and magnetic fields must be matched so that the stopping of electrons with the energy corresponding to the potential maximum occurs exactly in the median plane.

(v) The spectrometer resolution must be resistant to violations of the axial symmetry of the fields and also to some mismatching of the axes of symmetry.

The main problem is the degree of conservation of

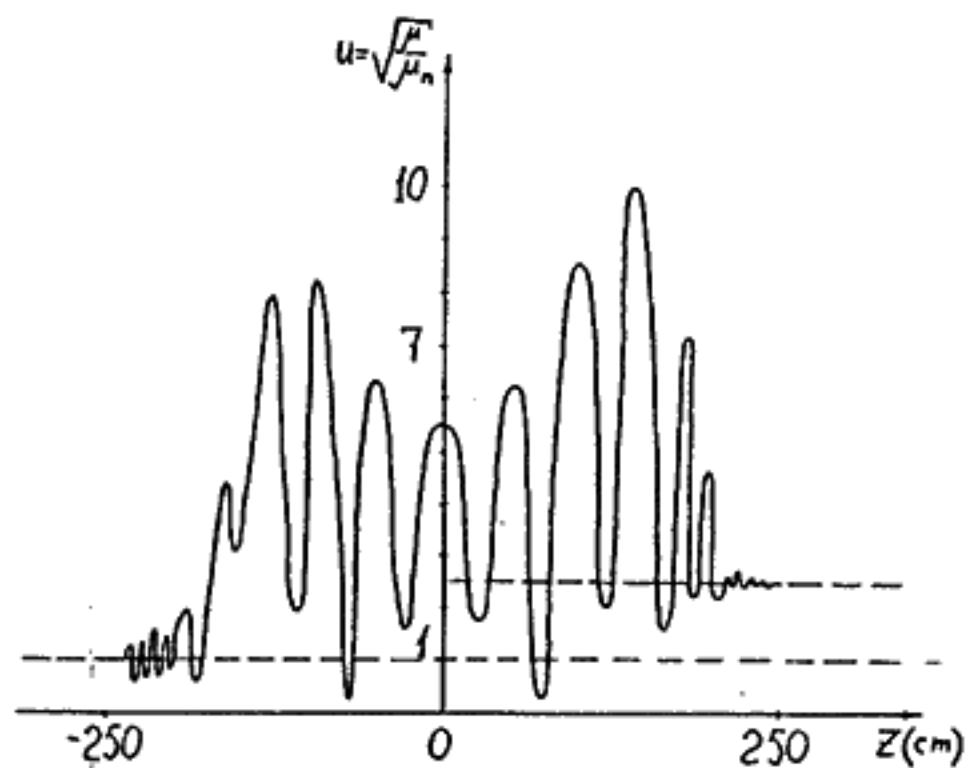


Fig. 2. Illustration of variations of the adiabatic invariant μ along the spectrometer at the zero stopping potential (the magnetic field at the minimum $H_0 = 60$ G and the initial conditions of electron motion: $E_n = 18$ keV, $r_n = 1$ cm and $\beta_n = 5^\circ$). This case corresponds to the maximal violation of adiabaticity.

the adiabatic invariant μ . Apparently the problem of adiabaticity of particle motion in so dramatically falling-off fields has never been considered in the literature. In the present paper we give the results of numerical experiments. The analytical calculations and estimates will be published somewhat later.

Calculation of the magnetic field was made by integrating the field from the current rings over the whole volume of solenoids. The electric potential was calculated by the numerical solution of the Laplace equation using the method of final elements. The dynamics of electrons in the spectrometer fields was analyzed by integrating the exact equations of motion.

The effective numerical integration of these equations requires formulas for the components of the fields. These expressions have been obtained by interpolation through a system of reference points. It should be noted that the analysis of the processes of adiabatic violation requires a high accuracy of interpolation and smooth coupling. This comes from the fact that the adiabatic invariant is equally sensitive to both jumps of the magnetic field and jumps of the curvature of a force line of motion. The required accuracy and smoothness of interpolation were obtained with the use of splines.

3. Numerical results

The basic characteristic of the spectrometer is its transmission function $f(E, E_0)$. It is defined as a fraction of all the tritium beta-decay electrons from those of energy E that have passed through the potential and have entered the detector. In this case the potential is set for the complete stopping of electrons having an energy less than E_0 .

In principle the transmission function can be determined by simulating a sufficiently large number of particle trajectories. However, each electron in its motion from a source to a detector makes about 10^4 Larmor oscillations so that such a method proved to be not feasible. Nevertheless the properties of the transmission function, including the detector resolution, can be determined indirectly with the use of a small number of trajectories.

The energy of tritium beta-decay electrons does not exceed 20 keV.

In the nonrelativistic case the energy integral for the exact equations of motion appears as

$$h = v^2(z) - \frac{2e}{m}\varphi(z) = \text{const.} \quad (1)$$

Here z is the coordinate along the force line of motion, v is the total velocity of the particle, $\varphi(z)$ is the electric potential along the line of motion, and e and m are the particle charge and mass respectively.

From eq. (1) one can obtain the function which

describes the stopping of the electron during its motion from the left neck to the median plane $z = 0$:

$$T(z) = \frac{v_{\parallel}^2(z)}{v_n^2} = 1 - \frac{v_{\perp}^2(z)}{v_n^2} - \frac{2e}{mv_n^2}\varphi(z),$$

where $v_n = v(z_n)$, and v_{\parallel} and v_{\perp} are the components of the total velocity v , which are, respectively, parallel and perpendicular to the force line, and $\varphi(z_n) = 0$.

If one considers only the electrons with the energies close to eV_0 (here adiabaticity conditions are completely fulfilled) then, taking into account the conservation of the adiabatic invariant μ , one can determine the stopping function as follows

$$T(z) = 1 - \frac{\sin^2\beta_n}{H_n}H(z) - \frac{2e}{mv_n^2}\varphi(z).$$

For the transit into the detector of all the electrons having energies greater than E_0 , one should set a potential maximum equal to

$$\varphi_m = \varphi(0) = \frac{mv_0^2}{2e} \left(1 - \frac{1}{\gamma}\right),$$

where $v_0 = v(E_0)$.

The condition of matching of the electric and magnetic fields along magnetic force lines can be written as

$$\varphi(z)/\varphi(0) < 1 - \frac{H(z) - H(0)}{H_n}.$$

In the median plane variations in $T(z, r)$ with r have to be much smaller than $1/\gamma$.

The distribution of the calculated electric potential in the median plane has a non-uniformity of about 5×10^{-6} , i.e. by one and a half orders lower than the resolution value, the fields being fully matched.

The degree of suppression of the nonadiabatic effects was determined numerically.

For the particles moving along the axis of symmetry of the field the adiabatic invariant is conserved accurately because the curvature of the central line is zero. The maximum violation of the adiabaticity conditions may take place for the particles moving along the boundary force line of the magnetic field. The non-adiabaticity is most pronounced for the particles with small angles of momenta with a force line since they intersect the gradient of the field and its curvature more rapidly than the others.

In the numerical integration of the equations of motion calculation was made of the functions $T(z)$ for the motion along the axis of symmetry and along the boundary force line: $T(0, z)$ and $T(r_n, z)$. The difference of the values of these functions just determines the contribution of the nonadiabatic effects to the spectrometer resolution.

The transmission function $f(E_n, E_0)$ can be calculated from the values of the stopping function in the

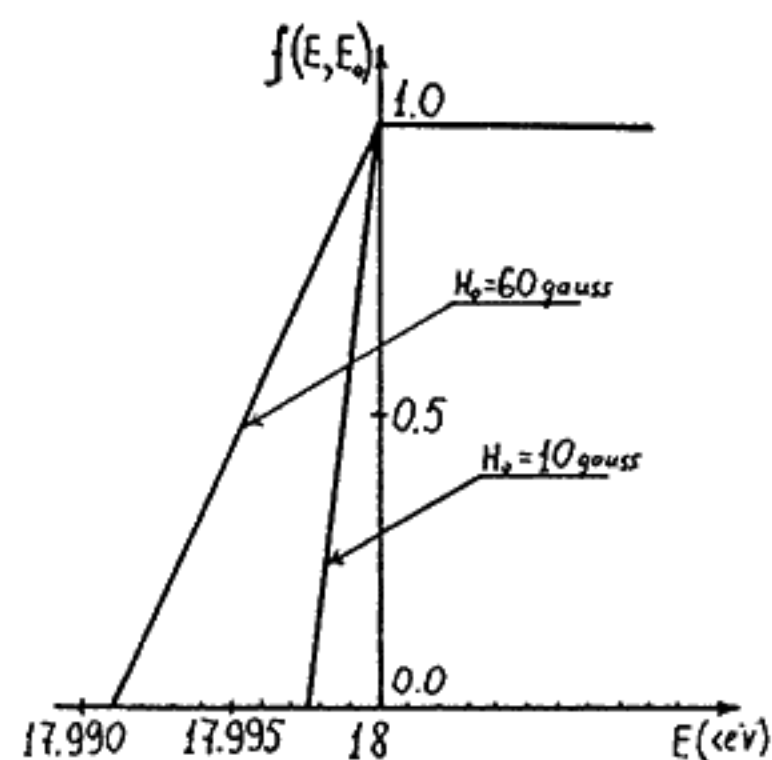


Fig. 3. The spectrometer transmission function ($E_0 = 18$ keV) for $H_0 = 60$ G and $H_0 = 10$ G.

median plane

$$T_{E_0}(0) = T_{E_0}(\beta_n, E_n).$$

The solution of the equation $T_{E_0}(\beta_n, E_n) = 0$ determines the solid angle in the left neck during the escape into which the electrons with the energy E_n pass through the potential E_0 .

On calculating the trajectories the potential maximum was set at 18 keV and consideration was given to the electrons with the energy $E_n = 18.2$ keV and with the initial angle $\beta_n = 10^\circ$. The spectrometer resolution was approximately 5 eV. For these parameters the values of the stopping function in the median plane are

$$T(0, 0) = 0.01034, \quad T(r_n, 0) = 0.01033.$$

This corresponds to the longitudinal energy spread of 0.2 eV for the electrons with $E_n - E_0 = 200$ eV, whereas for one that possess energies close to E_0 where the transmission function has a step, that is at $E_n - E_0 \sim 2$ eV, the distortion of the transmission function due to a nonadiabatic effect occurs within an interval smaller than 2×10^{-3} eV (fig. 3).

With a decrease in E_0 the contribution of the nonadiabatic effects will be still smaller due to the decrease in the Larmor radii of electrons. With increasing of the angle β_n the adiabaticity is approaching an ideal one.

This result shows that the spectrometer electric field

has a large margin as regards the suppression of the nonadiabaticity. A variant of the magnetic field with a minimum of $H_0 = 10$ G was also considered. In this case the spectrometer resolution is improved up to 1 eV (fig. 3). Numerical results have shown that for electrons with energies in the neighbourhood of $18 \text{ keV} \pm 50 \text{ eV}$ the nonadiabatic effects are suppressed completely. It is worth noting that with decreasing H_0 the aperture ratio of the source decreases. For the 90 cm diameter of the spectrometer operating area the radii of the "window" for electrons in the left neck are: $r_n(60 \text{ G}) = 1 \text{ cm}$ and $r_n(10 \text{ G}) = 0.4 \text{ cm}$.

The trajectory calculations made for axially symmetric fields enable one to judge the stability of the system to violations of axial symmetry. When moving from the source to the detector, in addition to Larmor oscillations the electrons perform a slow axial drift about the axis of symmetry over the force surface of the magnetic field. This drift is caused by the radial non-uniformity of the magnetic field, by the presence of the electric field component E_r and by the nonparallelism of the directions of the electric and magnetic fields. In the case of violations of axial symmetry the direction of the drift will be displaced relative to the ideal axially symmetric surface. This may lead to "missing" of the detector by some of the trajectories. The maximum value of the drift during the motion of an electron from the source to the detector turns out to be very small: $\varphi_D = 7^\circ - 8^\circ$. This suggests that the requirements for the axial symmetry of the fields are very nonstringent although as a whole the problem requires a more comprehensive investigation.

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References

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