Определение направления на источник антинейтрино по реакции обратного бета-распада

Ярослав Никитенко [1]

Институт Ядерных Исследований Российской Академии Наук

[1] metst13@gmail.com

План

1. Цели определения направления на источник $\overline{\nu}$
2. Double Chooz
3. Physical foundations. Direction preservation in IBD
4. Methods of direction reconstruction
5. Double Chooz preliminary results
6. Analytical models for direction reconstruction (personal studies, not collaboration)

Направление на сверхновую звезду с коллапсом ядра

Supernova explosion in our galaxy occurs once in $\sim$ 25-50 years.
However, humankind has recorded only $\sim$ 8 supernovae in our history (SN 185, 386, 393, 1006, 1054, 1181, 1572, 1604).
Most of supernovae are hidden for optical observations by galactic dust and gases.
Neutrinos' directionality reconstruction can be used to estimate SN location in order to:

- Know where supernova occured.
- Take into account MSW effect in the Earth.
- Point optical and other telescopes to observe the early times of a supernova explosion when possible.

The SuperNova Early Warning System (SNEWS) project includes members Borexino, Daya Bay, KamLAND, IceCube, LVD, and Super-Kamiokande. Its aim is "to provide the astronomical community with a prompt alert" of a SN.

Направление на сверхновую звезду с коллапсом ядра

Most of supernovae are hidden for optical observations by galactic dust and gases.
Neutrinos' directionality reconstruction can be used to estimate SN location in order to:

- Know where supernova occured.
- Take into account MSW effect in the Earth.
- Point optical and other telescopes to observe the early times of a supernova explosion when possible.

The SuperNova Early Warning System (SNEWS) project includes members Borexino, Daya Bay, KamLAND, IceCube, LVD, and Super-Kamiokande. Its aim is "to provide the astronomical community with a prompt alert" of a SN.
SNEWS involves no direction information, therefore it is important to implement online direction reconstruction in ν experiments.

Recent predictions

Adams et al., Observing the next galactic supernova (arXiv, astro-ph.HE 1306.0559v2):
We find, at very high probability (100%), that the next Galactic supernova will easily be detectable in the near-IR and that near-IR photometry of the progenitor star very likely (92%) already exists in the 2MASS survey.
Most ccSNe (98%) will be easily observed in the optical, but a significant fraction (43%) will lack observations of the progenitor due to a combination of survey sensitivity and confusion.
If neutrino detection experiments can quickly disseminate a likely position ($\sim 3^\circ$), we show that a modestly priced IR camera system can probably detect the shock breakout radiation pulse even in daytime (64% for the cheapest design).

Direction reconstruction for Earth sources

- Study the distribution of radioactive sources in the Earth.
- Locate a nuclear reactor.

Угловое распределение антинейтрино и физика Земли

В работе
Can Radiogenic Heat Sources Inside the Earth be located by their Antineutrino incoming Directions? // G. Domogatsky, V. Kopeikin, L. Mikaelyan, V. Sinev Phys.Atom.Nucl.69:1894-1898, 2006 arXiv:hep-ph/0411163
исследовалась возможность определения распределения р/а элементов в Земле по сигналу антинейтрино.
Использовалось 4000 сгенерированных Монте-Карло событий, что соответствует 5 годам сбора данных жидкосцинтилляционным детектором на Баксане массой 30 кт.
В работе анализировалось среднее смещение нейтрона (по расчетам авторов, этот метод эквивалентен анализу распределения событий по углам к вертикали).

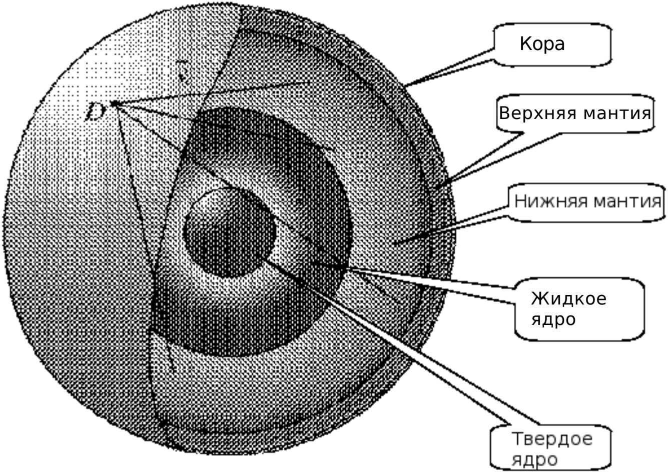


Рис.: Строение Земли. Телесный угол, с которого приходят антинейтрино в детектор, зависит от пространственного распределения р/а элементов.

Определение распределения р/а элементов в Земле по антинейтринным событиям

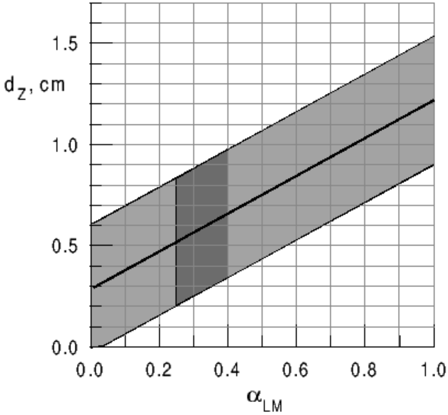


Рис.: Сплошная линия: среднее смещение нейтрона (см) в зависимости от α_{LM}, доли потока геонейтрино из нижней мантии в полном потоке (нижняя мантия + кора). Серым: область неопределенности (68%).

Why Double Chooz is the best experiment to study $\overline{\nu}$ directionality via IBD

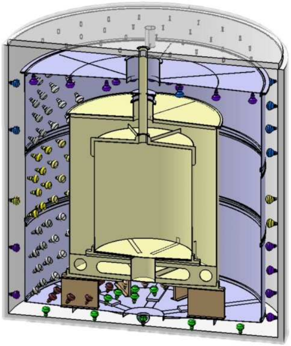


Рис.: Detector. $10.32 \mathrm{m}^3$ LS + Gd.

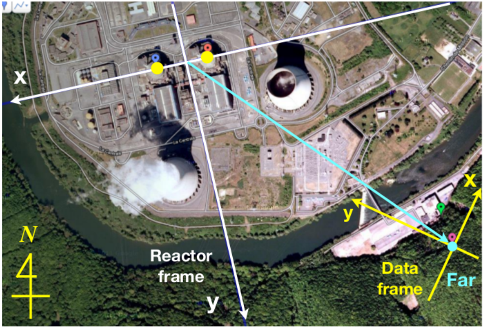


Рис.: Only 2 reactors. They are seen at only 6° from the far detector.

DC is the successor of Chooz, which measured $\overline{\nu}$ direction.
DC geometry is very good approximation for a point-like source.

Why Double Chooz is the best experiment to study $\overline{\nu}$ directionality via IBD

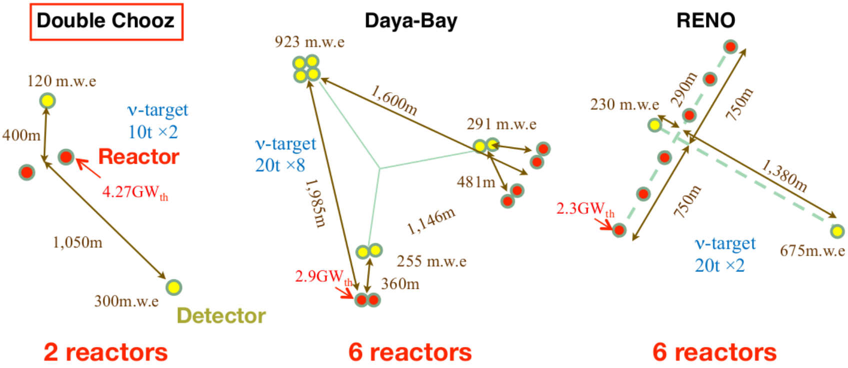


Рис.: Double Chooz experimental layout compared to some other ongoing reactor experiments (figure by K.Nakajima). DC 2-reactor setup has advantages: 1) geometrically small angle 2) sometimes one reactor stops and we have a single source.

Why inverse beta decay

$$\overline{\nu} + p \to e^+ + n$$

IBD advantages at energies of supernova neutrinos compared to other reactions:

- Largest cross section.
- Low background. Easy to detect using delayed coincidences:
  1. positron is almost immediately captured in scintillator with the release of > 1.022 MeV.
  2. neutron experiences moderation and diffusion and after $\sim 30\ \mu s$ is captured by Gd with the release of about 8 MeV.

While positrons are being born with almost isotropic momenta, neutrons on average conserve the momentum of the original $\overline{\nu}$:
[Vogel, Beacom, Phys.Rev. D, 1999. hep-ph/9903554v1]

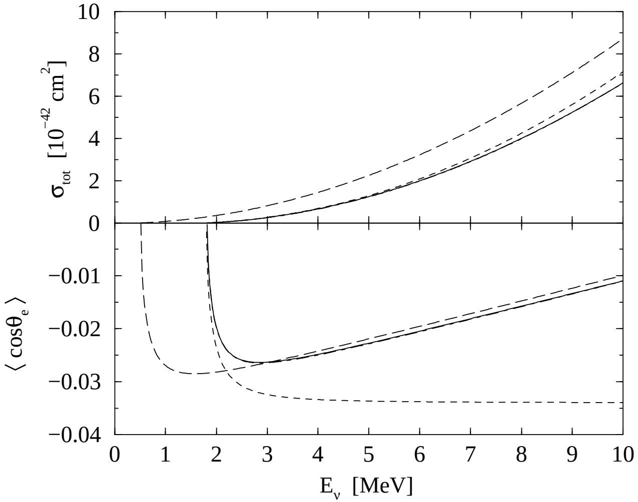


Рис.: Average cosine of the angle between $\overline{\nu}$ and positron as a function of $\overline{\nu}$ energy. Almost isotropic distribution.

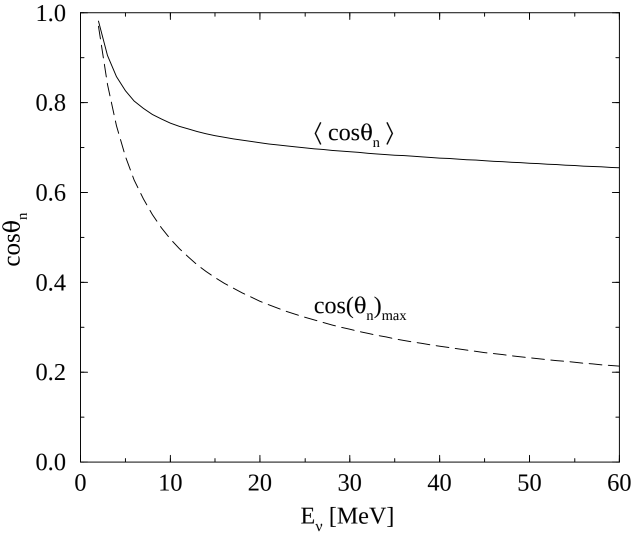


Рис.: Average cosine of the angle between $\overline{\nu}$ and neutron as a function of $\overline{\nu}$ energy. At low $\overline{\nu}$ energy the neutron is emitted almost in the direction of $\overline{\nu}$.

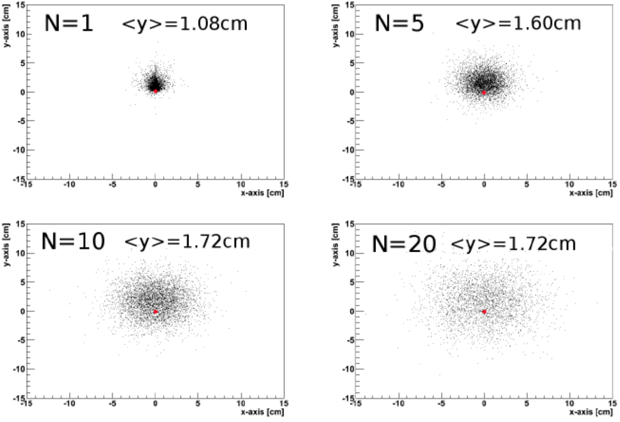


Рис.: Neutron displacement as a function of the number of collisions N (Monte-Carlo [2]). Average neutron displacement is preserved during the diffusion.

[2] C. Langbrandtner, Ph. D. Dissertation, 2011. www.mpi-hd.mpg.de/personalhomes/conradin/PhD_Thesis_Langbrandtner.pdf

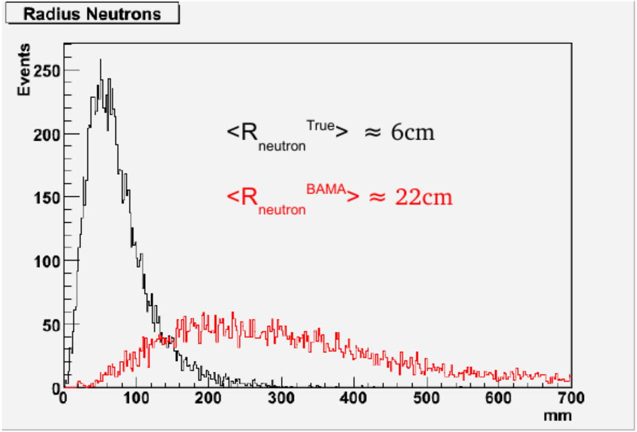


Рис.: Average neutron displacement from the origin (MC, Langbrandtner). After the coordinate reconstruction neutrons' distribution is much broader than their true distribution.

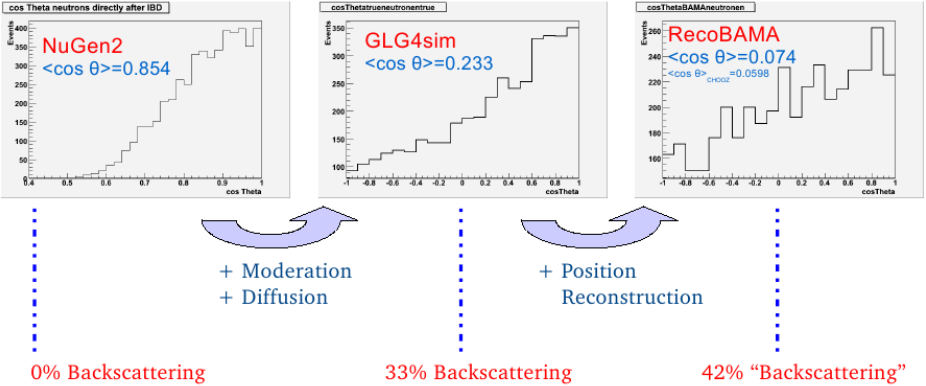


Рис.: Distribution of cosines of the angles between displacement vectors $\mathbf{r}_i = \mathbf{r}_{neutron} - \mathbf{r}_{positron}$ and the true direction (MC, Langbrandtner).

Direction reconstruction

Input data:

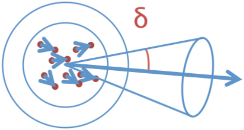
- Displacement vectors, $\mathbf{r}_i = \mathbf{r}_{neutron} - \mathbf{r}_{positron} \in \mathbb{R}^3$,
- Normalised displacement vectors, $\mathbf{u}_i = \frac{\mathbf{r}_i}{r_i} \in S^2$

Direction estimators:

- Simple mean of $\mathbf{r}_i$: $\hat{\mathbf{r}}_n = \frac{\sum_{i=1}^{n} \mathbf{r}_i}{n}$
- Mean of normalised vectors: $\hat{\mathbf{u}}_n = \frac{\sum_{i=1}^{n} \mathbf{u}_i}{n}$.
  About 10% more precise, but uneasy to calculate analytically.

Precision of a direction reconstruction

The precision δ of a direction reconstruction at the confidence level *cl* (e.g. cl=68%) is defined as the half-aperture angle of the cone inside which *cl* of directions are reconstructed:



The precision is the confidence interval in angular space:

$$\mathrm{CDF}_{estimator}(\cos\delta) = cl,$$

$$\cos\delta = \mathrm{CDF}^{-1}(cl).$$

Special aspects of direction reconstruction

- The distribution is non-gaussian, as was assumed in Chooz.
- *Directional statistics* should be used for a not very large number of events (e.g. less than 1000, which is important for SN).
- No understanding yet why mean normalised estimator is the best and hence how to improve that.

Chooz spatial distribution

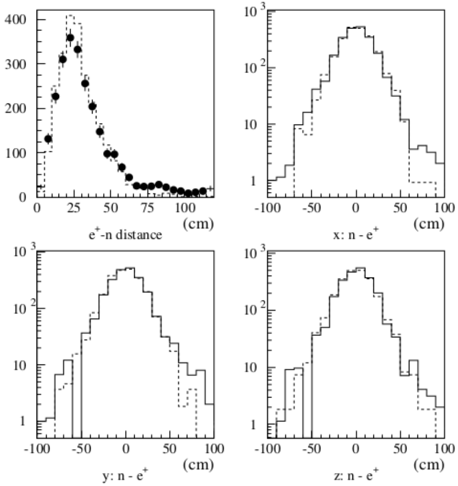


Рис.: Exponential spatial distribution in Chooz.

Double Chooz preliminary results

Determination of antineutrino direction using large number of events is possible.
The precision is of the order

$$6^\circ \text{ for 8249 events } ^{3\ 4} \text{ or}$$
$$9.4^\circ \text{ for 8246 events } ^{5}$$

[3] Erica Caden, Applied Antineutrino Physics conference, 10.2012. www.phys.hawaii.edu/~hanohano/post/AAP2012/AAP2012_EricaCaden.pdf

[4] Kyohei Nakajima, Neutrino Geoscience, 03.2013 www.awa.tohoku.ac.jp/geoscience2013/wp-content/uploads/2012/08/NGS2013_Mar22_Nakajima.pdf

[5] Romain Roncin, AAP 11.2013. indico.cern.ch/getFile.py/access?contribId=28&sessionId=1&resId=0&materialId=slides&confId=245969

Angular distribution

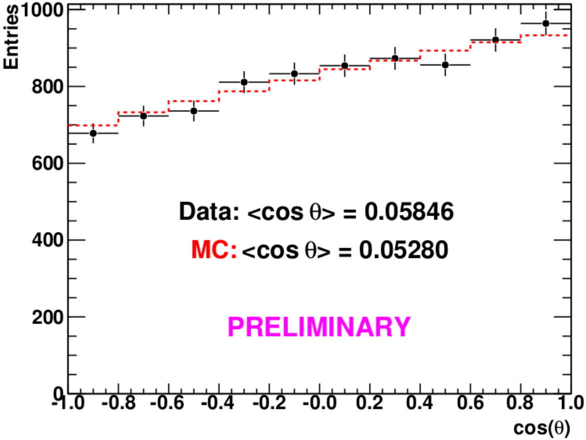


Рис.: Cosines of angles between the reconstructed displacement vectors and the bisector of the directions to the reactors (Caden). Most of the events are in the direction of $\overline{\nu}$, but many are backward. The average cosine is quite low.

Pdf models

Input data is a set of $\mathbf{r}_i = \mathbf{r}_{neutron} - \mathbf{r}_{positron} \in \mathbb{R}^3$

The direction estimator is the simple mean of $\mathbf{r}_i$: $\hat{\mathbf{r}}_n = \frac{\sum_{i=1}^{n} \mathbf{r}_i}{n}$

Mathematical distribution models to find accuracy of the estimators:

- Normal distribution $\mathrm{pdf}_{G,1}(x, y, z) = \frac{1}{(2\pi\sigma^2)^{\frac{3}{2}}} e^{-\frac{\left(x^2+y^2+z^2\right)}{2\sigma^2}}$
- Exponential distribution $\mathrm{pdf}_{e,1}(x, y, z) = \frac{1}{8\pi l^3} e^{-\frac{\sqrt{x^2+y^2+z^2}}{l}}$

$\mathrm{CDF}(\cos\theta)$

$$\begin{pmatrix} x & = & r\cos\phi\sin\theta \\ y & = & r\sin\phi\sin\theta \\ z & = & r\cos\theta \end{pmatrix}, \theta \in [0, \pi].$$

$$\mathrm{CDF}(\cos\theta) = \int_{\cos\theta}^{1} \mathrm{d}\cos\theta' \int_0^{\infty} r^2 \mathrm{d}r \cdot \\ \cdot \int_0^{2\pi} \mathrm{d}\phi \; \mathrm{pdf}\left((x, y, z - r_0)(r, \phi, \cos\theta')\right)$$

Pdf is shifted on "average neutron displacement" $\mathbf{r}_0 = (0, 0, r_0)$.

Confidence interval for normal distribution

$$\mathrm{CDF}_{G,\hat{\mathbf{r}}_n}(\cos\theta) = \frac{1}{2}\Bigg(1 + \mathrm{erf}\left(\frac{\sqrt{n}r_0}{\sqrt{2}\sigma}\right) - e^{-\frac{nr_0^2}{2\sigma^2}(1-cos^2\theta)}\cos\theta\left(1 + \mathrm{erf}\left(\frac{\sqrt{n}r_0}{\sqrt{2}\sigma}\cos\theta\right)\right)\Bigg),$$

where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$.
$\mathrm{CDF_G}$ depends only on one combination of parameters $\sqrt{n}\frac{r_0}{\sigma}$.
For $\sqrt{n}\frac{r_0}{\sigma} \gg 1$ and $\theta \ll 1$ the confidence interval

$$\theta_G \approx \frac{\sqrt{-2\ln(1-cl)}\sigma}{\sqrt{n}r_0}$$

$\mathrm{pdf}(r_n)$

The probability density function (pdf) $f_{\mathbf{r}_1+\mathbf{r}_2}(\mathbf{r})$ of the sum of two independent variables in $\mathbb{R}^{\mathrm{d}}$ is given by the *convolution* of their pdfs:

$$f_{\mathbf{r}_1+\mathbf{r}_2}(\mathbf{r}) = (f_1 * f_2)(\mathbf{r}) = \int_{\mathbb{R}^{\mathrm{d}}} f_1(\mathbf{r}') f_2(\mathbf{r} - \mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

The Fourier transform

$$\hat{f}(\mathbf{p}) = \int_{\mathbb{R}^{\mathrm{d}}} \frac{e^{-i\mathbf{pr}}}{(2\pi)^{d/2}} f(\mathbf{r})\,\mathrm{d}^d\mathbf{r},$$

and

$$\widehat{f * g}(\mathbf{p}) = (2\pi)^{d/2} \hat{f}(\mathbf{p}) \hat{g}(\mathbf{p}).$$

Fourier transform for exponential distribution

$$\hat{f}_e(\mathbf{p}) = \frac{e^{-i\mathbf{p}\mathbf{r}_0}}{(2\pi)^{\frac{3}{2}}(1 + l^2 p^2)^2}$$

then

$$f_n(\mathbf{r}) = \frac{1}{2\pi^2|\mathbf{r} - n\mathbf{r}_0|} \int_0^\infty \frac{p \sin(p|\mathbf{r} - n\mathbf{r}_0|)}{(1 + l^2 p^2)^{2n}} \, \mathrm{d}p$$

$$\int_0^\infty \frac{x \sin(ax) \, \mathrm{d}x}{(x^2 + \beta^2)^{n+1}} = \begin{cases} \dfrac{\pi a e^{-a\beta}}{2^{2n} n! \beta^{2n-1}} \displaystyle\sum_{k=0}^{n-1} \dfrac{(2n-k-2)!(2a\beta)^k}{k!(n-k-1)!} \\ \dfrac{\pi}{2} e^{-a\beta} \qquad\qquad\qquad\qquad [n = 0, \beta \geq 0] \end{cases}$$

[6]

[6] Gradshteyn, I.S. and Ryzhik, I.M., *Table of Integrals, Series, and Products*, 7th ed., Academic Press, 2007.

Pdf of the mean of exponential distribution

$$E_n(\mathbf{r}_n) = \frac{n^3}{\pi l^3} \frac{e^{-\frac{n}{l}|\mathbf{r}_n - \mathbf{r}_0|}}{2^{4n-1}(2n-1)!} \sum_{k=0}^{2n-2} \frac{(4n-4-k)!(2\frac{n}{l}|\mathbf{r}_n - \mathbf{r}_0|)^k}{k!(2n-2-k)!}$$

Confidence interval for exponential distribution

$$\mathrm{CDF}_{E,\hat{\mathbf{r}}_n}(\cos\theta) = \sum_{k=0}^{2n-2} \frac{(4n-4-k)!2^k(k+1)}{(2n-2-k)!2^{4n-2}(2n-1)!}\Bigg(2(k+2) - e^{-\frac{n}{l}r_0}\sum_{i=0}^{k+1}(k+2-i)\frac{\left(\frac{n}{l}r_0\right)^i}{i!} - \cos\theta\int_{\frac{n}{l}r_0}^{\infty}\frac{\sum_{i=0}^{k+1}\frac{x^{i+1}}{i!}}{\sqrt{x^2-\frac{n^2}{l^2}r_0^2(1-\cos^2\theta)}}e^{-x}\,\mathrm{d}x - 2\Theta_{Heaviside}(\cos\theta)\cos\theta\int_{\frac{n}{l}r_0\sqrt{1-\cos^2\theta}}^{\frac{n}{l}r_0}\frac{\sum_{i=0}^{k+1}\frac{x^{i+1}}{i!}}{\sqrt{x^2-\frac{n^2}{l^2}r_0^2(1-\cos^2\theta)}}e^{-x}\,\mathrm{d}x\Bigg)$$

For n large the spatial distribution tends to gaussian with $\sigma_{E,n} = \frac{2l}{\sqrt{n}}$.
In the limit of a very large number of events ($\sqrt{n}\frac{r_0}{2l} \gg 1$ and $\theta \ll 1$)

$$\theta_E \approx \frac{2l\sqrt{-2\ln(1-cl)}}{r_0\sqrt{n}}.$$

CDF for exponential distribution

$$a = \frac{nr_0}{l}, c_k = \frac{(4n-4-k)!}{(2n-2)!}\frac{2^k}{2^{4n-2}(2n-2-k)!}, S_{a,k} = \sum_{i=0}^{k}\frac{a^i}{i!},$$

$$\begin{aligned}\mathrm{CDF}_E(\cos\theta) = 1 - \frac{e^{-a}}{2n-1}\sum_{k=0}^{2n-2} c_k(k+1)(k+2)S_{a,k+1} + \\ -\frac{e^{-a}a^2}{2n-1}\sum_{k=0}^{2n-2} c_k\frac{a^k}{k!} + (1-\cos^2\theta)\frac{e^{-a}a}{2n-1}\sum_{k=0}^{2n-2} c_k(k+1)S_{a,k+1} \\ -\cos\theta\int_a^\infty \frac{x}{2n-1}\left(\sum_{k=0}^{2n-2} c_k\frac{x^k}{k!}\right)e^{-x}\sqrt{x^2-a^2(1-\cos^2\theta)}\,\mathrm{d}x \\ -2\Theta(\cos\theta)\cos\theta\int_{a\sqrt{1-\cos^2\theta}}^{a} \frac{x}{2n-1}\left(\sum_{k=0}^{2n-2} c_k\frac{x^k}{k!}\right)e^{-x}\sqrt{x^2-a^2(1-\cos^2\theta)}\,\mathrm{d}\end{aligned}$$

Small n / single event

To have a good angular precision,

$$\sqrt{n}\frac{r_0}{\sigma} \gg 1.$$

$r_0 \sim 1.6\mathrm{cm}, \sigma \sim 18\mathrm{cm}$,

$$\frac{r_0}{\sigma} \sim \frac{1}{10},$$
$$n \gg 100.$$

For a small number of events (and for 1 event) direction reconstruction with a big confidence level is not possible.
Per-event background reduction using $\overline{\nu}$ direction estimate is practically impossible.

Mean values for symmetrical shifted normal distribution

The multivariate central limit theorem states that

$$\sqrt{n}\left(\overline{\mathbf{X}}_n - \boldsymbol{\mu}\right) \xrightarrow{D} N_k(0, \boldsymbol{\Sigma})$$

where the covariance matrix $\boldsymbol{\Sigma}$ is equal to

$$\boldsymbol{\Sigma} = \begin{bmatrix} \operatorname{Var}(X_{1(1)}) & \operatorname{Cov}(X_{1(1)}, X_{1(2)}) & \operatorname{Cov}(X_{1(1)}, X_{1(3)}) & \cdots & \operatorname{Cov}(X_{1(1)}, X_{1(k)} \\ \operatorname{Cov}(X_{1(2)}, X_{1(1)}) & \operatorname{Var}(X_{1(2)}) & \operatorname{Cov}(X_{1(2)}, X_{1(3)}) & \cdots & \operatorname{Cov}(X_{1(2)}, X_{1(k)} \\ \operatorname{Cov}(X_{1(3)}, X_{1(1)}) & \operatorname{Cov}(X_{1(3)}, X_{1(2)}) & \operatorname{Var}(X_{1(3)}) & \cdots & \operatorname{Cov}(X_{1(3)}, X_{1(k)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \operatorname{Cov}(X_{1(k)}, X_{1(1)}) & \operatorname{Cov}(X_{1(k)}, X_{1(2)}) & \operatorname{Cov}(X_{1(k)}, X_{1(3)}) & \cdots & \operatorname{Var}(X_{1(k)}) \end{bmatrix}$$

https://en.wikipedia.org/wiki/Central_limit_theorem#Multidimensional_CLT

Mean values for symmetrical shifted normal distribution

$$\langle z^2 \rangle = 1 - \frac{1}{a^2} + \frac{e^{-a^2}}{2a^3}\frac{\sqrt{\pi}}{i}\mathrm{erfi}(a)$$

As a tends to 0,

$$\langle z^2 \rangle = \frac{1}{3} + \frac{4}{15}a^2 + O(a^3)$$

$$\langle z \rangle = \mathrm{erf}(a) + \left(\frac{e^{-a^2}}{a\sqrt{\pi}} - \frac{\mathrm{erf}(a)}{2a^2}\right)$$

As a tends to 0,

$$\langle z \rangle = \frac{2}{\sqrt{\pi}}\left(\frac{2}{3}a - \frac{2}{15}a^3 + O(a^5)\right)$$

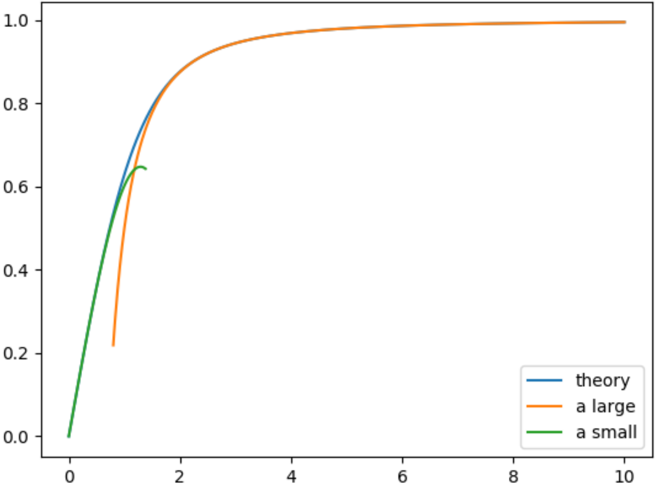


Рис.: $\cos\theta$ mean, approximations

Non-symmetrical shifted gaussian

$$a = \frac{r_0}{\sigma_x\sqrt{2}}, s = \frac{\sigma_x}{\sigma_z}$$

$$\mathrm{CDF}_{\tilde{G}} = \frac{1}{2}\Bigg[1 + \mathrm{erf}(a)$$

$$-\cos\theta \frac{s}{\sqrt{1+\cos^2\theta(s^2-1)}} e^{-a^2 \frac{1-\cos^2\theta}{1+\cos^2\theta(s^2-1)}} \left(1 + \mathrm{erf}\left(\frac{as\cos\theta}{\sqrt{1+\cos^2\theta(s^2-1)}}\right)\right)$$

As n tends to infinity, the angular confidence interval at the confidence level cl can be estimated as

$$\theta = \frac{\sigma_x^2\sqrt{2}}{\sigma_z\sqrt{n}r_0}\sqrt{-\ln(1-cl)} + O(a^{-3})$$

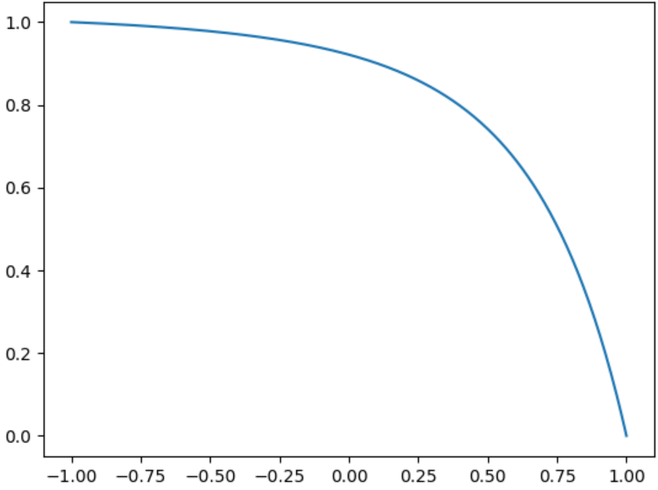


Рис.: $CDF(\cos\theta)$, gauss

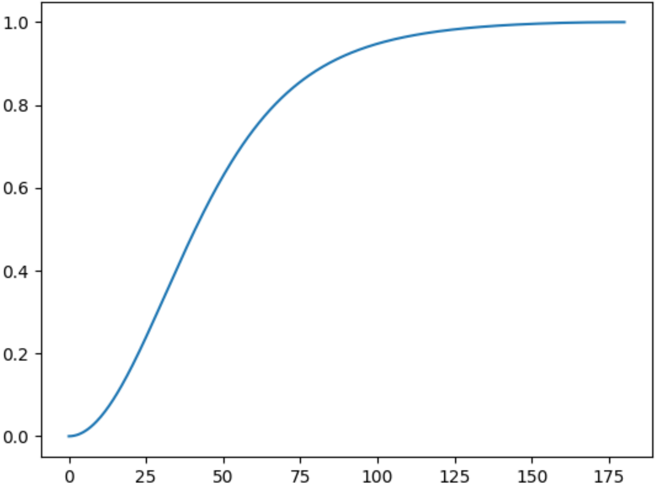


Рис.: $CDF(\theta(deg))$, gauss

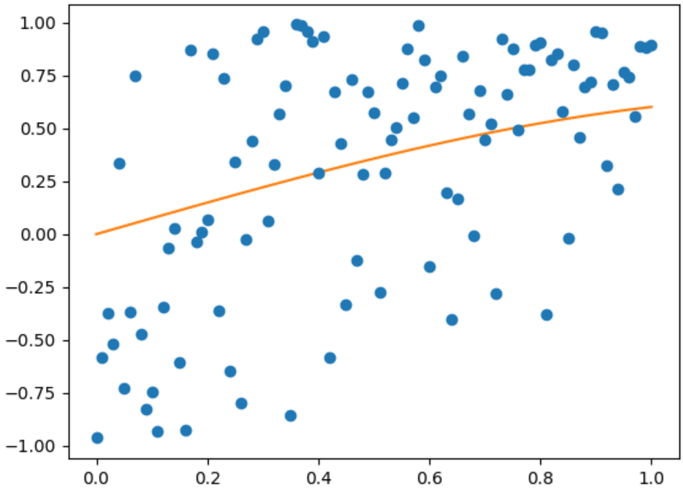


Рис.: gauss, mean $\cos\theta$

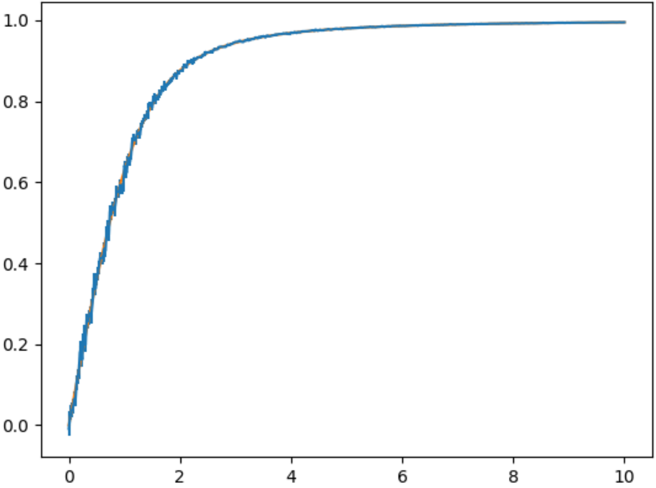


Рис.: gauss, mean $\cos\theta$ (averaged on 1024 samples)

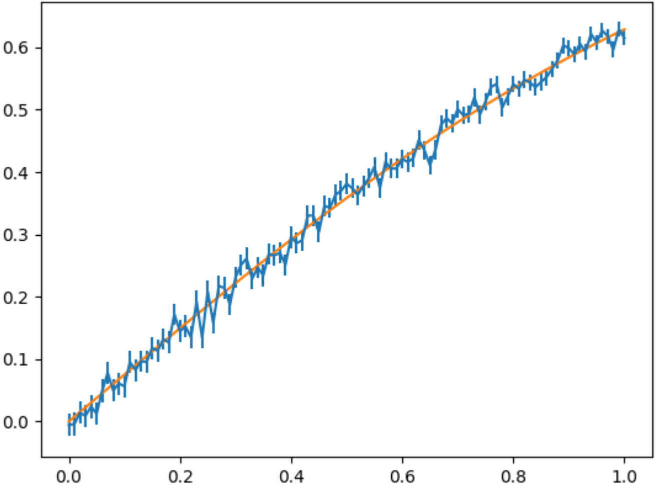
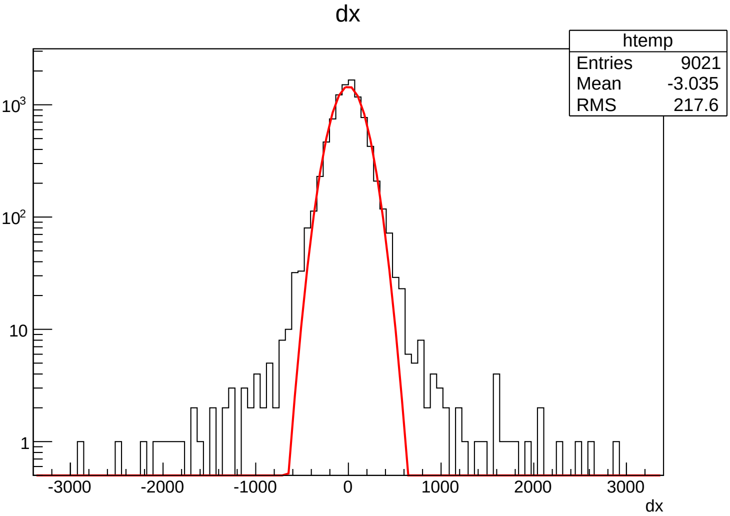
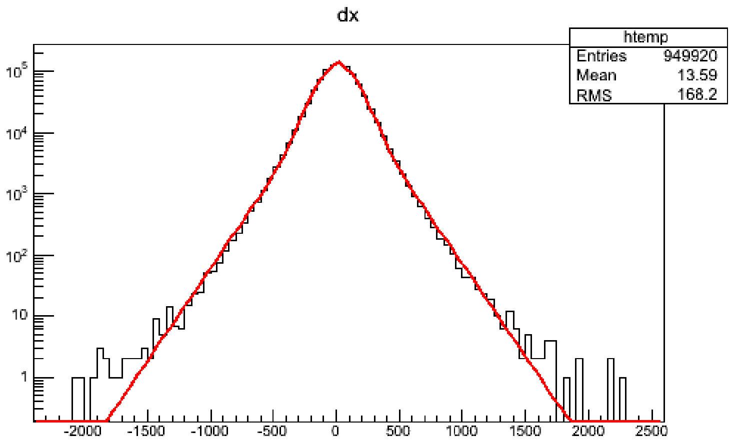


Рис.: $\cos\theta$ mean, theor vs generated, a small

dx, real data


dx
htemp
Entries 9021
Mean -3.035
RMS 217.6
10^3
10^2
10
1
-3000
-2000
-1000
0
1000
2000
3000
dx

dx, Monte-Carlo



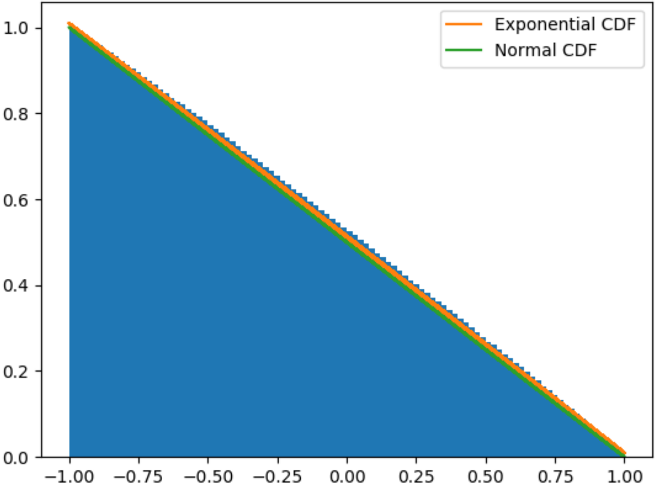


Рис.: $CDF_{exp,gauss}(\cos\theta)$

Conclusions

Current results:

- Inverse beta decay allows reconstruction of the direction to the source of $\overline{\nu}$ using a large number of events.
- Double Chooz can locate $\overline{\nu}$ source and can be used to test direction reconstruction methods.
- Analytical formulae for the precision of direction reconstruction (for two reconstruction methods) have been obtained.

Future work:

- Find more precise estimators using mathematical models.
- Create online supernova monitoring system with direction reconstruction using existing $\overline{\nu}$ detectors.

bibliography

- Determination of the direction to a source of antineutrinos via inverse beta decay in Double Chooz. Ya. Nikitenko, Physics of Particles and Nuclei, November 2016, Volume 47, Issue 6, pp 1009–1013.
- Determination of the direction to a source of antineutrinos via inverse beta decay in Double Chooz. Yaroslav Nikitenko, talk on behalf of the Double Chooz collaboration at the International Workshop on Prospects of Particle Physics "Neutrino Physics and Astrophysics", February 05, 2015, Valday, Russia http://www.inr.ac.ru/ school/talks/Nikitenko.pdf
- Estimation of the directional parameter of the offset exponential and normal distributions in three-dimensional space using the sample mean. Yaroslav Nikitenko. https://arxiv.org/abs/1505.04647