Cosmology

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Outline

- Lecture 1
  - Expanding Universe
  - Dark matter: evidence
  - WIMPs
- Lecture 2
  - Axions
    - Theory
    - Cosmology
    - Search
  - Warm dark matter
    - Sterile neutrino
    - Gravitino
  - Dark matter summary
  - Baryon asymmetry of the Universe
    - Generalities.
    - Electroweak baryon number non-conservation

Outline, cont'd

- Lecture 3
  - Electroweak baryogenesis. What can make it work?
  - Before the hot epoch
    - Cosmological perturbations
      - Regimes of evolution
      - Acoustic oscillations: evidence for pre-hot epoch
    - Inflation and alternatives
    - BICEP-2 saga
  - Conclusions

Appendices

- Calculation of WIMP mass density
- Estimating axion mass density
- Wave equation in expanding Universe
- Dark energy
- CMB anisotropies, BAO and recent Universe
- Anthropic principle/Environmentalism

Lecture 1

Expanding Universe

- The Universe at large is homogeneous, isotropic and expanding.

  3d space is Euclidean (observational fact!)

  Sum of angles of a triangle = 180^o, even for triangles as large as the size of the visible Universe.

  All this is encoded in space-time metric (Friedmann–Lemâitre–Robertson–Walker)

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

  $\mathbf{x}$: comoving coordinates, label distant galaxies.

  $a(t)dx$: physical distances.

  $a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!) Space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 = (67.3 \pm 1.2)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

1 Mpc $= 3 \cdot 10^6$ light yrs $= 3 \cdot 10^{24}$ cm

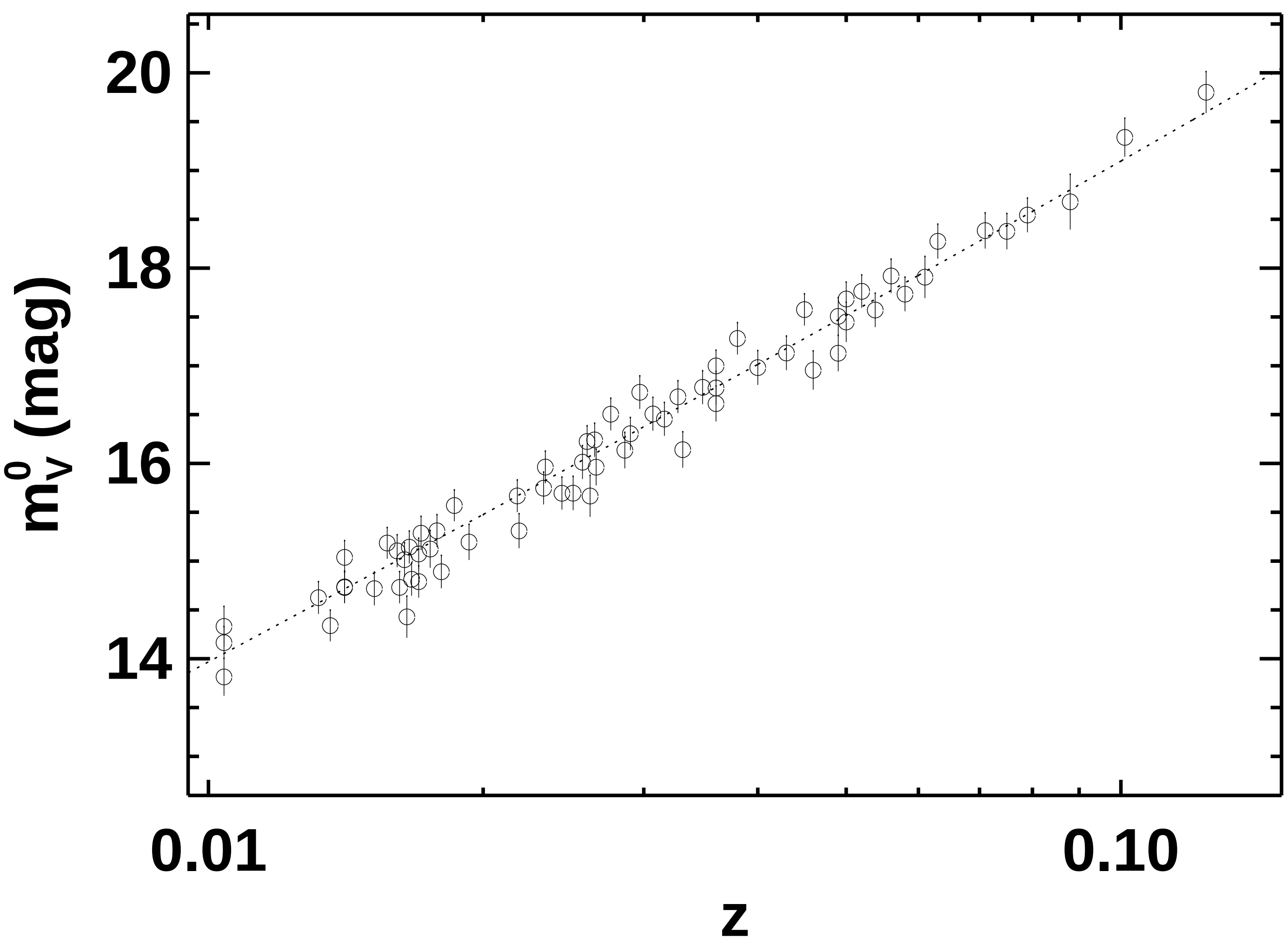
- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Figs. a,b,c

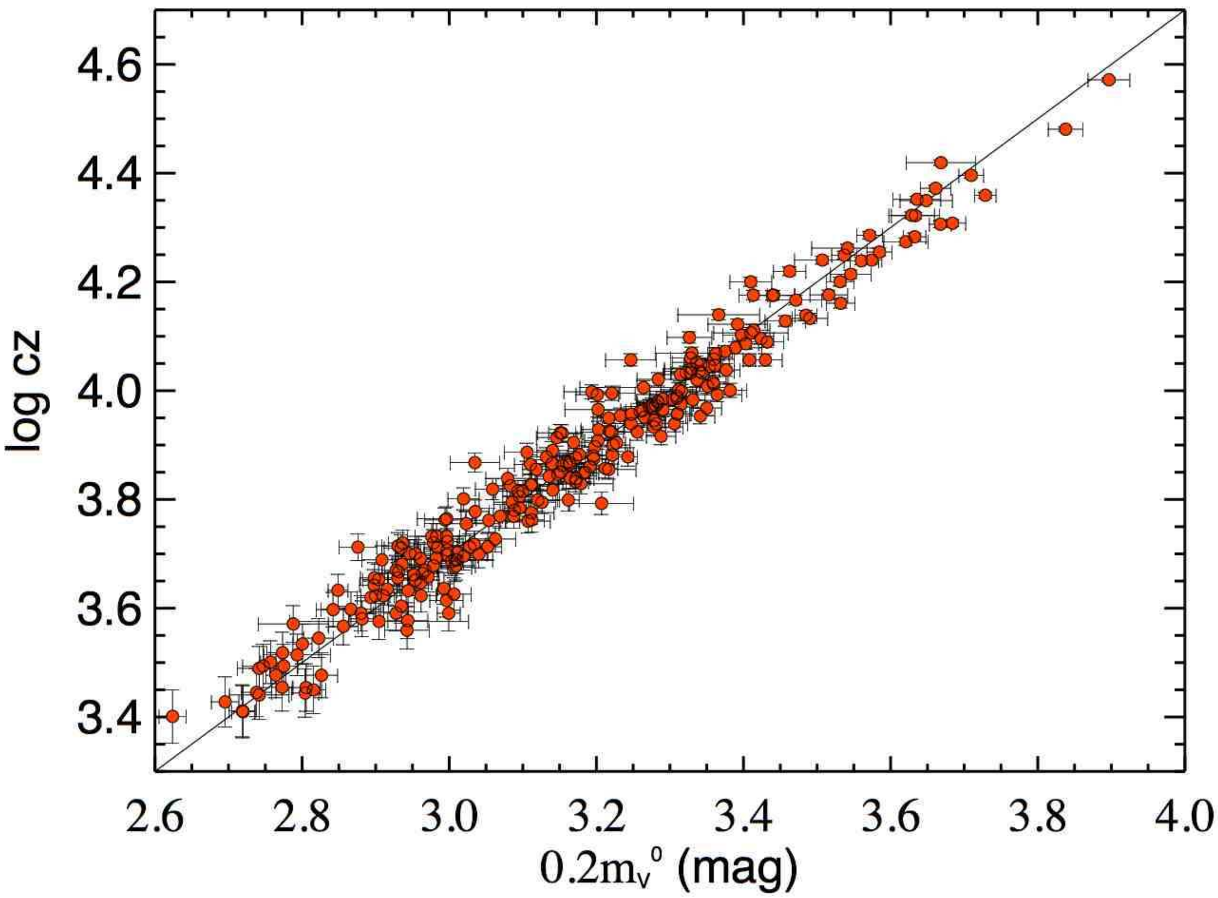
Problem: prove the Hubble law

Hubble diagram for SNe1a

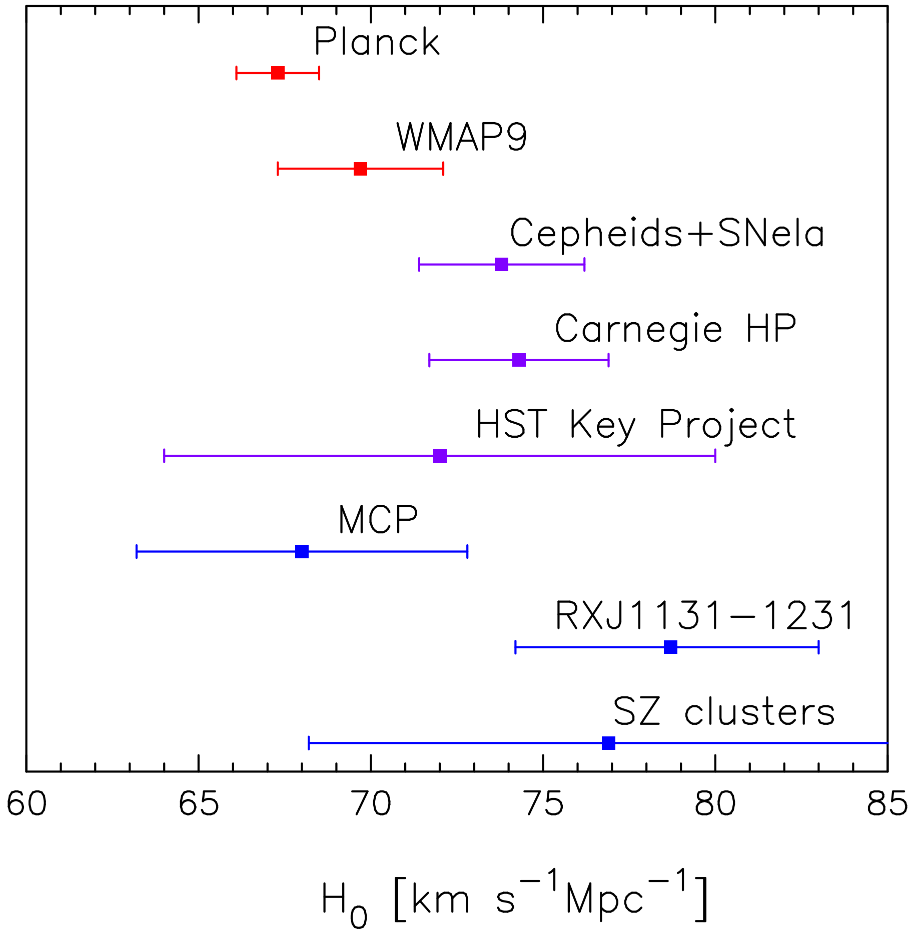


mag $= 5\log_{10} r +$ const

Hubble diagram, recent



Systematics still large



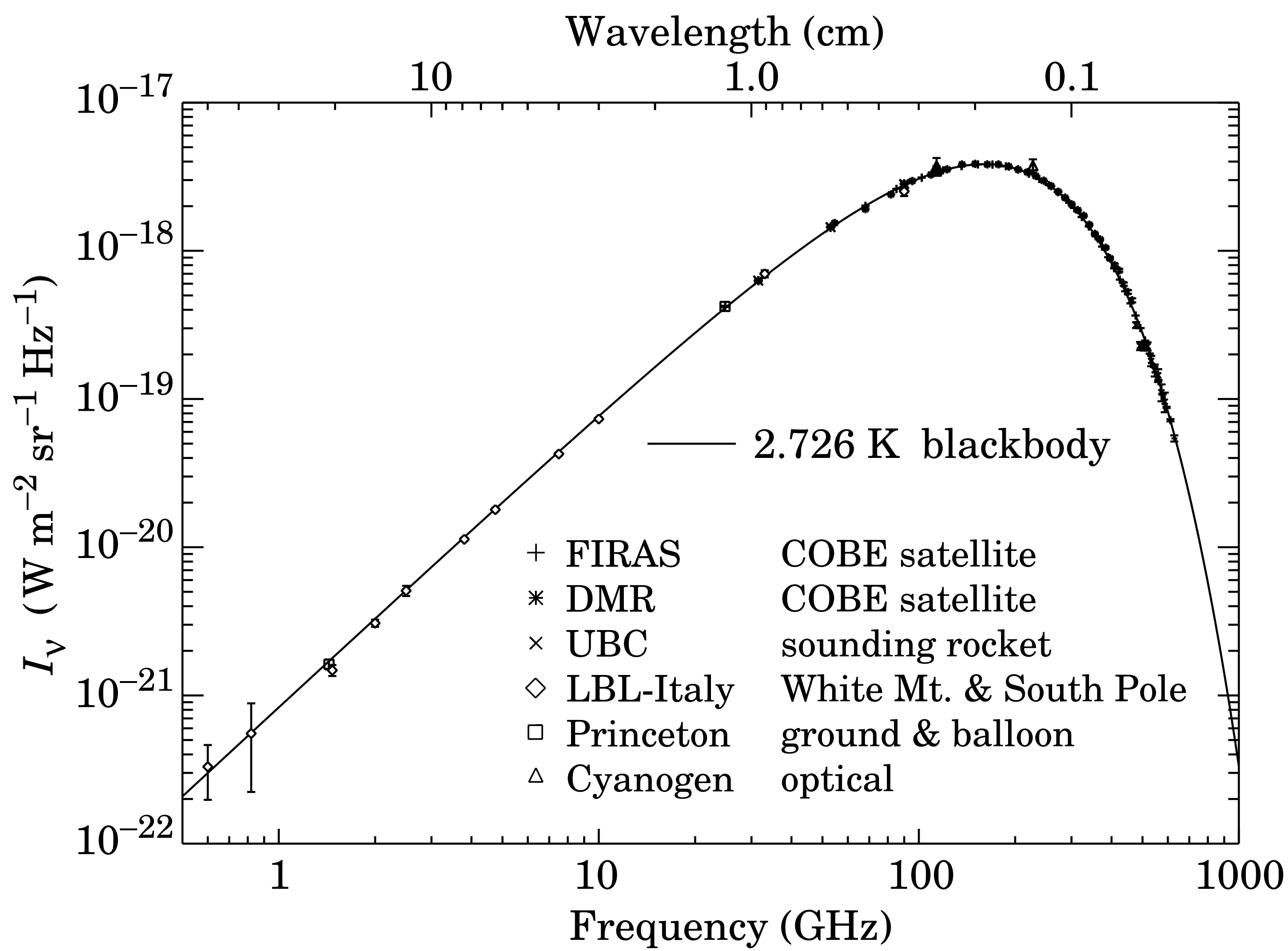
- The Universe is warm: CMB temperature today

$$T_0 = 2.7255 \pm 0.0006 \text{ K}$$

Fig.

It was denser and warmer at early times.

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\text{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3}$$

In early Universe (Bose–Einstein, Fermi–Dirac)

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$; fermions contribute with factor $7/8$.

Slow expansion $\Longrightarrow$ entropy conservation $\Longrightarrow$ Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

Dynamics of expansion

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi}H_0^2 = 5\cdot 10^{-6}\ \frac{\text{GeV}}{\text{cm}^3}$$

$$= 5\frac{m_p}{\text{m}^3}$$

$\hbar = c = k_B = 1$ in what follows

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.685$
  ρ_Λ stays (almost?) constant in time
- Non-relativistic matter: $\Omega_M = 0.315$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.265$
  - Usual matter (baryons): $\Omega_B = 0.050$
- Relativistic matter (radiation): $\Omega_{rad} = 8.6 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

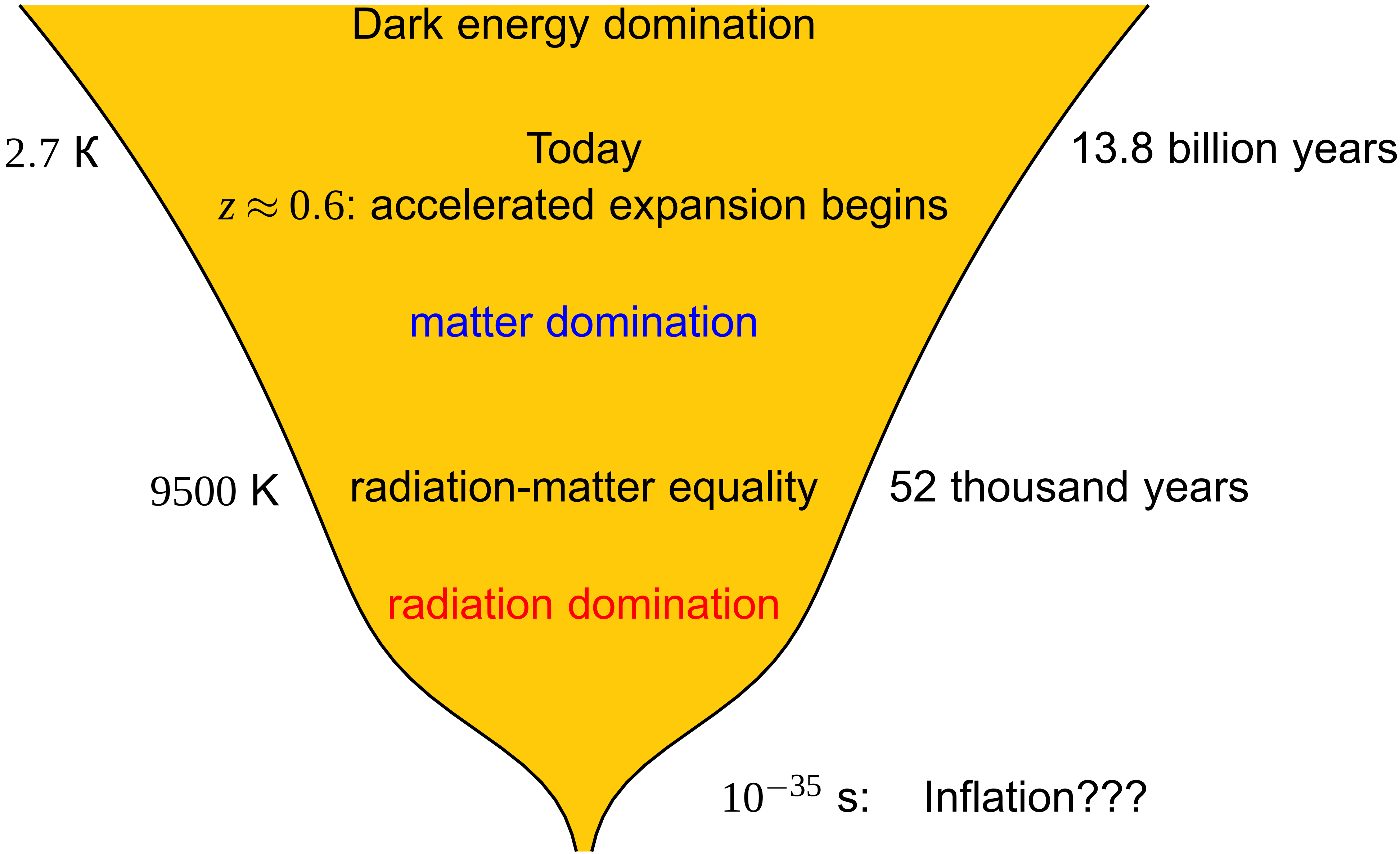
$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

$z_{eq} = 3500$

$T_{eq} = 9500$ K $= 0.8$ eV

$t_{eq} = 52 \cdot 10^3$ yrs

now



Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
matter domination
9500 K
radiation-matter equality
52 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

  Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.

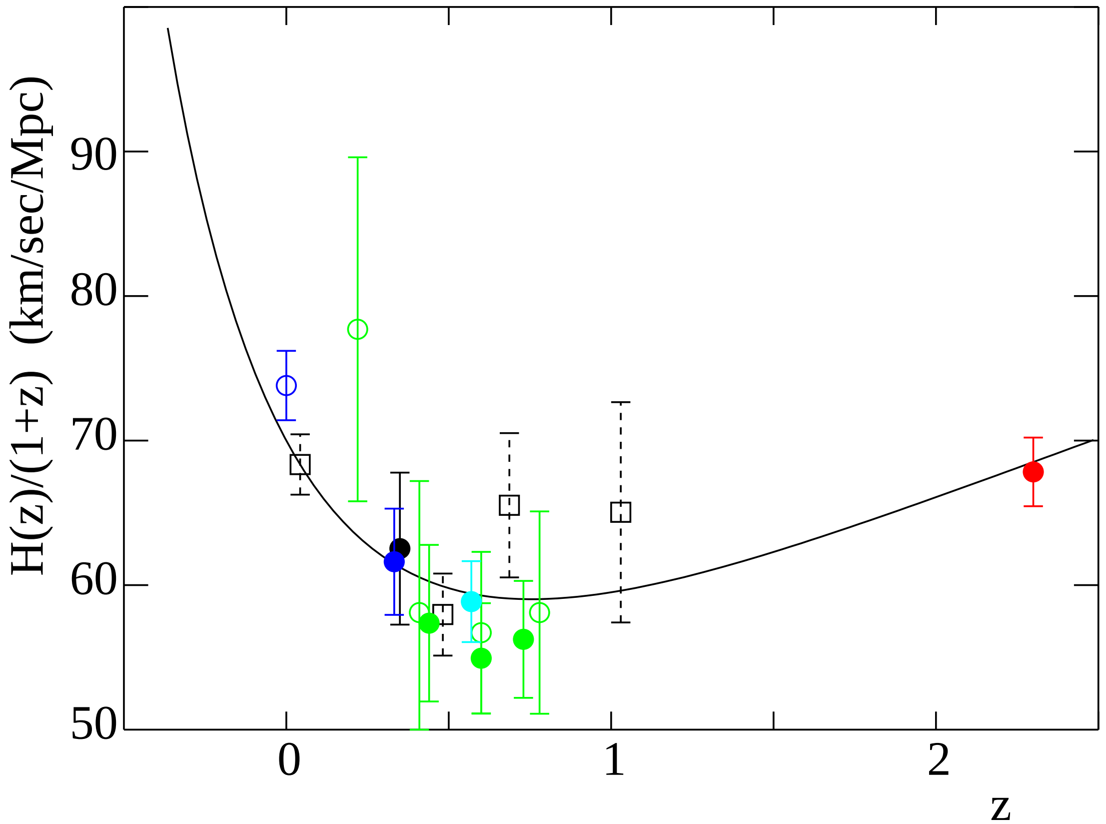
NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion

Fig.

Deceleration to acceleration



$$\frac{H}{1+z} = \frac{\dot{a}}{a} \cdot \frac{a}{a_0} = \frac{\dot{a}}{a_0} = \dot{a}(z)\ ;$$ large $z \leftrightarrow$ early time

Cosmological (particle) horizon

Light travels along $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = 0 \Longrightarrow dx = dt/a(t)$.
If emitted at $t = 0$, travels finite coordinate distance

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \sqrt{t} \quad \text{at radiation domination}$$

$\eta \propto \sqrt{t} \Longrightarrow$ visible Universe increases in time

Fig.

Physical size of causally connected region at time t (horizon size)

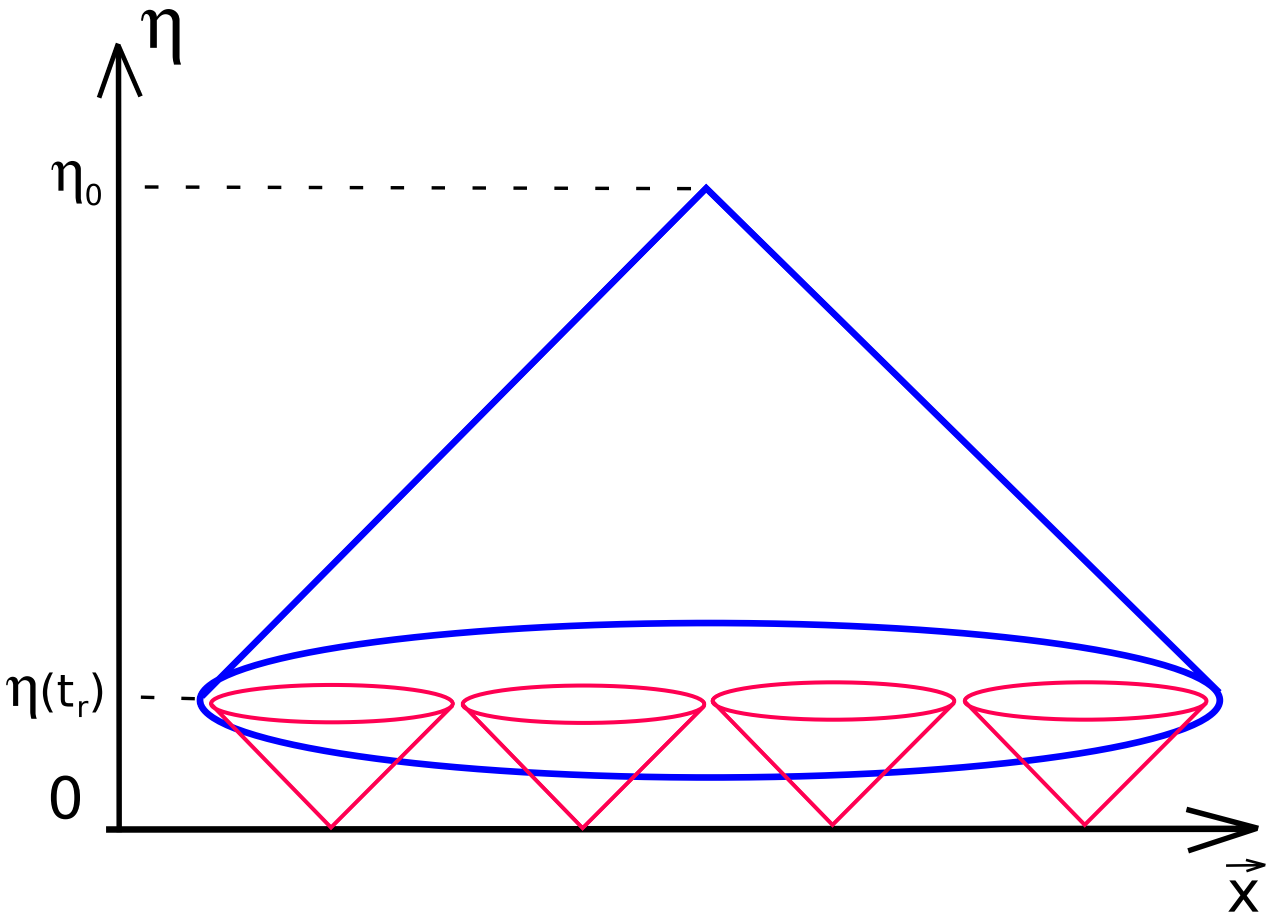
$$l_{H,t} = a(t) \int_0^t \frac{dt'}{a(t')} = 2t \quad \text{at radiation domination}$$

In hot Big Bang theory at both radiation and matter domination

$$l_{H,t} \sim t \sim H^{-1}(t)$$

Today $l_{H,t_0} \approx 15$ Gpc $= 4.5 \cdot 10^{28}$ cm

Causal structure of space-time in hot Big Bang theory



We see many regions that were causally disconnected by time t_r.
Why are they all the same?

Cornerstones of thermal history

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{aligned} p+n &\to {}^2H \\ {}^2H+p &\to {}^3He \\ {}^3He+n &\to {}^4He \\ \text{up to} & \quad {}^7Li \end{aligned}$$

  Abundances of light elements: measurements vs theory

  $T = 10^{10} \to 10^9$ K, $\quad t = 1 \to 300$ s

  Earliest time in thermal history probed so far Fig.

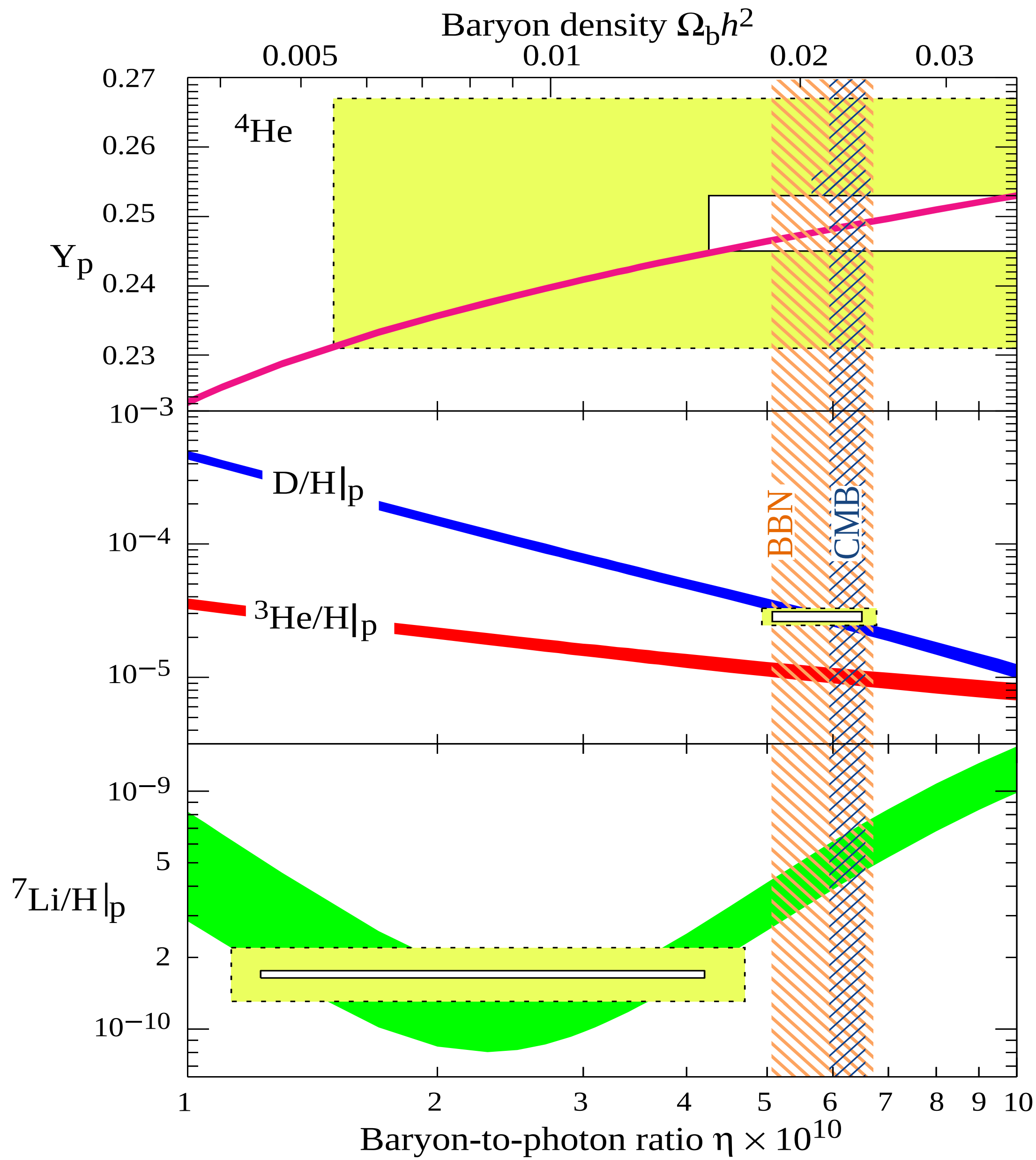
- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $\quad t = 380\,000$ years

  Last scattering of CMB photons Fig.

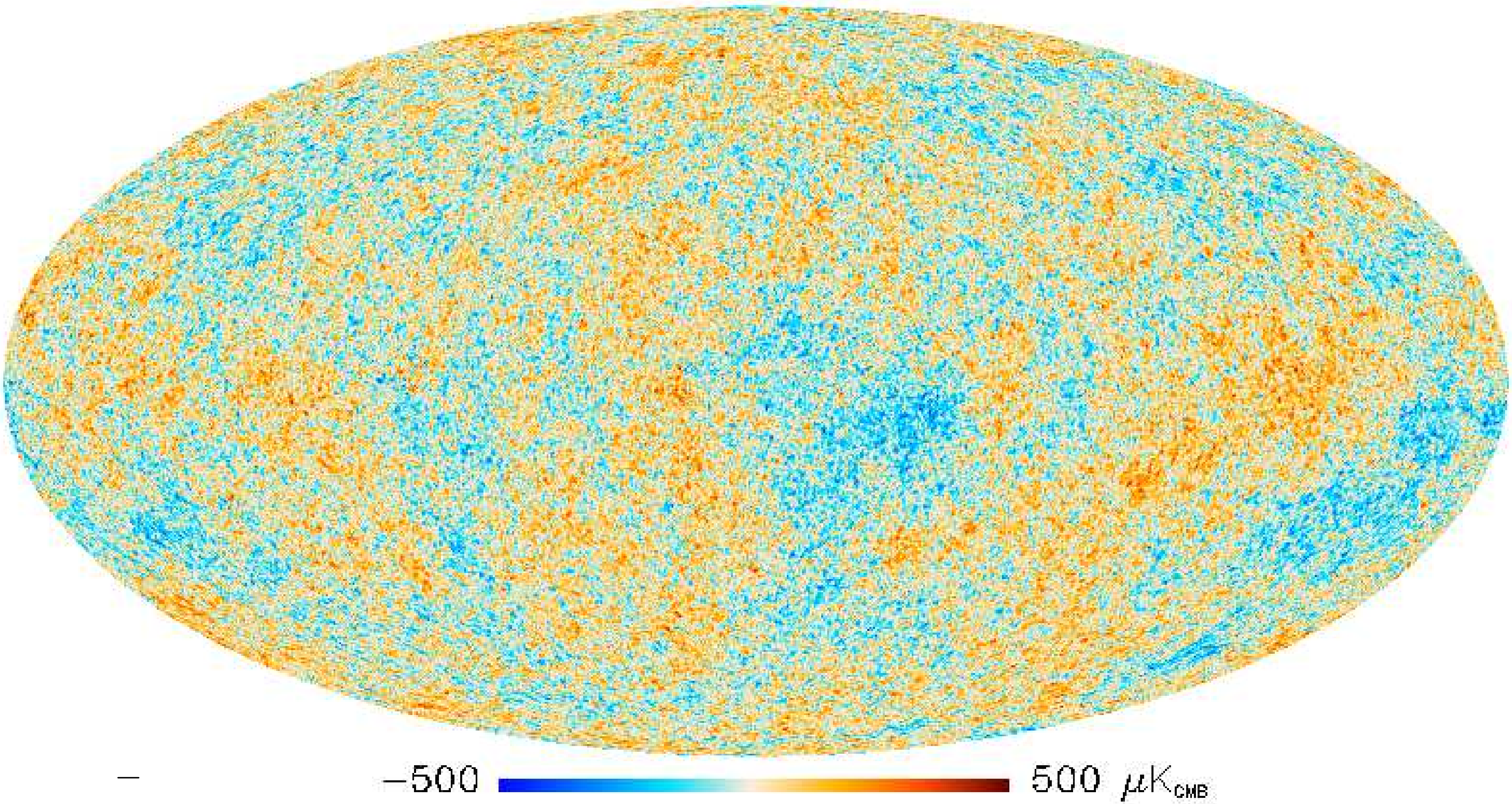
- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $\quad t \sim 0.1-1$s
- Generation of dark matter*
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch

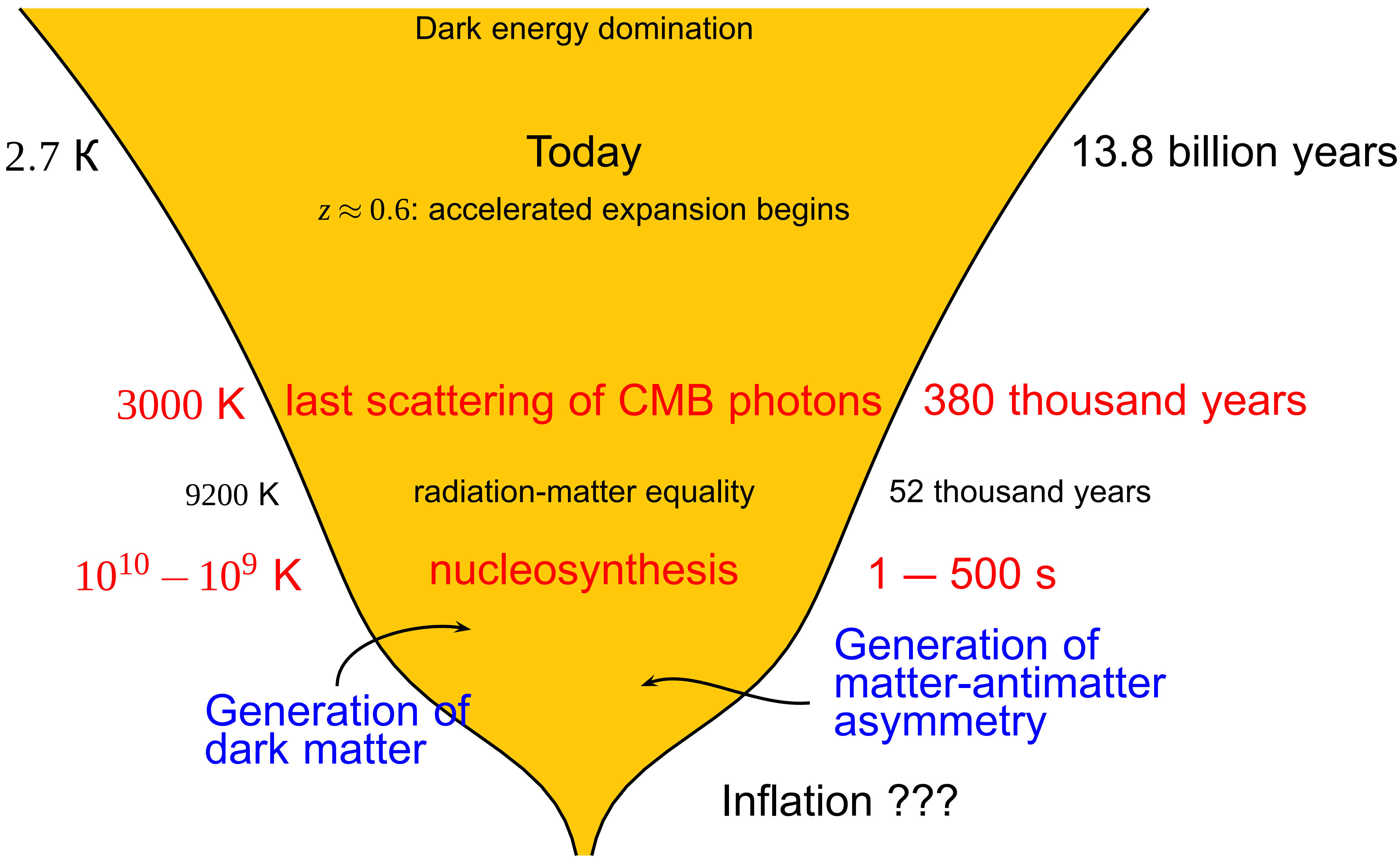


$\eta_{10} = \eta \cdot 10^{-10}$ = baryon-to-photon ratio. Consistent with CMB determination of η

$$T = 2.726^{\circ}K,\ \ \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck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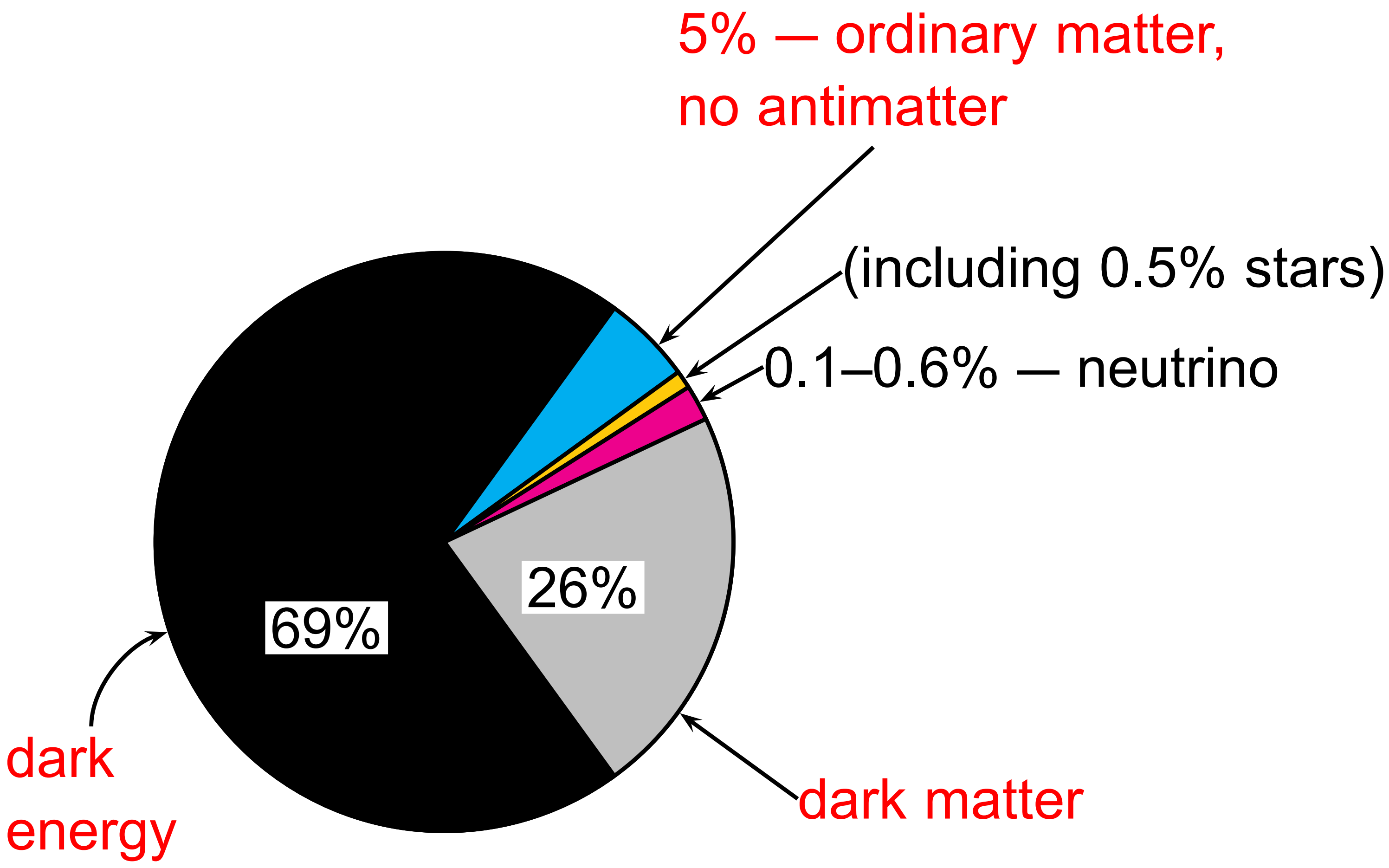
52 thousand years
$10^{10} - 10^{9}$ K
nucleosynthesis
1 — 500 s
Generation of matter-antimatter asymmetry
Generation of dark matter
Inflation ???

Unknowns



5% — ordinary matter,
no antimatter
(including 0.5% stars)
0.1–0.6% — neutrino
26%
69%
dark matter
dark
energy

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

    Fig.
  - Velocities of galaxies in clusters

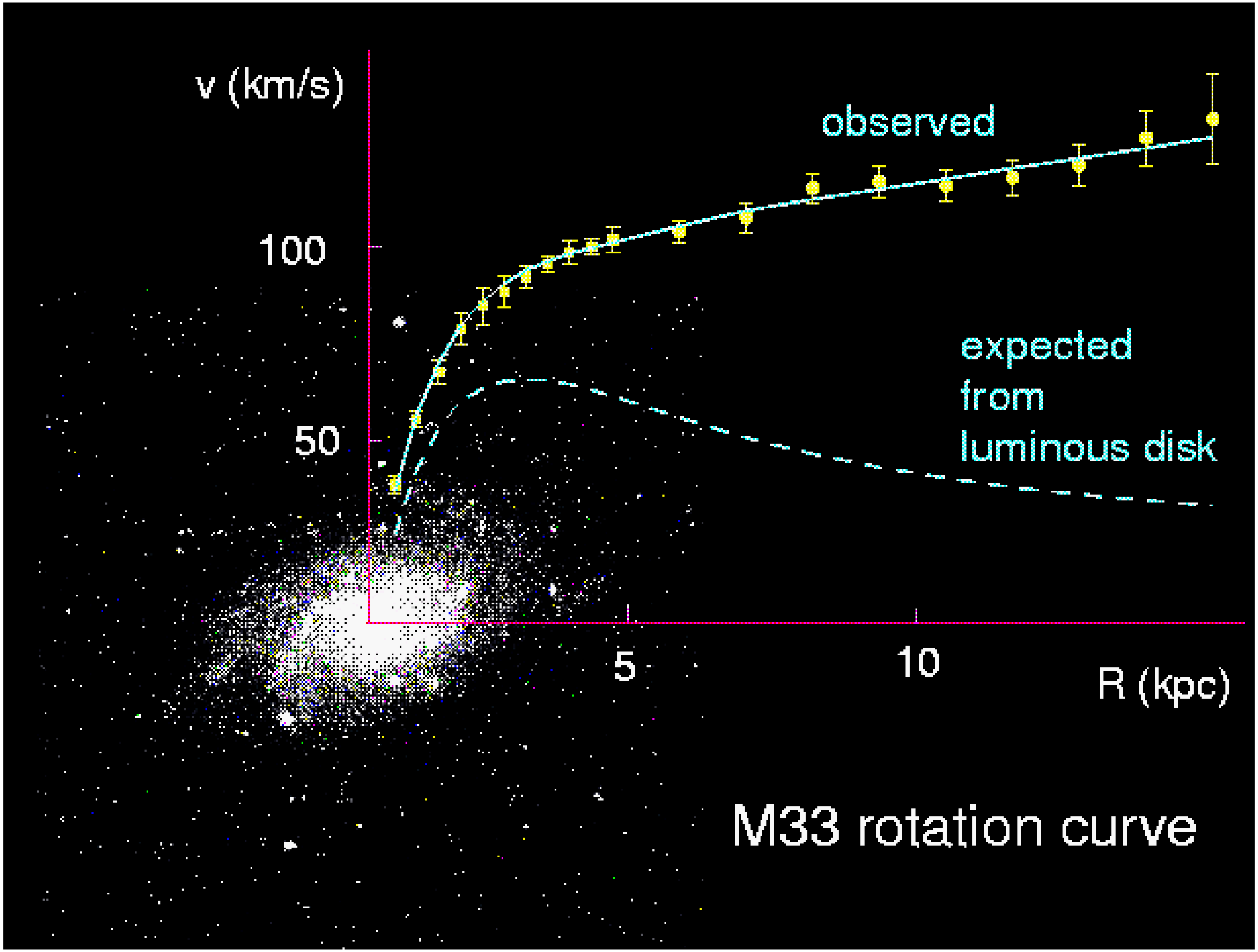
    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

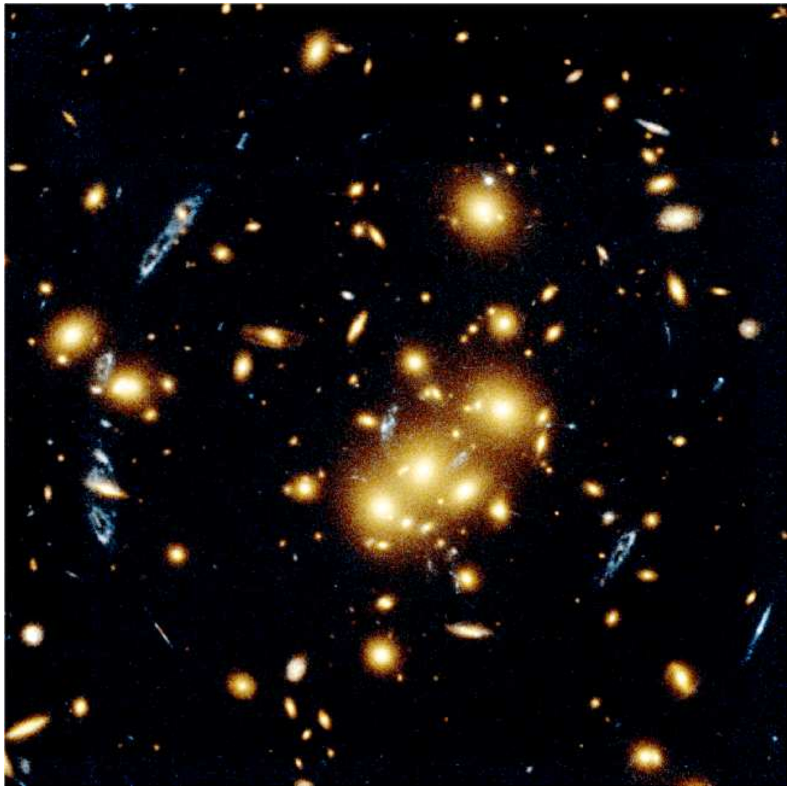
  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters

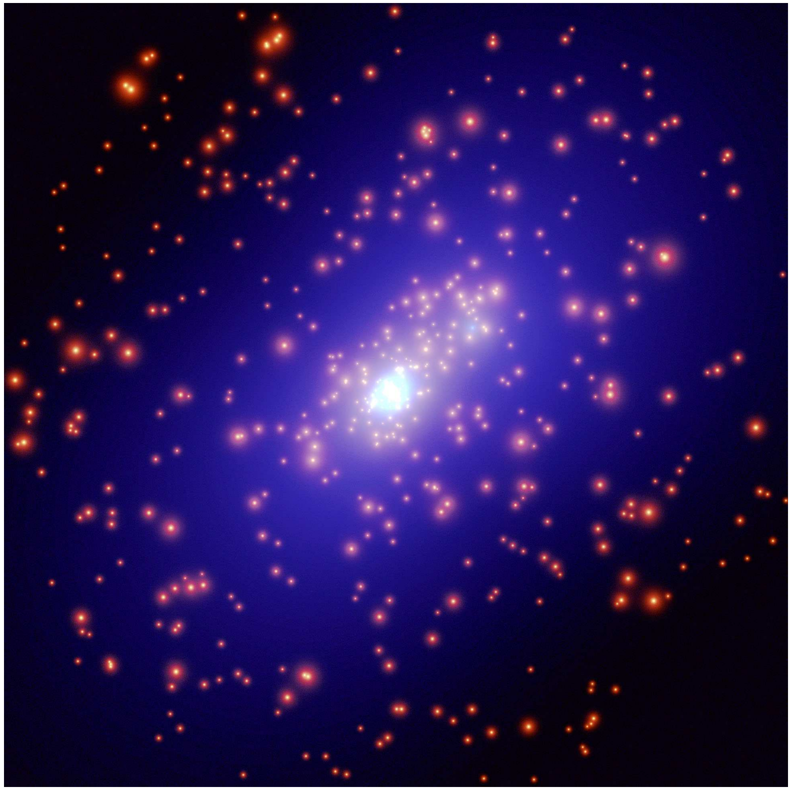
    Fig.
  - Etc.

Rotation curves

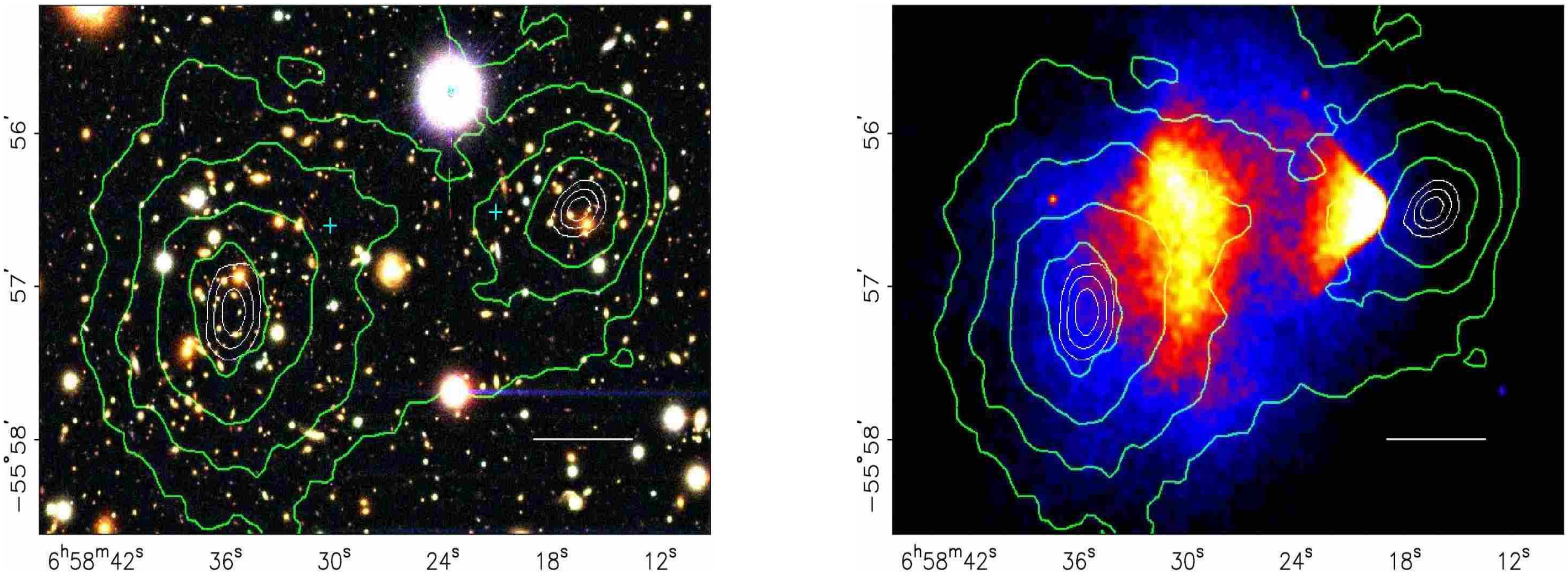


Gravitational lensing





Bullet cluster



Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters

NB: only 10 % of galaxies sit in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.05$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.26$.

Physical parameter: mass-to-entropy ratio. Stays constant in time. Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.26 \cdot 0.5 \cdot 10^{-6}\ \text{GeV cm}^{-3}}{3000\ \text{cm}^{-3}} \simeq 4 \cdot 10^{-10}\ \text{GeV}$$

Cosmological evidence: growth of structure

CMB anisotropies: baryon density perturbations at recombination $\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \sim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

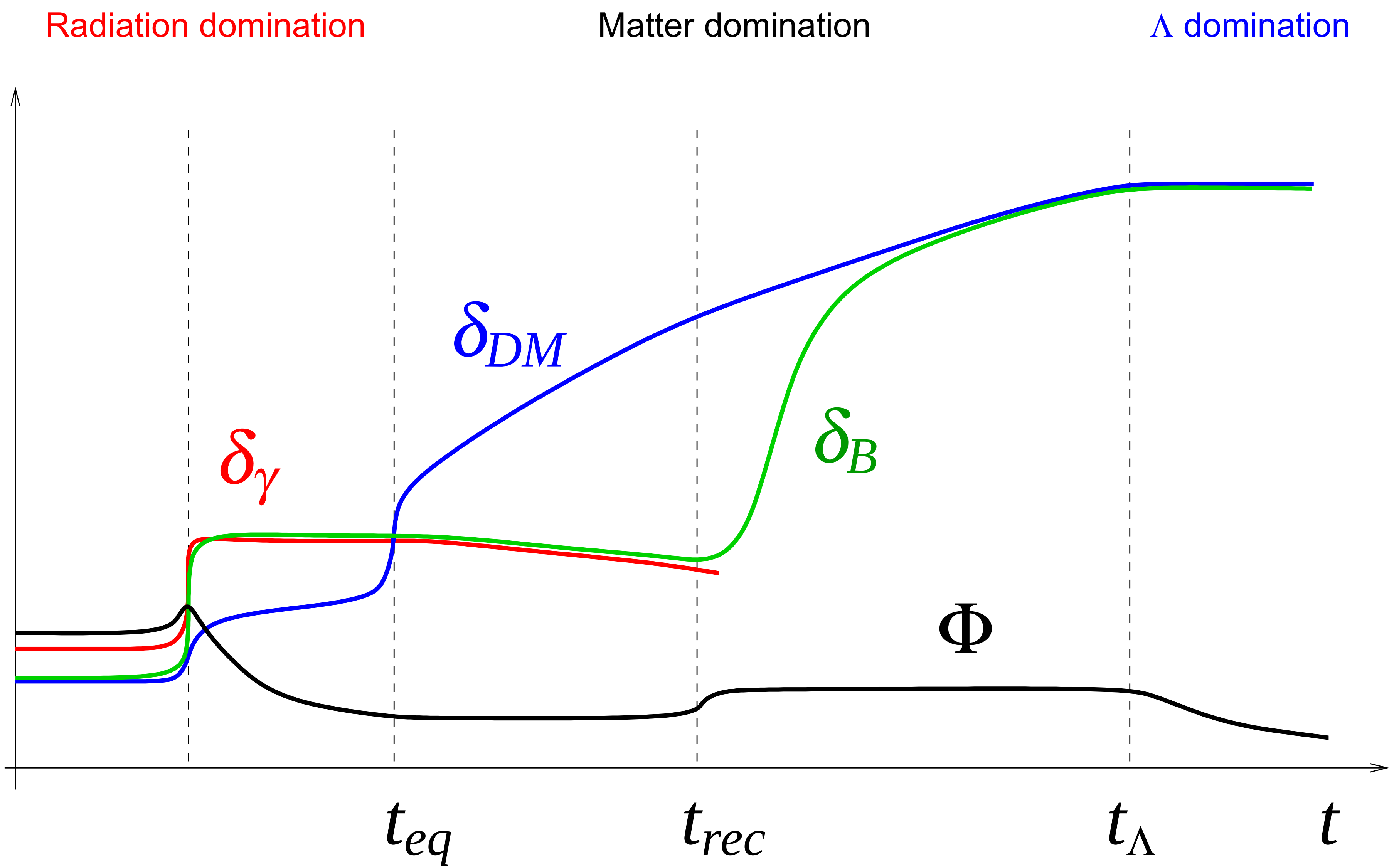
$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...
Perturbations in dark matter start to grow much earlier
(already at radiation-dominated stage)

Growth of perturbations (linear regime)



NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu \lesssim 0.1$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

If thermal relic:

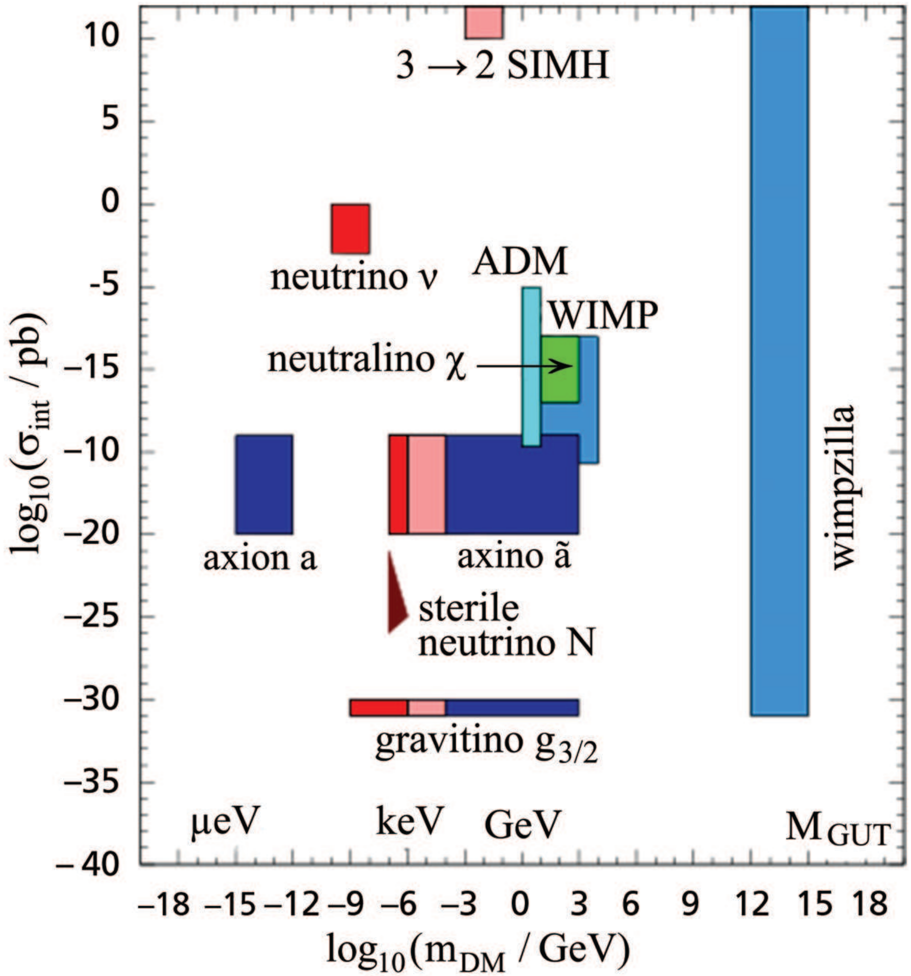
Cold dark matter, CDM

$$m_{DM} \gtrsim 100 \text{ keV}$$

Warm dark matter

$$m_{DM} \simeq 1 - 30 \text{ keV}$$

Canidates for Dark Matter particles are numerous



WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}\text{ ,etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity σ
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density

Outcome: mass to entropy ratio

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \langle\sigma v\rangle)}{\langle\sigma v\rangle \sqrt{g_*(T_f)} M_{Pl}} \; ; \qquad \# = \frac{3\sqrt{5}}{\sqrt{\pi}}$$

- Correct value, mass-to-entropy$= 4 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle\sigma v\rangle = (1 \div 2) \cdot 10^{-36}\ \text{cm}^2 = (1 \div 2)\ \text{pb}$$

  - Weak scale cross section.

    Gravitational physics and EW scale physics combine into

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W}\right)^2 \simeq 10^{-10}\ \text{GeV}$$

  - Mass M_X should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates

SUSY: LSP neutralinos, $X = \chi$

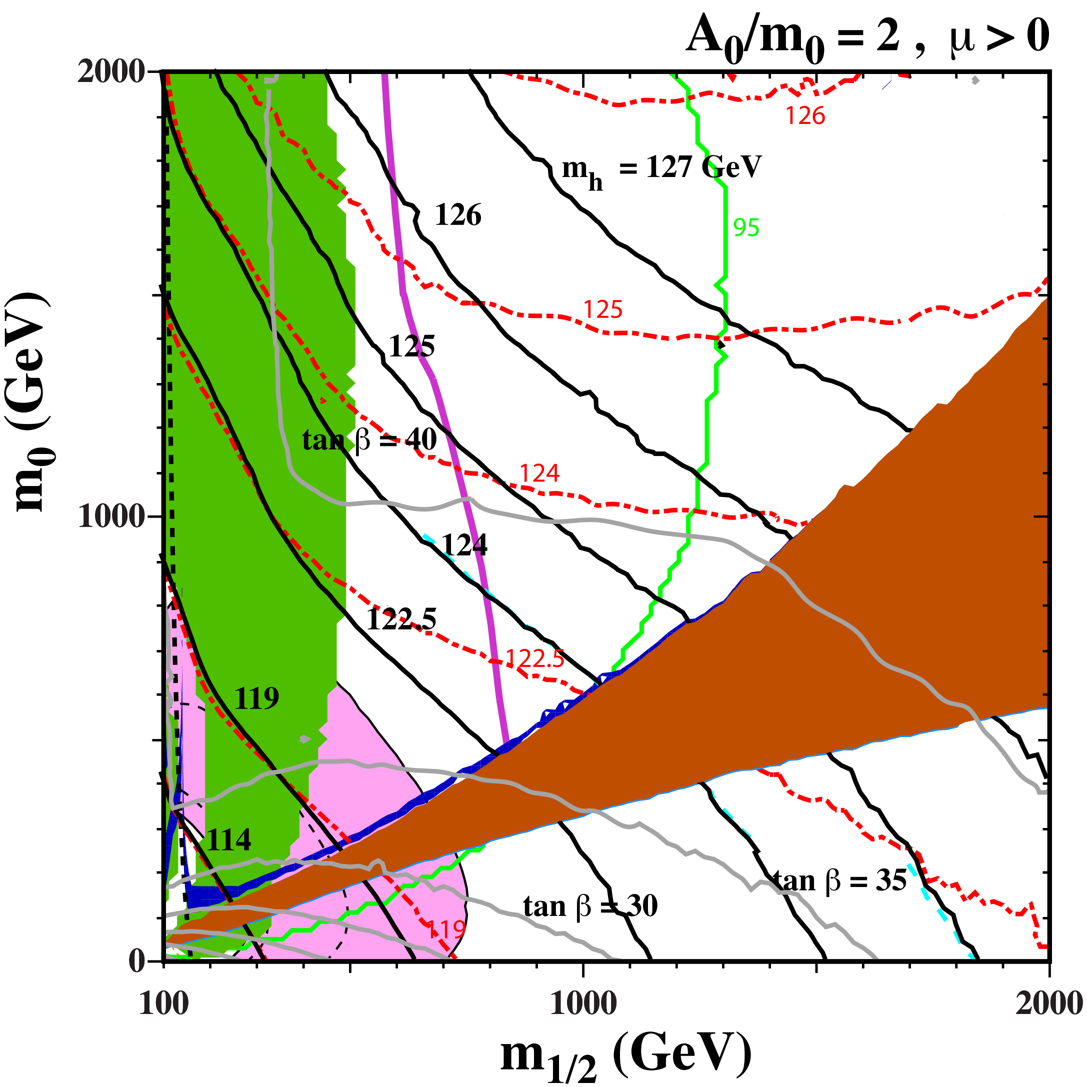
But situation is rather tense already: annihilation cross section is often too low

Important supperssion factor: $\langle \sigma v \rangle \propto v^2 \propto T/M_\chi$ because of p-wave annihilation in case $\chi\chi \to Z^* \to f\bar{f}$:

Relativistic $f\bar{f}$ $\Longrightarrow$ total angular momentum $J = 1$

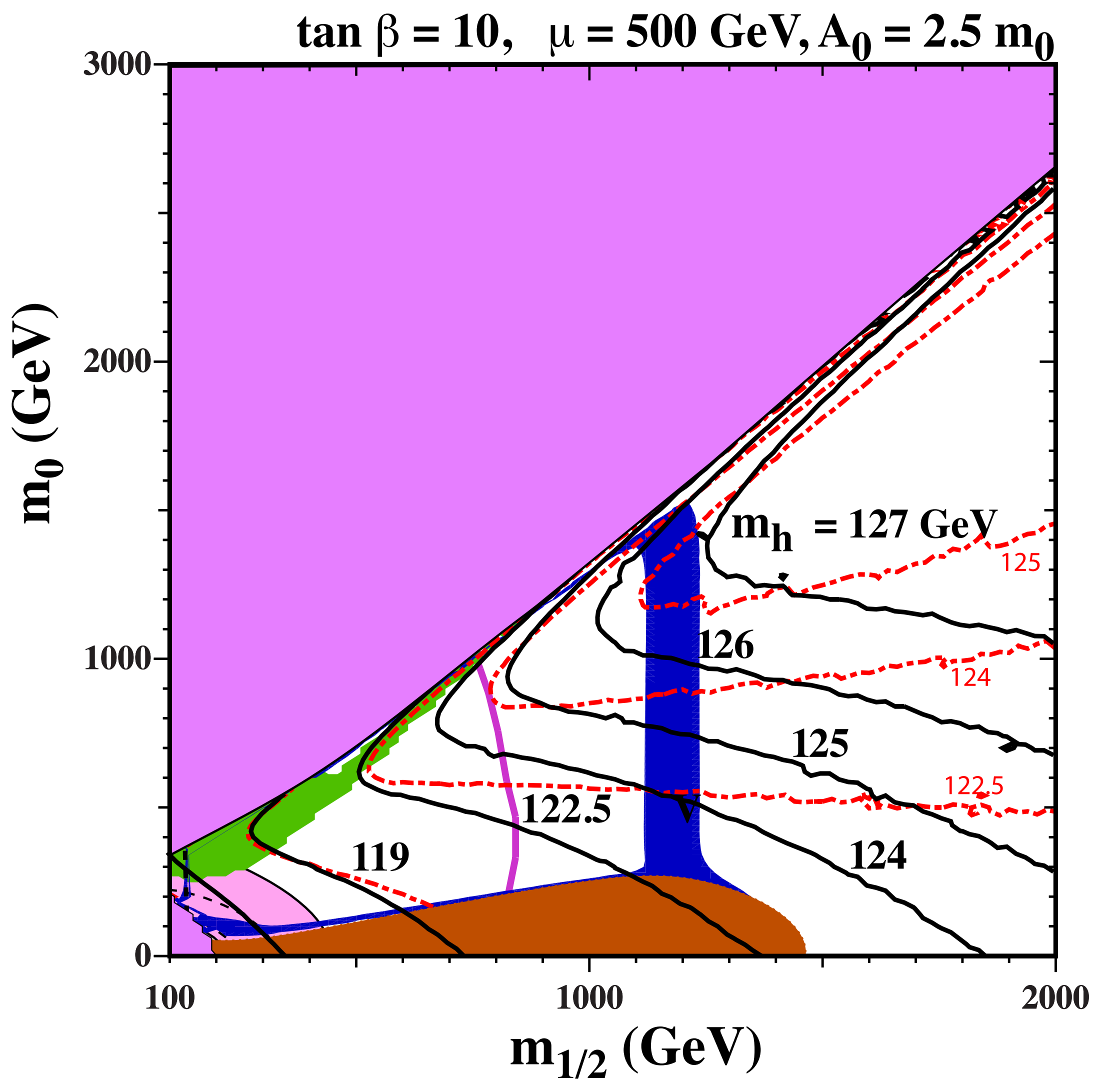
$\chi\chi$: identical fermions $\Longrightarrow$ $L = 0$, parallel spins impossible $\Longrightarrow$ p-wave

Constrained MSSM



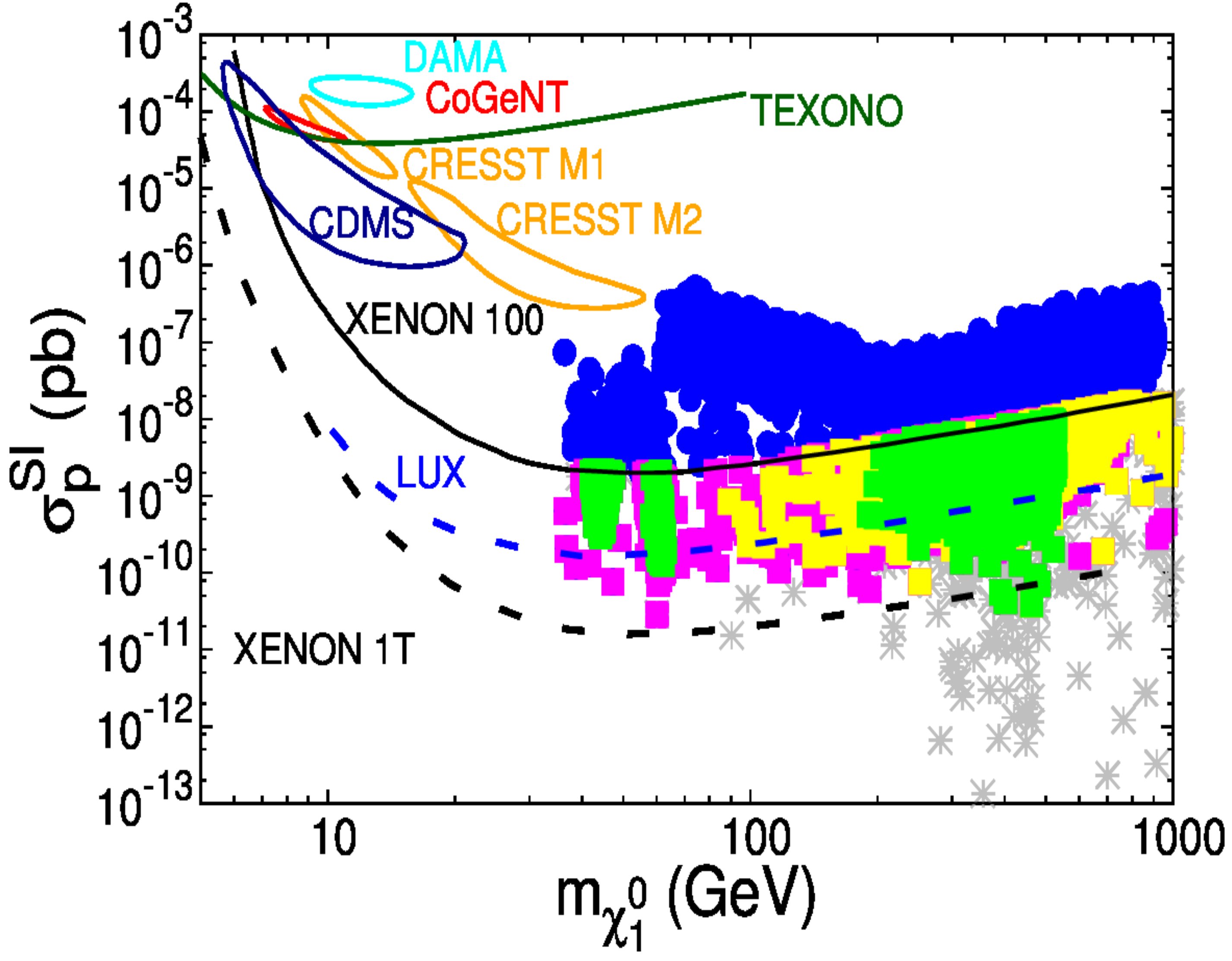
Buchmuller et.al.' 2013

Less constrained MSSM



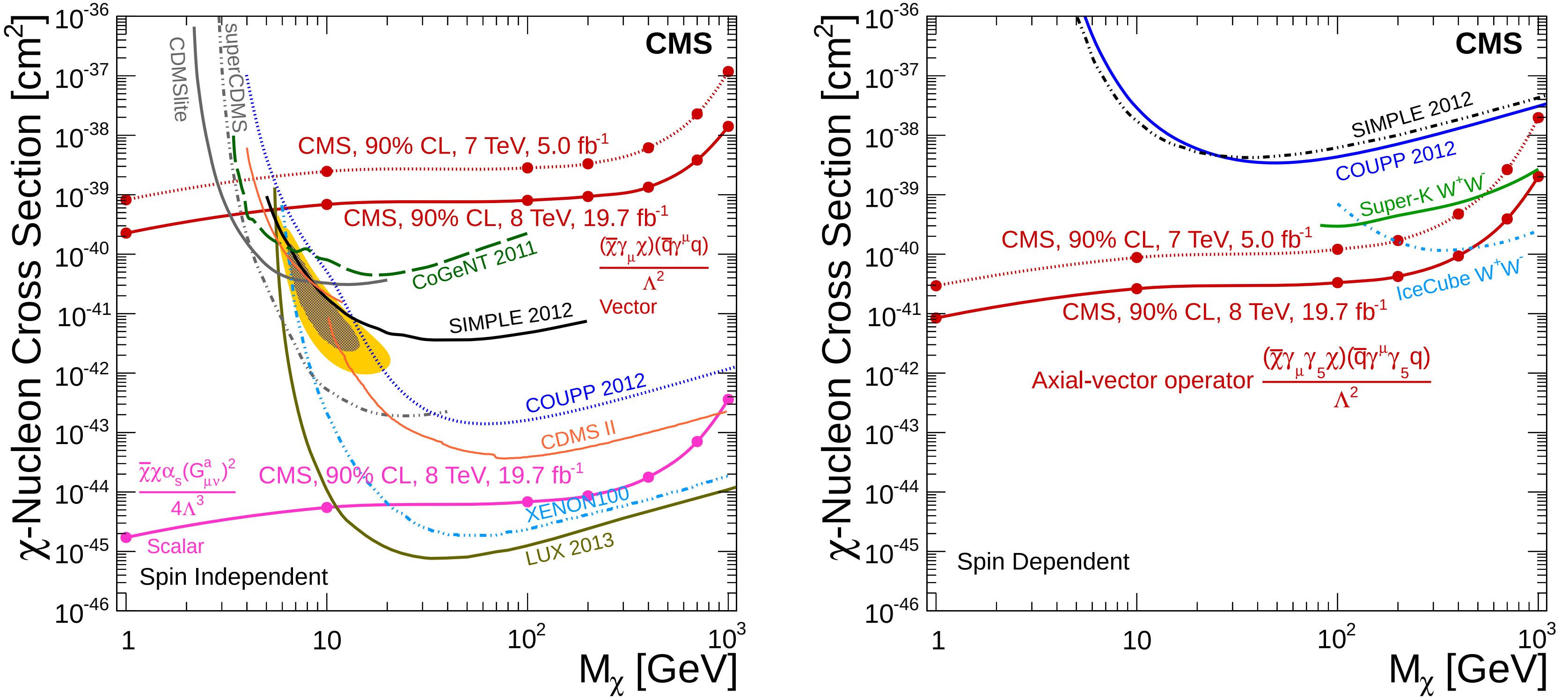
Buchmuller et.al.’ 2013

Direct searches are sensitive to SUSY



Han et.al.' 2013

The LHC becomes sensitive too



Khachatryan et.al.' 2014

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. Other good DM candidates:

axion, sterile neutrino, gravitino.

Plus a lot of exotica...

Crucial impact of the LHC to cosmology, direct and indirect dark matter searches

Lecture 2

Outline of Lecture 2

- Axions
  - Theory
  - Cosmology
  - Search
- Warm dark matter
  - Sterile neutrino
  - Gravitino
- Dark matter summary
- Baryon asymmetry of the Universe
  - Generalities.
  - Electroweak baryon number non-conservation

Axions

Motivation: solution of strong CP problem

What's the problem?

Point No. 1: global symmetries of QCD in chiral limit

$m_u = m_d = m_s = 0$

$$L_{QCD,m=0} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_i \bar{q}_i i\gamma^\mu D_\mu q_i$$

$$= -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_i \left(\bar{q}_{L,i} i\gamma^\mu D_\mu q_{L,i} + \bar{q}_{R,i} i\gamma^\mu D_\mu q_{R,i}\right)$$

Naively: symmetry under independent $SU(3) \times U(1)$ rotations of left and right quarks,

$$SU(3)_L \times U(1)_L \times SU(3)_R \times U(1)_R = SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$$

$U(1)_B$: $q_i \to e^{i\alpha} q_i$; $U(1)_A$: $q_i \to e^{i\beta\gamma^5} q_i$

Symmetry partially broken due to quark condensate in QCD vacuum,

$$\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle = \frac{1}{2}\langle \bar{q} q \rangle = \text{real} \sim \Lambda_{QCD}^3$$

Remaining symmetry $SU(3)_V$: rotates left and right quarks together; $U(1)_B$

Expect 9 Nambu–Goldstone bosons, 8 from $SU(3)_L \times SU(3)_R \to SU(3)_V$ plus 1 from $U(1)_B \times U(1)_A \to U(1)_B$.

But there are only 8 in Nature: $\pi^{\pm}$, π^0, $K^{\pm}$, K_0, $\bar{K}^0$, η.

NB: $m_\pi^2 = m_{u,d} \langle \bar{q} q \rangle / f_\pi^2$

η' is heavy, does not behave like Nambu–Goldstone boson.

Reason: $U(1)_A$ is not a symmetry in QCD

Axial current has triangle anomaly, $\partial_\mu J_\mu^A \neq 0$.

Point No. 2

Quark Yukawa interactions $\Longrightarrow$ quark mass matrix

$$L_Y = y_{ij}^{(d)} \bar{Q}_L^i H d_R^j + y_{ij}^{(u)} \bar{Q}_L^i H^* u_R^j + \text{h.c.} \Longrightarrow L_m = m_{ij}^{(d)} \bar{d}_L^i d_R^j + m_{ij}^{(u)} \bar{u}_L^i u_R^j + \text{h.c.}$$

$m_{ij}^{(u,d)} = y_{ij}^{(u,d)} v/\sqrt{2}$ complex; $i, j = 1, 2, 3 =$ generation label.

Standard lore: diagonalize $\Longrightarrow$ CKM matrix, 3 angles, 1 phase.

This is not quite true

One more phase: common phase of all Yukawa couplings/masses,

$$m_{ij} = \mathrm{e}^{i\theta} \cdot m_{ij}^{CKM} \qquad \text{or} \qquad \theta = (\text{Arg det } m)/3$$

At first sight: rotate away,

$$q_L^i \to \mathrm{e}^{i\theta/2} q_L^i, \quad q_R^i \to \mathrm{e}^{-i\theta/2} q_R^i, \quad \text{i.e.,} \quad q^i \to \mathrm{e}^{-i\gamma^5\theta/2} q^i$$

But this is not allowed: $U(1)_A$ is not a symmetry!

θ is a physical parameter

Vacuum energy density for non-zero θ, call it $V(\theta)$ (useful in what follows). Keep only u, d-quarks, take their masses equal for simplicity, $m_u = m_d \equiv m_q \sim 10$ MeV (heavier quarks less important),

$$L_m = \mathrm{e}^{i\theta} m_q(\bar{u}_L u_R + \bar{d}_L d_R) + \text{h.c.}$$

Perturbation theory in m_q: $V(\theta) = -\langle L_m \rangle$. Recall quark condensate

$$\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle = \frac{1}{2}\langle \bar{q}q \rangle = \text{real} \sim \Lambda^3_{QCD}$$

NB: No arbitrary phase here, otherwise η' would be pseudo-Nambu–Goldstone boson! Get

$$V(\theta) = -\langle L_m \rangle = -m_q \langle \bar{q}q \rangle \cos\theta = -\frac{m_\pi^2 f_\pi^2}{4} \cos\theta$$

θ is a physical parameter! Violates CP.

NB: Minimum of $V(\theta)$ is at $\theta = 0$. No use if θ is just a parameter.

CP-violation within QCD. Neutron edm $d_n < 3 \cdot 10^{-26} e \cdot \mathrm{cm} \Longrightarrow$

$$\theta \lesssim 10^{-10}$$

Strong CP problem. Fine tuning? Mechanism that ensures $\theta = 0$

Peccei–Quinn: promote θ to a field.

Simple version: two Englert–Brout–Higgs fields

$$L = y^{(d)} \bar{Q}_L H_1 d_R + y^{(u)} \bar{Q}_L H_2 u_R + |D_\mu H_1|^2 + |D_\mu H_2|^2 - V(H_1, H_2)$$

Classical level: require global $U(1)_{PQ}$ symmetry (PQ symmetry)

$$q^i \to \mathrm{e}^{i\gamma^5 \theta/2} q^i \ , \quad H_1 \to \mathrm{e}^{i\theta} H_1 \ , \quad H_2 \to \mathrm{e}^{i\theta} H_2$$

Vev's: $\langle H_1 \rangle = v_1/\sqrt{2}$, $\langle H_2 \rangle = v_2/\sqrt{2}$, break PQ symmetry spontaneously.

Parametrize

$$H_1 = \mathrm{e}^{i\theta(x)} v_1/\sqrt{2}\,, \quad H_2 = \mathrm{e}^{i\theta(x)} v_2\sqrt{2}$$

If not for QCD, $\theta(x)$ would be a massless Nambu–Goldstone boson, axion. Kinetic term

$$\frac{1}{2}v_1^2(\partial_\mu\theta)^2 + \frac{1}{2}v_2^2(\partial_\mu\theta)^2 = \frac{1}{2}f_{PQ}^2(\partial_\mu\theta)^2$$

Quark masses $m_{d,u} = y_{d,u}v_{1,2}\mathrm{e}^{i\langle\theta(x)\rangle}$.

Turn on QCD: shift $\theta \to \theta + \mathrm{const}$ is NOT a symmetry.

Consequences

- $\langle\theta(x)\rangle$ is such that $V(\theta)$ is at minimum $\Longrightarrow \theta = 0$ automatically. Strong CP problem solved.
- $\theta(x)$ gets a mass

$$L_\theta = \frac{1}{2} f_{PQ}^2 (\partial_\mu \theta)^2 - V(\theta)\,, \qquad V(\theta) \simeq -m_q \langle \bar{q} q \rangle \cos\theta = \frac{1}{2} m_q \langle \bar{q} q \rangle \theta^2$$

Axion field $\theta(x) = a(x)/f_{PQ}$:

$$m_a^2 \simeq \frac{m_q \langle \bar{q} q \rangle}{f_{PQ}^2} \simeq \frac{m_q \Lambda_{QCD}^3}{f_{PQ}^2} \quad \Longrightarrow \quad m_a = 0.6 \text{ eV} \cdot \left(\frac{10^7 \text{ GeV}}{f_{PQ}} \right)$$

Interactions:

- Axion-photon-photon

$$C_{a\gamma\gamma} \frac{\alpha}{16\pi} \theta \cdot \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} = C_{a\gamma\gamma} \frac{\alpha}{2\pi} \frac{a(x)}{f_{PQ}} (\vec{E} \cdot \vec{H})\,, \qquad C_{a\gamma\gamma} \sim 1 \text{ roughly}$$

- Two Englert–Brout–Higgs fields are not enough

  $f_{PQ} = \sqrt{v_1^2 + v_2^2} = 246$ GeV too small:

  (Weinberg–Wilczek) axion is too heavy ($m_a \sim 15$ keV), its couplings too large, ruled out experimentally.

  Add heavy fields, make f_{PQ} large.

  Dine–Fischler–Srednicki–Zhitnitsky (DFSZ);
  Kim–Shifman–Vainshtein–Zakharov (KSVZ)

  Light axions, interact very weakly

To summarize

Peccei–Quinn solution to strong CP problem predicts axion with mass

$$m_a = 0.6 \text{ eV} \cdot \left(\frac{10^7 \text{ GeV}}{f_{PQ}} \right)$$

and $a\gamma\gamma$ interaction

$$C_{a\gamma\gamma} \frac{\alpha}{2\pi} \frac{a(x)}{f_{PQ}} (\vec{E} \cdot \vec{H})$$

where $C_{a\gamma\gamma} \sim 1$ is model-dependent, and f_{PQ} is the only free parameter. Larger $f_{PQ} \Longrightarrow$ smaller m_a, weaker interactions.

Why is this interesting for cosmology?

- Axion is practically stable:

$$\Gamma(a \to \gamma\gamma) = C_{a\gamma\gamma}^2 \left(\frac{\alpha}{8\pi}\right)^2 \frac{m_a^3}{4\pi f_{PQ}^2} \quad \Longrightarrow \quad \tau_a = 10^{17} \left(\frac{\text{eV}}{m_a}\right)^5 \text{ yrs}$$

- Interacts very weakly $\Longrightarrow$ dark matter candidate
- May never be in thermal equilibrium $\Longrightarrow$ cold dark matter if momenta are negligibly small.

Q. How can one arrange for negligibly small momenta for particles with sub-eV masses?

A. One way: Condensates (not the only option)

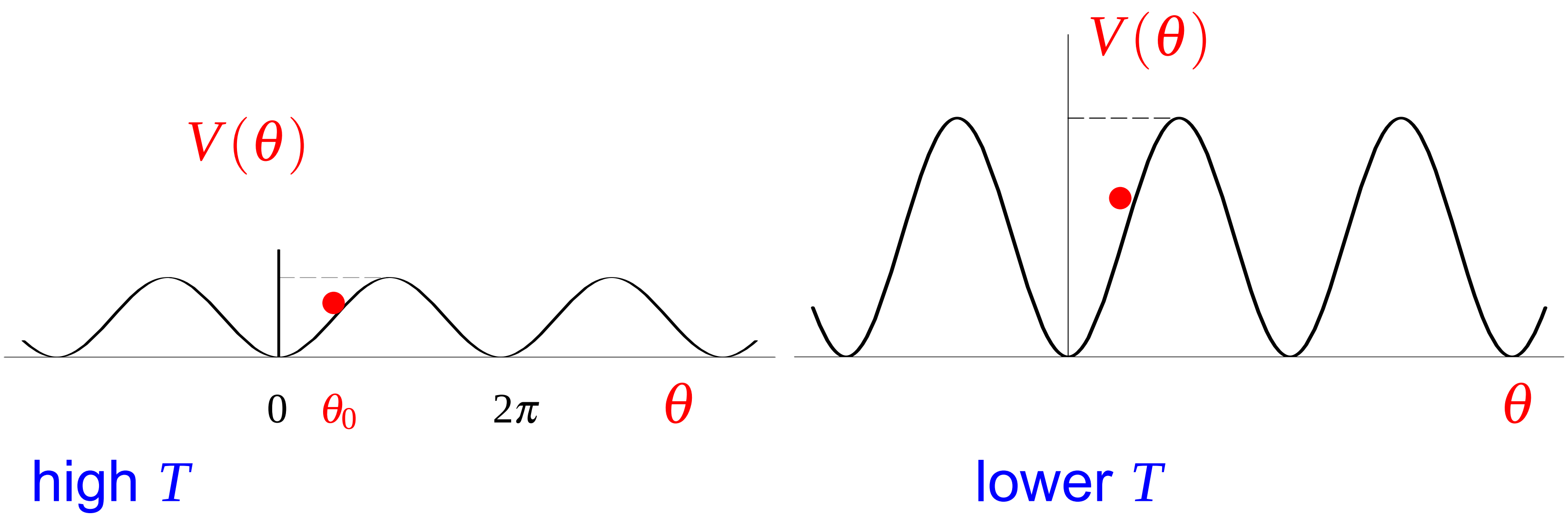
Axion production: misalignment

Recall $V(\theta) \simeq -m_q \langle \bar{q}q \rangle \cos\theta$

Early Universe, high T: $\langle \bar{q}q \rangle = 0 \Longrightarrow V(\theta) = 0$.

No preferred value of $\theta \Longrightarrow$ Initial condinion θ_0 anywhere between $-\pi$ and π.

At QCD epoch ($T \sim 200$ MeV) potential $V(\theta)$ builds up. θ starts to roll down.



Rolling down starts when $m_a(T) \sim H(T)$: before that time scale of rolling m_a^{-1} is larger than the cosmological time scale $\sim H^{-1}$.

After initial rolling, θ oscillates about minimum $\theta = 0$.

Homogeneous oscillating field $=$ condensate $=$ collection of quanta with zero spatial momentum. Just what we need for cold dark matter!

Estimate for present mass-to-entropy ratio

$$\frac{\rho_a}{s} = \# \frac{1}{M_{Pl} T_{QCD}} m_a a_0^2 = \# \frac{1}{M_{Pl} T_{QCD}} m_a f_{PQ}^2 \theta_0^2 \,, \qquad \# \sim 1 \,.$$

Recall $m_a f_{PQ}^2 \propto m_a^{-1}$: the lighter axions, the more dark matter.

$\rho_{DM}/s \sim 4 \cdot 10^{-10}$ GeV is obtained for $m_a = 10^{-5} - 10^{-6}$ eV (for $\theta_0 = \pi/2 - 0.1$).

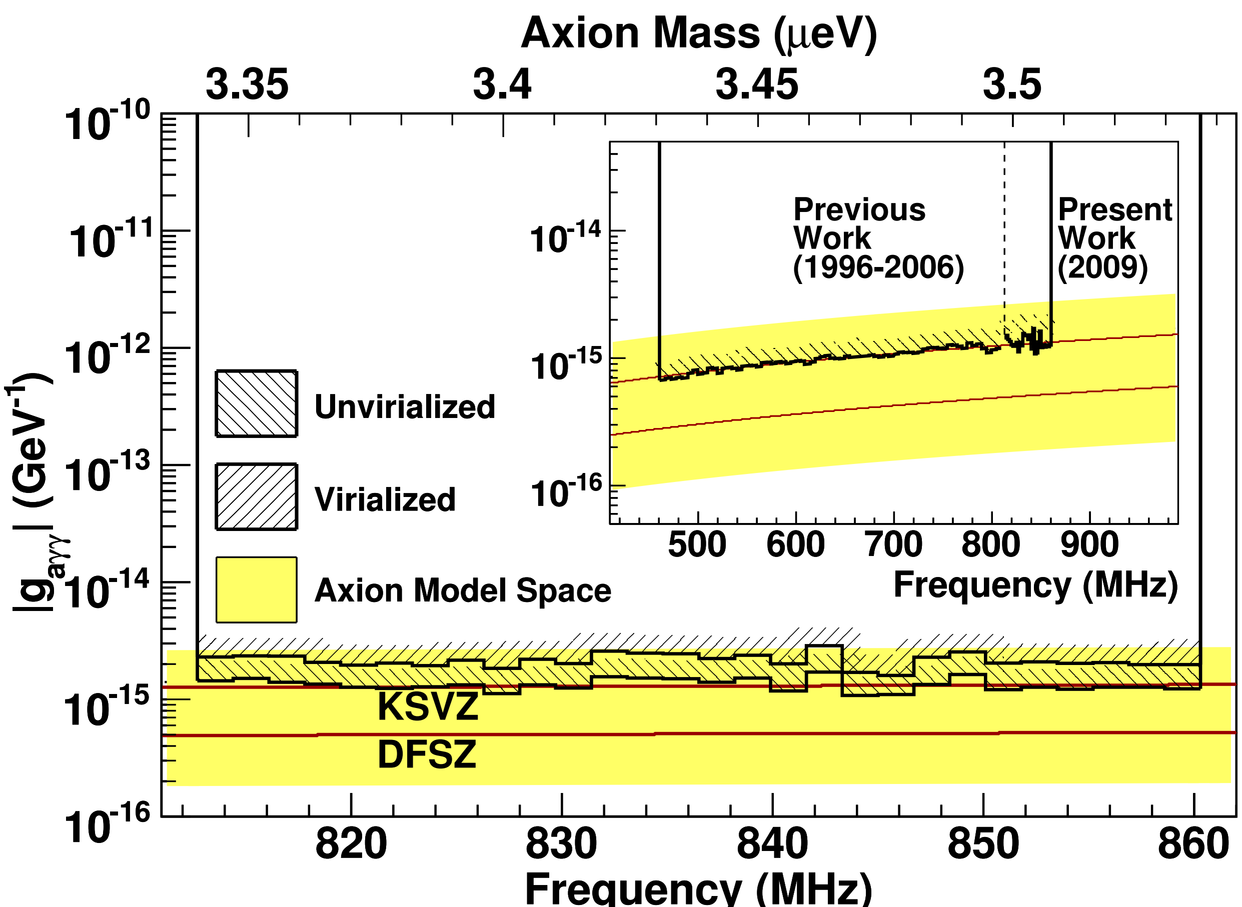
Axions of mass $(1 - 10)$ μeV are good cold dark matter candidates.

NB: Misalignment is not the only possible production mechanism.

Search

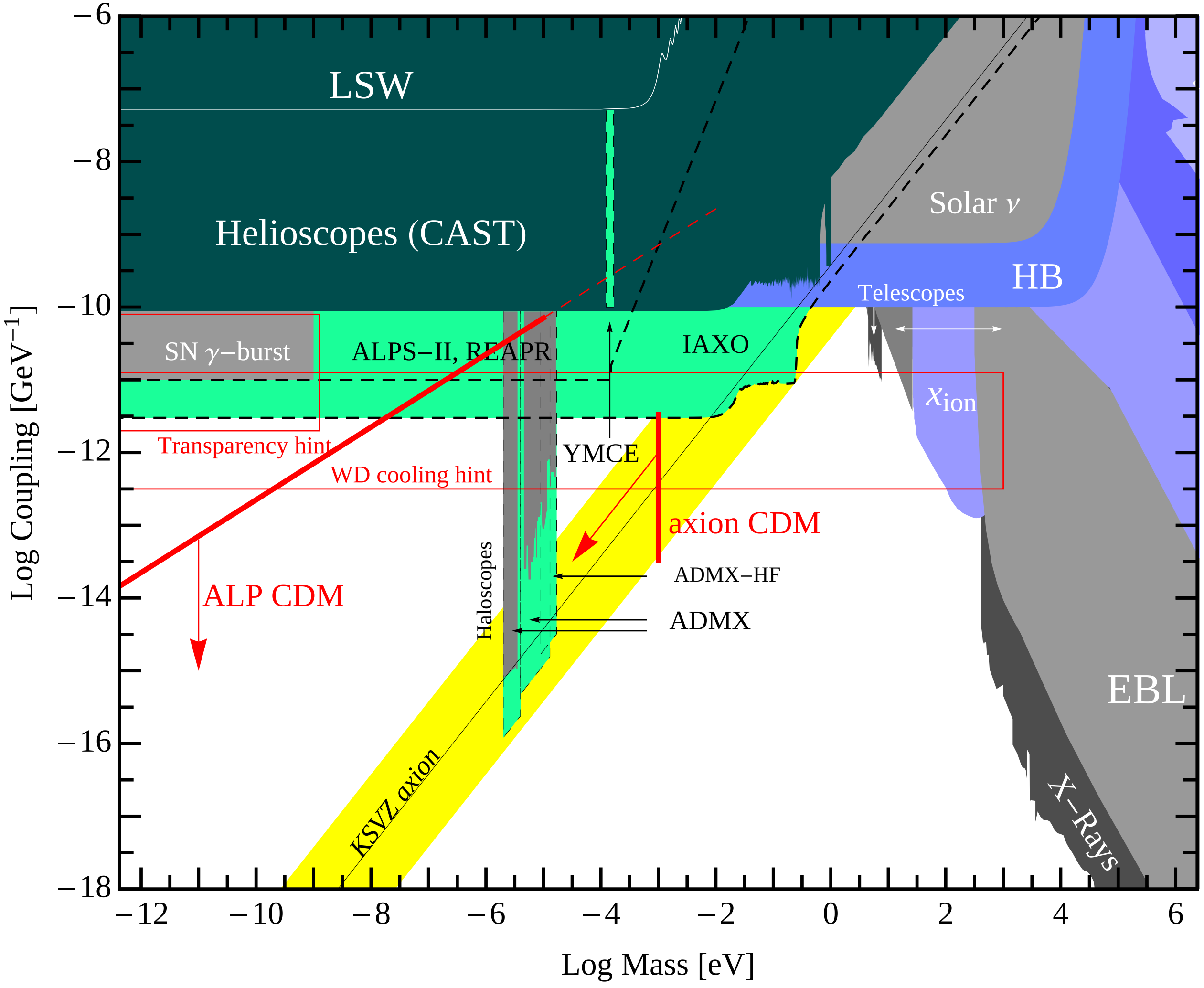
$a\gamma\gamma$ interaction $$C_{a\gamma\gamma}\frac{\alpha}{2\pi}\frac{a(x)}{f_{PQ}}(\vec{E}\cdot\vec{H})$$

Conversion of DM axion into photon in magnetic field in a resonant cavity. 10^{-6} eV/2π = 240 MHz. Need high Q resonator to collect photons, narrow bandwidth, go small steps in m_a. Long story.



ADMX, PRL ’2010

Stay tuned ... and stay ... and stay ...



Ringwald' 2012

Warm dark matter

- Clouds over CDM

  Numerical simulations of structure formation with CDM show

  - Too many dwarf galaxies

    A few hundred satellites of a galaxy like ours —

    But just over dozen observed so far

  - Too high density in galactic centers (“cusps”)

- No serious worry yet

  But what if one really needs to suppress small structures?

High initial momenta of DM particles $\Longrightarrow$ Warm dark matter

Warm dark matter

- Decouples when relativistic, $T_f \gg m$.
- Remains relativistic until $T \sim m$ (assuming thermal distribution). Does not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\Longrightarrow$ small size objects do not form ("free streaming")
- Horizon size at $T \sim m$

$$l(T) = H^{-1}(T \sim m)$$

Digression: expansion at radiation domination

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

  - Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M_{Pl}^*}$$

with $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

Back to warm dark matter

- Assuming thermal velocities, relativistic until $T \sim m$
- Horizon size at $T \sim m$

$$l_H(T) = H^{-1}(T \sim m) \sim \frac{M^*_{Pl}}{T^2} = \frac{M^*_{Pl}}{m^2}$$

Present size of this region

$$l_c = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{m T_0}$$

(modulo g_* factors).

Objects of initial comoving size smaller than l_c are less abundant

- Initial size of dwarf galaxy $l_{dwarf} \sim 100$ kpc $\sim 3 \cdot 10^{23}$ cm
Require

$$l_c \simeq \frac{M_{Pl}}{mT_0} \sim l_{dwarf}$$

$\Longrightarrow$ obtain mass of DM particle

$$m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}$$

($M_{Pl} = 10^{19}$ GeV, $T_0^{-1} = 0.1$ cm).

- Particles of masses in 3 – 30 keV range
are good warm dark matter candidates (assuming they had thermal velocities)

Sterile neutrinos

- Needed to give masses to ordinary neutrinos

See lectures by Zhi-Zhong Xing
and Koichi Hamaguchi.

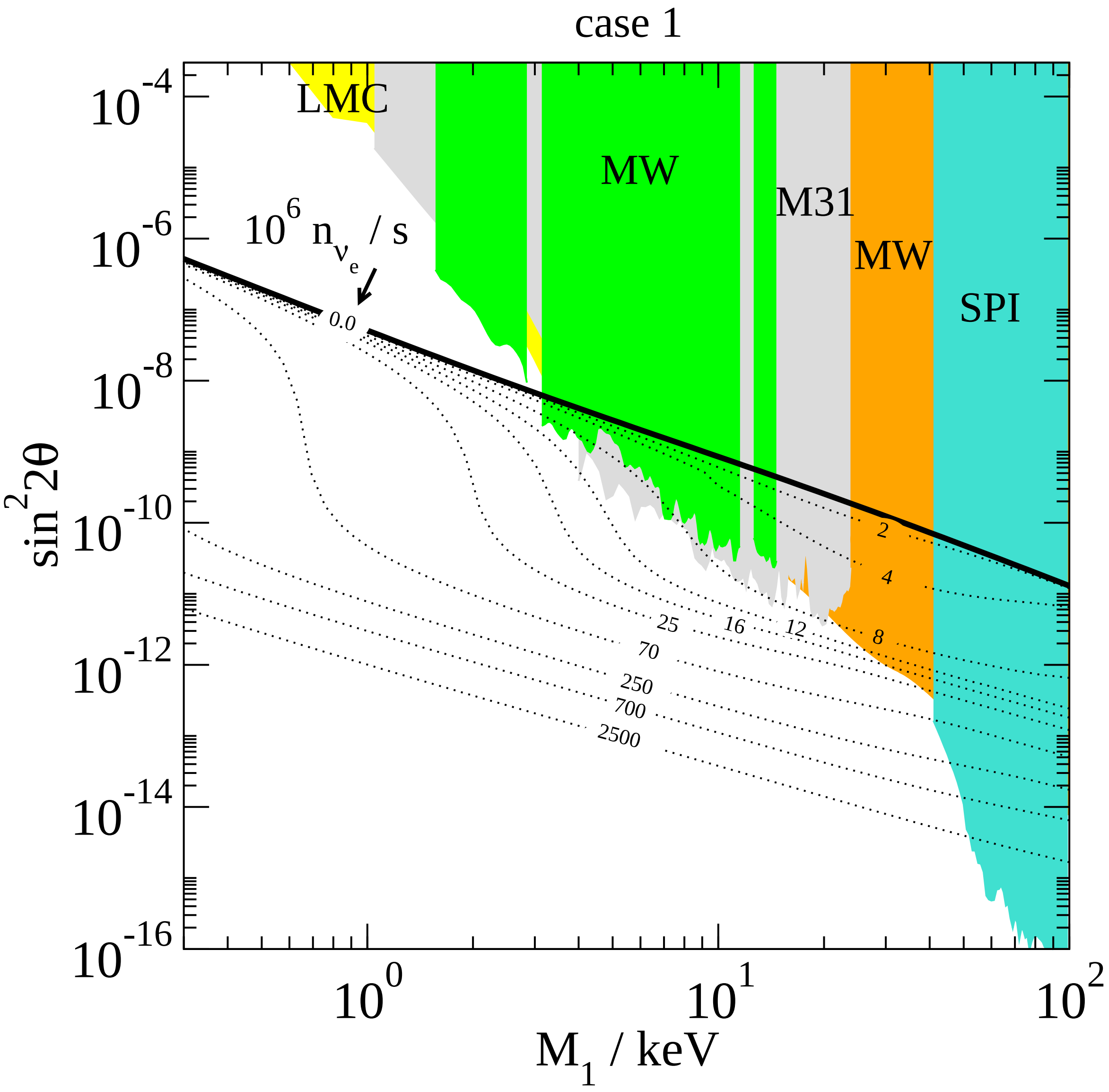
- Nothing wrong with $m_{\nu_s} = 3 - 10$ keV
- Created in early Universe at $T \sim 200$ MeV due to mixing with ordinary neutrinos, mixing angle θ_s. Without lepton asymmetry

$$\Omega_s \simeq 0.2 \cdot \left(\frac{\sin 2\theta_s}{10^{-4}}\right)^2 \cdot \left(\frac{m_{\nu_s}}{1 \text{ keV}}\right)$$

- Long lifetime: $\tau_{\nu_s} \gg 10^{10}$ yrs for $m_{\nu_s} = 3 - 10$ keV, $\sin 2\theta_s = 10^{-4} - 10^{-5}$
- $\nu_s \to \nu\gamma \Longrightarrow$ Search for photons with $E = m_{\nu_s}/2$ from sky.

  Straightforward version of scenario ruled out
  But more contrived (assuming lepton asymmetry) does not

Search for for photons with $E = m_{\nu_s}/2$



Laine' 2009

Gravitinos

- Mass $m_{3/2} \simeq F/M_{Pl}$
  $\sqrt{F} =$ SUSY breaking scale.
  $\Longrightarrow$ Gravitinos light for low SUSY breaking scale.
  E.g. gauge mediation
- Light gravitino = LSP $\Longrightarrow$ Stable
- Correct present mass density for $m_{3/2} \sim 10$ keV, provided that
  - some superpartners are light, $M_{\tilde{S}} \simeq 100 \div 300$ GeV
  - maximum temperature in the Uiverse is low, $T_{max} \lesssim$ (a few) TeV to avoid overproduction in collisions of superpartners (and in decays of heavy squarks and gluinos)

Rather contrived scenario, but generating warm dark matter is always contrived

NB: $\Gamma_{NLSP} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \Longrightarrow c\tau_{NLSP} =$ a few $\cdot$ mm $\div$ a few $\cdot 100$ m

for $m_{3/2} = 1 \div 10$ keV, $M_{\tilde{S}} = 100 \div 300$ GeV

Dark matter summary

- WIMP, signal at the LHC:
  - Strongest possible motivation for direct and indirect detection
    - Inferred interactions with baryons $\Longrightarrow$ strategy for direct detection
  - A handle on the Universe at

$$T = (\text{a few}) \cdot 10 \text{ GeV} \div (\text{a few}) \cdot 100 \text{ GeV}$$

$$t = 10^{-11} \div 10^{-8} \text{ s}$$

cf. $T = 1$ MeV, $t = 1$ s   at nucleosynthesis

- Gravitino-like
  - Find supersymmetry at the LHC first
  - A lot of work to make sure that LSP is gravitino and it is indeed DM particle
  - Hard time for direct and indirect searches
- No signal at the LHC
  - Good guesses: axion, sterile neutrino
  - If not, need more hints from cosmology and astrophysics

Changing geers

Baryon asymmetry of the Universe

- There is matter and no antimatter in the present Universe.
- Baryon-to-photon ratio, almost constant in time:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

Baryon-to-entropy, constant in time: $n_B/s = 0.9 \cdot 10^{-10}$

What's the problem?

Early Universe ($T > 10^{12}$ K $= 100$ MeV):
creation and annihilation of quark-antiquark pairs $\Rightarrow n_q, n_{\bar{q}} \approx n_\gamma$
Hence

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

How was this excess generated in the course of the cosmological evolution?

Sakharov'67, Kuzmin'70

Sakharov conditions

To generate baryon asymmetry, three necessary conditions should be met at the same cosmological epoch:

- B-violation
- C- and CP-violation
- Thermal inequilibrium

NB. Reservation: L-violation with B-conservation at $T \gg 100$ GeV would do as well $\Longrightarrow$ Leptogenesis.

Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.
"Sphalerons".

Non-perturbative effect

Hint: triangle anomaly in baryonic current B^μ:

$$\partial_\mu B^\mu = \left(\frac{1}{3}\right)_{B_q} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

$F^a_{\mu\nu}$: $SU(2)_W$ field strength; g_W: $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_\mu L^\mu_n = \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

Large field fluctuations, $F^a_{\mu\nu} \propto g_W^{-1}$ may have

$$Q \equiv \int d^3x dt \; \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \neq 0$$

Then

$$B_{fin} - B_{in} = \int d^3x dt \; \partial_\mu B^\mu = 3Q$$

Likewise

$$L_{n,\; fin} - L_{n,\; in} = Q$$

B is violated, $B - L$ is not.

How can baryon number be not conserved without explicit B-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\vec{A}(\mathbf{x},t)$ (gauge $A_0 = 0$). Let $\vec{A}(\mathbf{x},t)$ start from vacuum value and end up in vacuum.

NB: This can be a fluctuation

Dirac equation

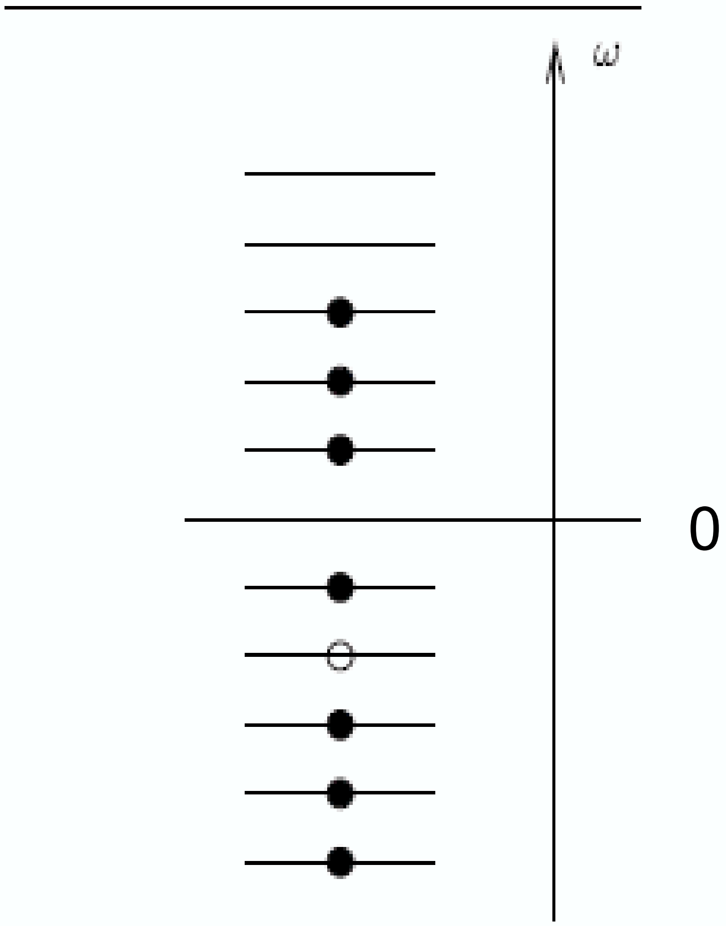
$$i\frac{\partial}{\partial t}\psi = i\gamma^0\vec{\gamma}(\vec{\partial} - ig\vec{A})\psi = H_{Dirac}(t)\psi$$

Suppose for the moment that $\vec{A}$ slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{Dirac}(t)\psi_n = \omega_n(t)\psi_n$$

How do eigenvalues behave in time?

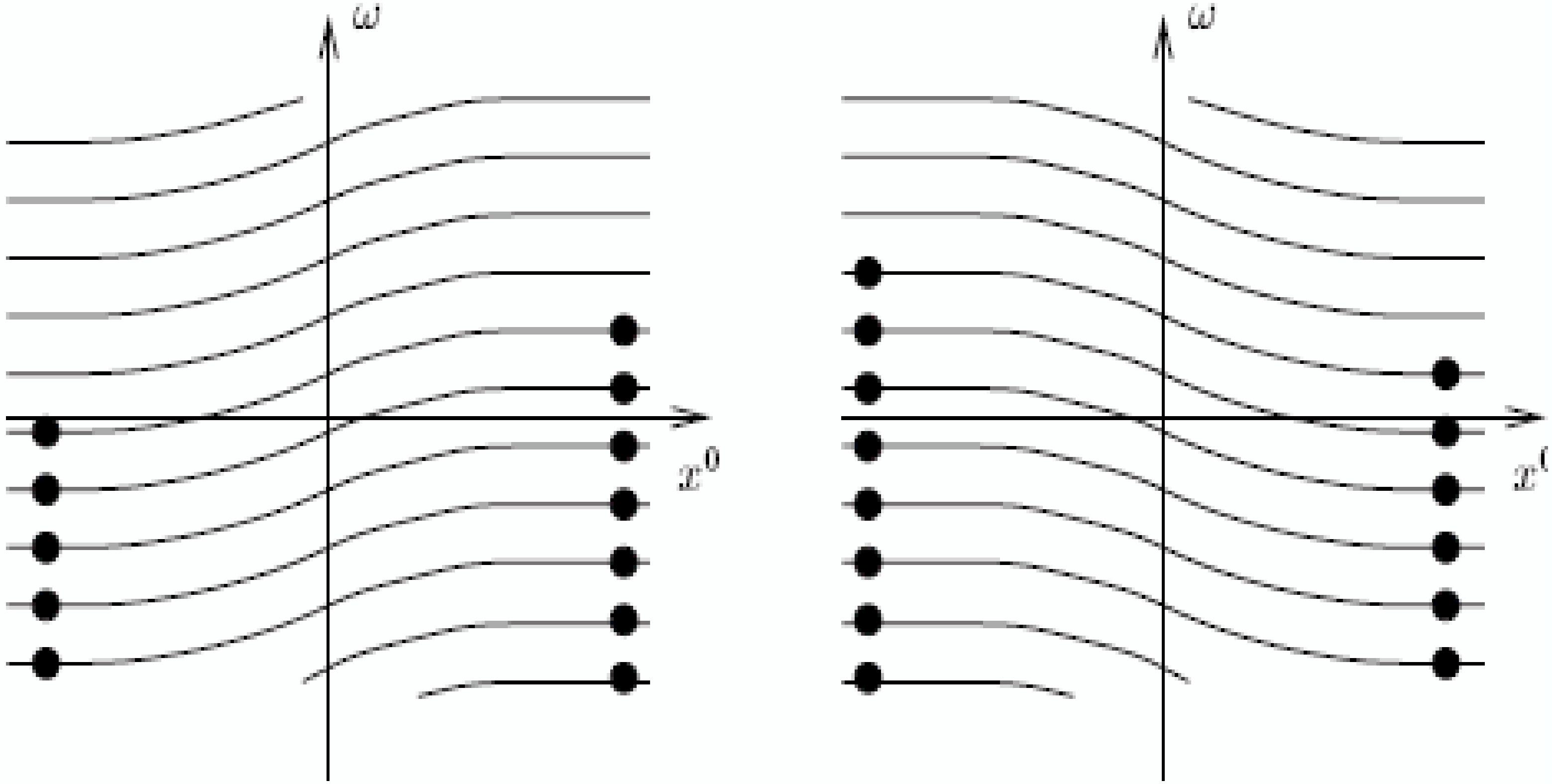
Dirac picture at $\vec{A} = 0$, $t \to \pm\infty$



TIME EVOLUTION OF LEVELS

IN SPECIAL (TOPOLOGICAL) GAUGE FIELDS

Left-handed fermions   Right-handed

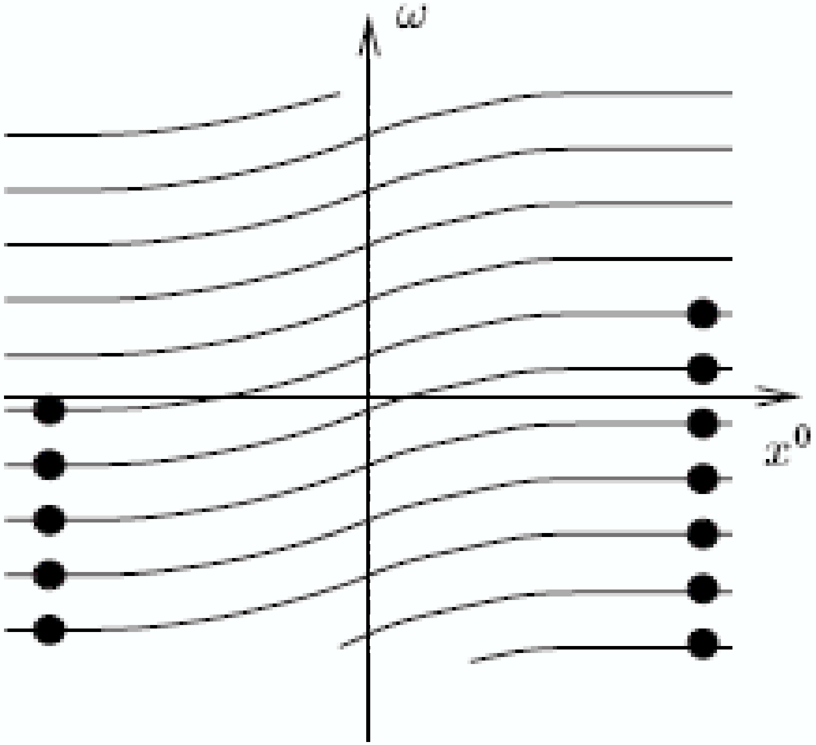


The case for QCD

$B = N_L + N_R$ is conserved, $Q^5 = N_L - N_R$ is not

If only left-handed fermions interact with gauge field,

then number of fermions is not conserved



The case for $SU(2)_W$

Fermion number of every doublet changes by equal amount

NB: Non-Abelian gauge fields only (in 4 dimensions)

QCD: Violation of Q^5 is a fact.

In chiral limit $m_u, m_d, m_s \to 0$,
global symmetry is $SU(3)_L \times SU(3)_R \times U(1)_B$,
not symmetry of Lagrangian $SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$

Need large field fluctuations. At zero temprature their rate is suppressed by

$$e^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

High temperatures: large thermal fluctuations (“sphalerons”).

B-violation rapid as compared to cosmological expansion at

$$\langle \phi \rangle_T < T$$

$\langle \phi \rangle_T$: Higgs expectation value at temperature T.

Lecture 3

Outline of Lecture 3

- Electroweak baryogenesis. What can make it work?
- Before the hot epoch
  - Cosmological perturbations
    - Regimes of evolution
    - Acoustic oscillations: evidence for pre-hot epoch
  - Inflation and alternatives
  - BICEP-2 saga
- Conclusions

Baryogenesis, cont'd

- Bottom line from Lecture 2:

  B and L are violated in electroweak interactions
  $(B-L)$ is conserved

- Non-perturbative effect, needs large field fluctuations. At zero temprature their rate is suppressed by

$$\mathrm{e}^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

  High temperatures: large thermal fluctuations ("sphalerons").

  B-violation rapid as compared to cosmological expansion at

$$\langle\phi\rangle_T < T$$

  $\langle\phi\rangle_T$: Higgs expectation value at temperature T.

  Possibility to generate baryon asymmetry at electroweak epoch, $T_{EW} \sim 100$ GeV ?

Problem: Universe expands slowly. Expansion time

$$H^{-1} \sim 10^{-10} \text{ s}$$

Too large to have deviations from thermal equilibrium?

The only chance: 1st order phase transition, highly inequilibrium process

Electroweak symmetry is restored, $\langle\phi\rangle_T = 0$ at high temperatures

Just like superconducting state becomes normal at "high" T

Transition may in principle be 1st order

Fig

1st order phase transition occurs from supercooled state via spontaneous creation of bubbles of new (broken) phase in old (unbroken) phase.

Bubbles then expand at $v \sim 0.1c$

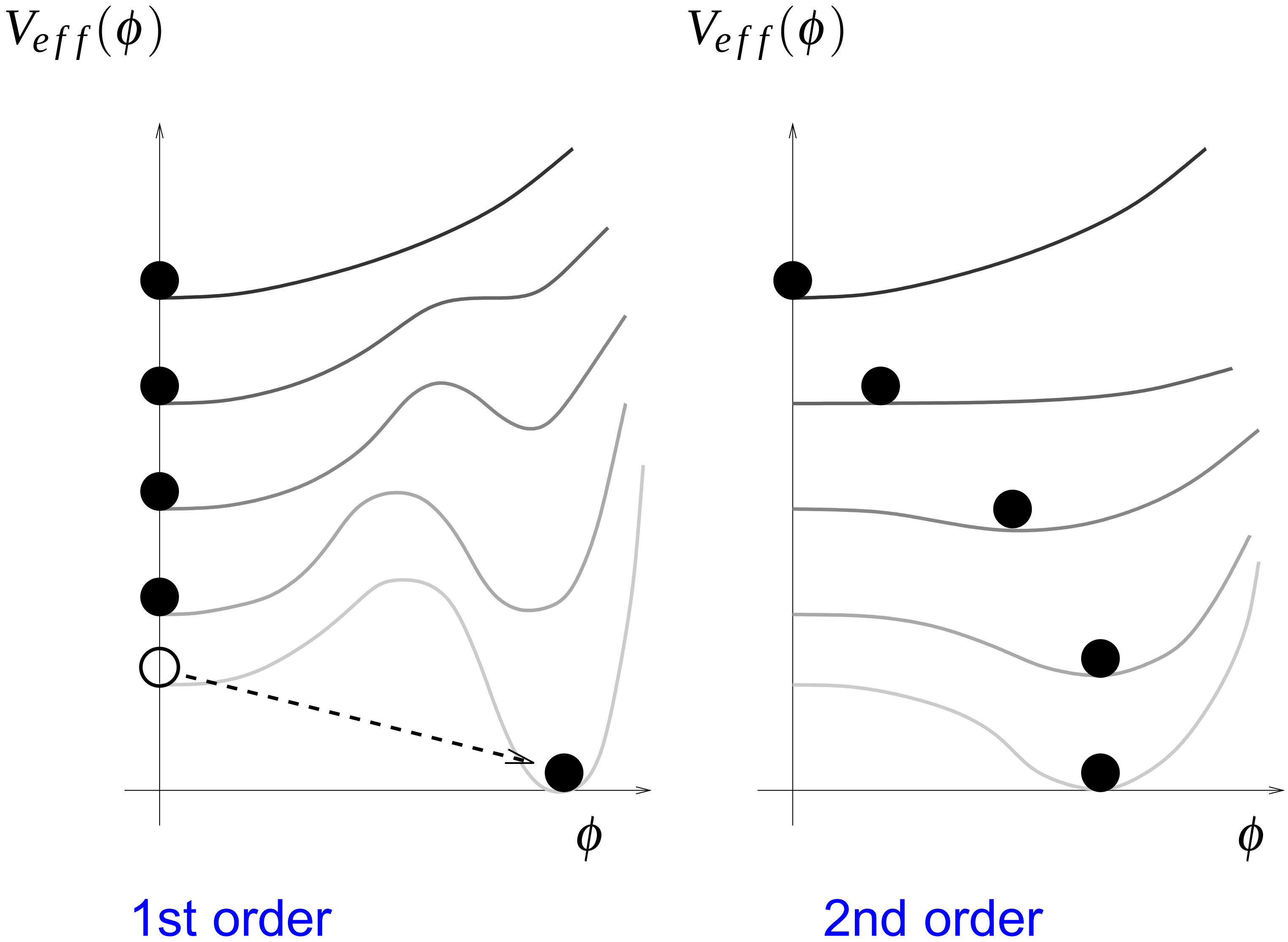
Fig

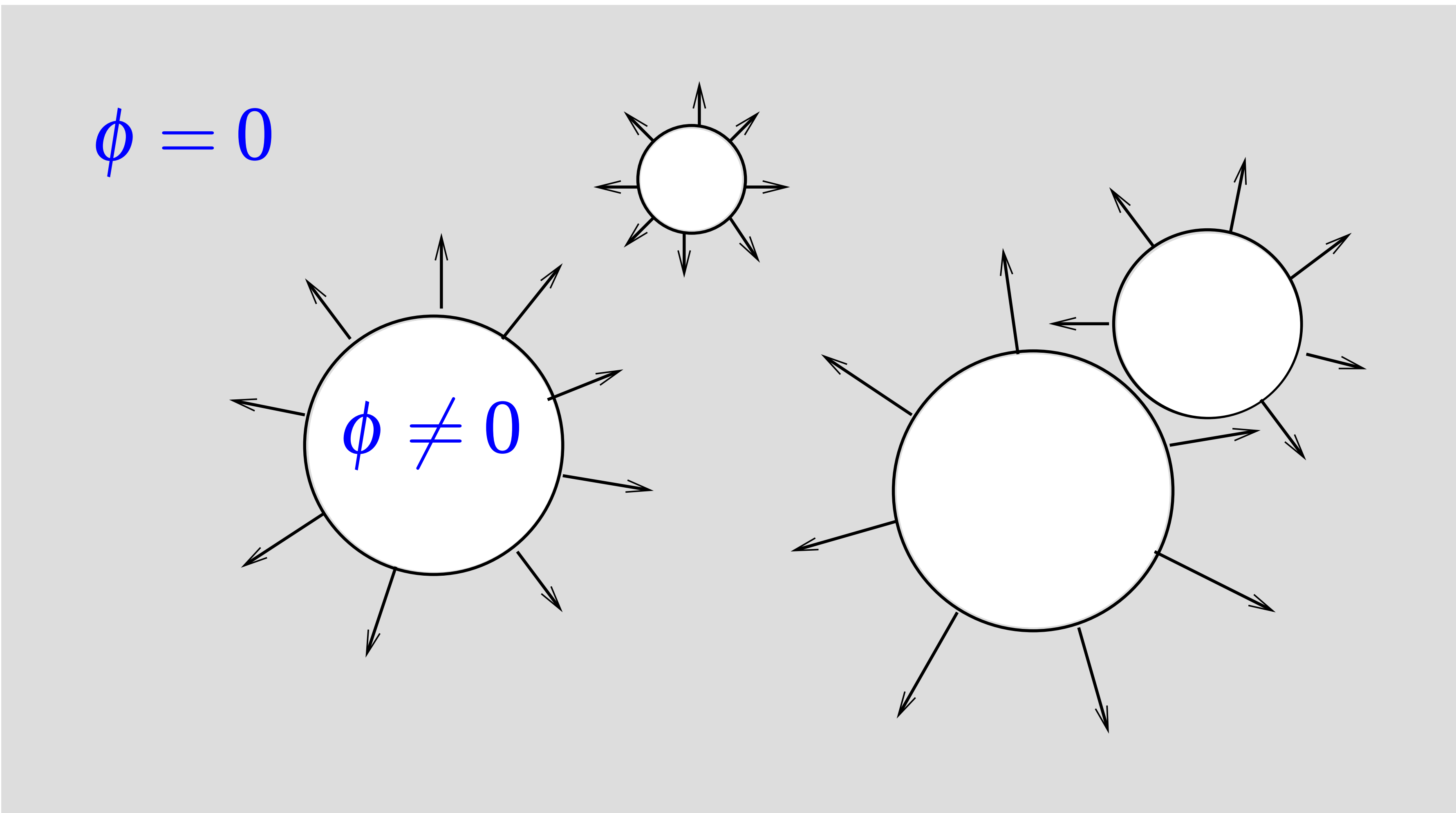
Beginning of transition: about one bubble per horizon

Bubbles born microscopic, $r \sim 10^{-16}$ cm, grow to macroscopic size, $r \sim 0.1H^{-1} \sim 1$ mm, before their walls collide

Boiling Universe, strongly out of equilibrium

$V_{eff}(\phi)$ = free energy density




$\phi = 0$
$\phi \neq 0$

Baryon asymmetry may be generated in the course of 1st order phase transition, provided there is enough C- and CP-violation.

Does this really happen?

Not in SM

- Given the Higgs boson mass

$$m_H = \sqrt{2\lambda} v = 126 \text{ GeV}$$

No phase transition at all; smooth crossover

- Way too small CP-violation

What can make EW mechanism work?

- Extra bosons
  - Should interact fairly strongly with Higgs(es)
  - Should be present in plasma at $T \sim 100$ GeV

    $\Longrightarrow$ not much heavier than 300 GeV

  E.g. light stop

- Plus extra source of CP-violation.

  Better in Englert–Brout–Higgs sector $\Longrightarrow$ Several scalar fields

More generally, EW baryogenesis requires
complex dynamics in EW symmetry breaking sector
at $E \sim$ (a few) $\cdot 100$ GeV

LHC's FINAL WORD

Is EW the only appealing scenario?

By no means!

— Leptogenesis

— Something theorists never thought about

Why $\Omega_B \approx \Omega_{DM}$?

Changing geers

Before the hot epoch

With Big Bang nucleosynthesis theory and observations

we are confident of the theory of the early Universe

at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the LHC, we hope to be able to go

up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Linear analysis appropriate.

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We know that this is not the whole story!

Properties of perturbations in conventional ("hot") Universe.

Reminder:

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^{\,2}$$

$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV, $t \simeq 60$ thousand years)
$a(t) \propto t^{2/3}$ at matter domination stage (until recently).

Cosmological horizon at time t (assuming that nothing preceeded hot epoch): distance that light travels from Big Bang moment,

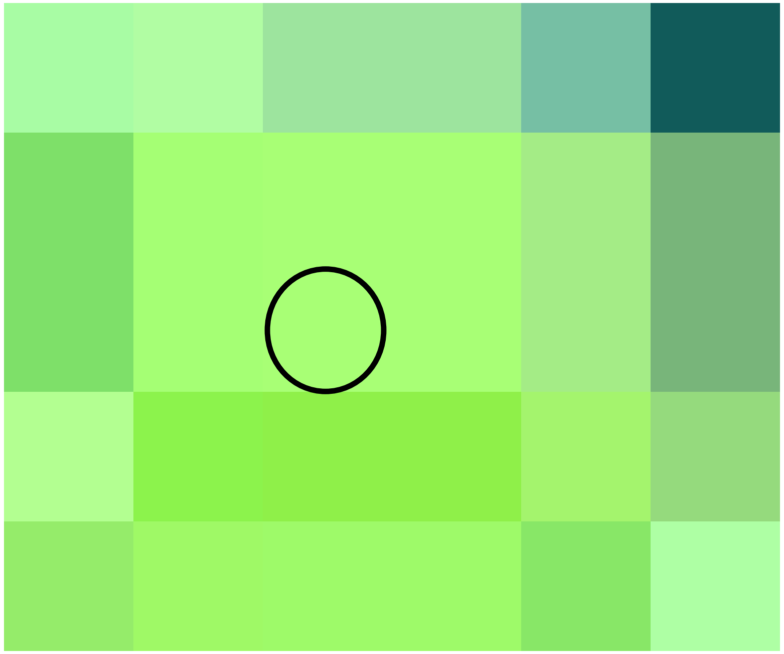
$$l_{H,t} \sim H^{-1}(t) \sim t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

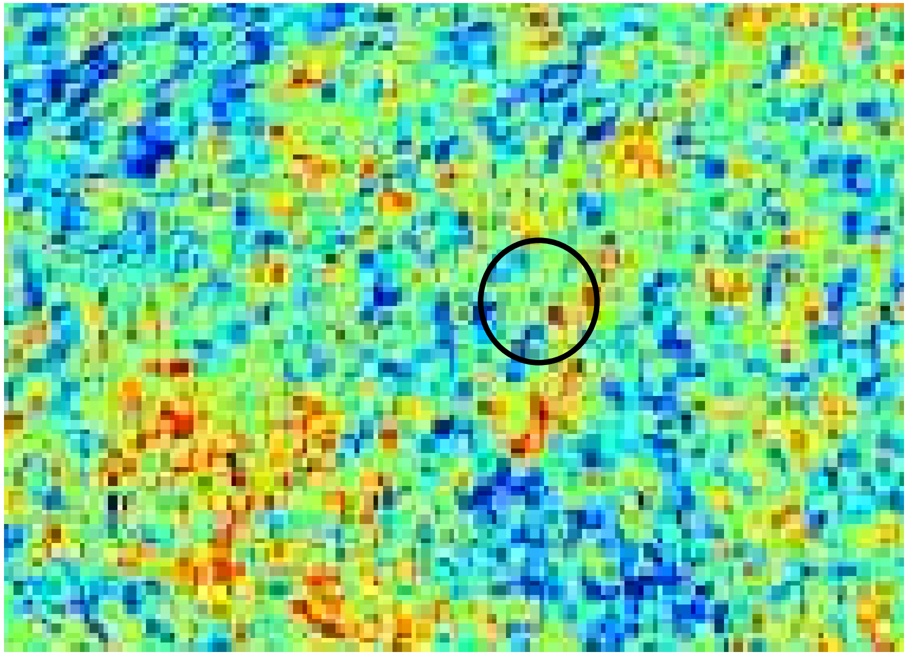
$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_{H,t} \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



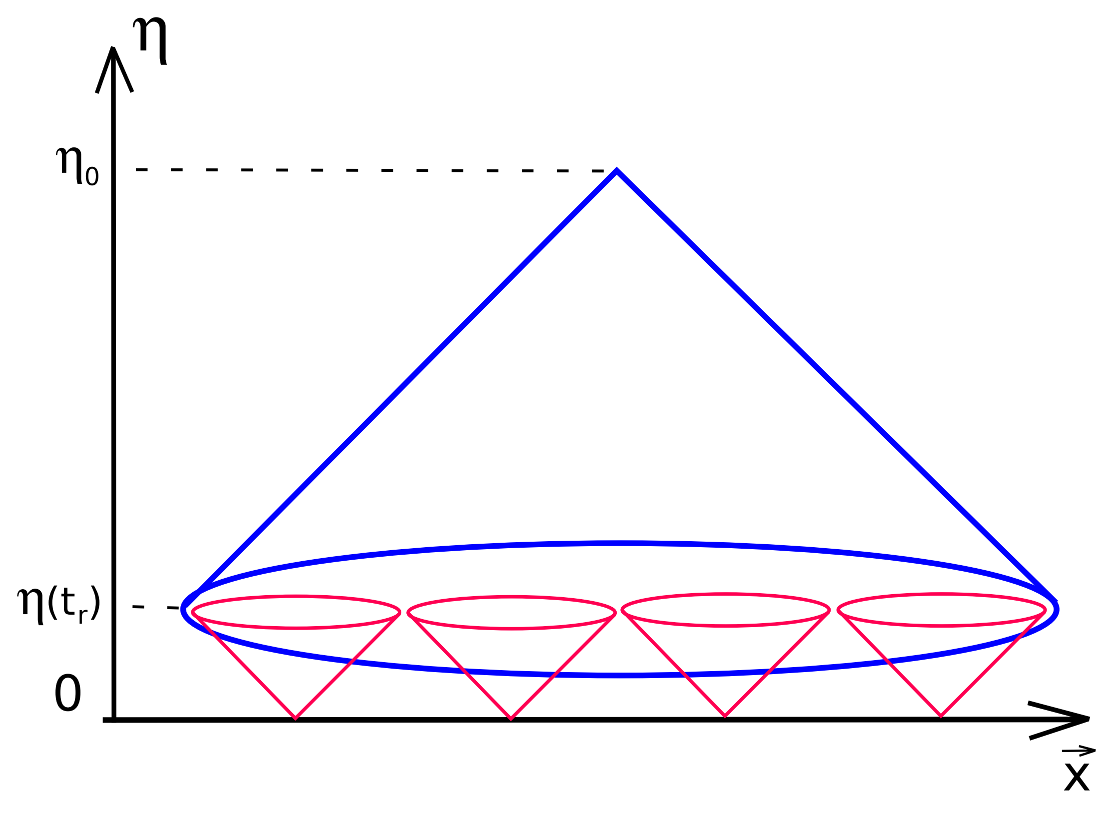
superhorizon mode



subhorizon mode

Causal structure of space-time in hot Big Bang theory (no inflation or anything else before the hot epoch)

$$\eta = \int \frac{dt}{a(t)} \,,$$ conformal time



Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  The only possibility, if expansion started from hot Big Bang.

No longer an option!

- Hot epoch was preceeded by some other epoch.
  Perturbations were generated then.

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase. Why?

Subhorion regime (late times): acoustic oscillations

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\mathrm{e}^{i\vec{k}\vec{x}}\cos\left(\int_0^t v_s\,\frac{k}{a(t)}\,dt + \psi\right), \qquad \psi = \text{ arbitrary phase}$$

NB: Physicsl distance $dl = adx \Longleftrightarrow$ physical momentum k/a, gets redshifted.

Sound velocity $v_s \approx 1/\sqrt{3}$.

Solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \quad \text{and} \quad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. Consistency of the picture: the Universe was not very inhomogeneous at early times, the initial condition is (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations well defined.

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\mathrm{e}^{i\vec{k}\vec{x}}\cos\left(\int_0^t v_s \frac{k}{a(t)}\,dt\right),$$

no arbitrary phase

Perturbations come to the time of photon last scattering (= recombination) at different phases, depending on wave vector:

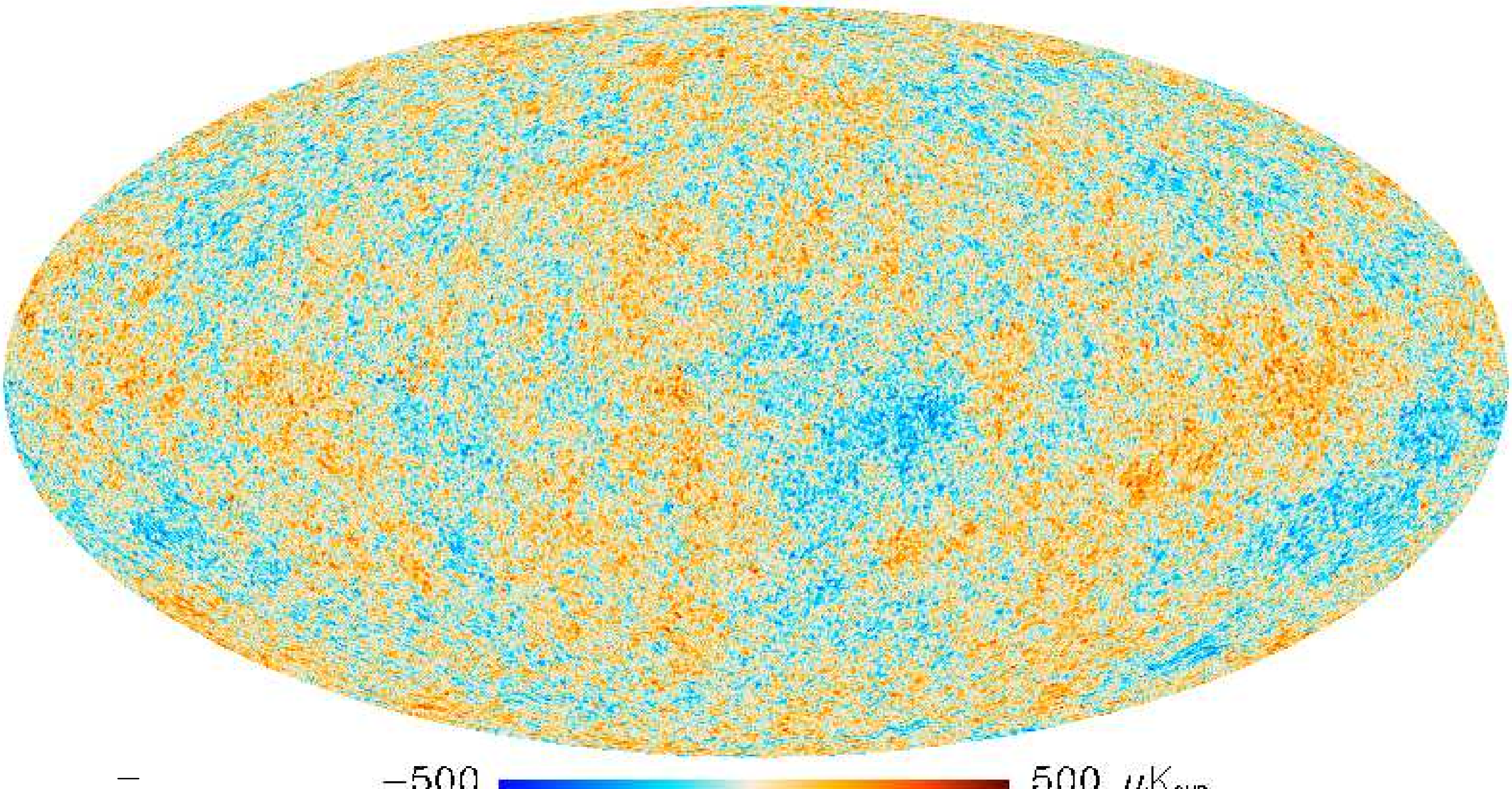
$$\delta(t_r) \equiv \frac{\delta\rho}{\rho}(t_r) \propto \cos\left(k\int_0^{t_r} dt\ \frac{v_s}{a(t)}\right) = \cos(kr_s)$$

r_s: sound horizon at recombination, $a_0 r_s = 150$ Mpc.

Waves with $k = \pi n / r_s$ have large $|\delta\rho|$, while waves with $k = (\pi n + 1/2)/r_s$ have $|\delta\rho| = 0$ in baryon-photon component.

This translates into oscillations in CMB angular spectrum

$$T = 2.726^\circ K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

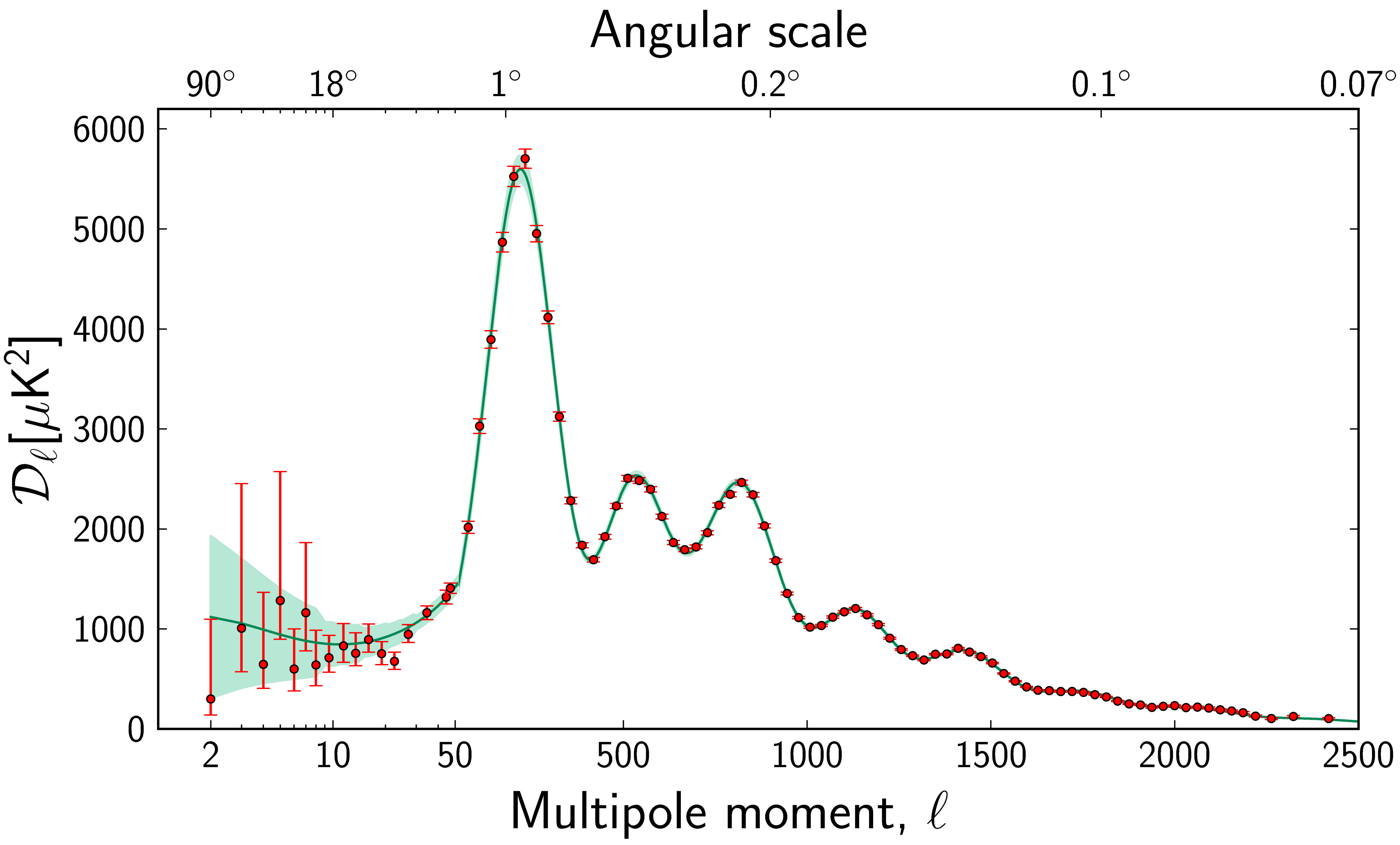
Fourier decomposition of temperatue fluctuations:

$$\frac{\delta T}{T}(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

a_{lm}: independent Gaussian random variables, $\langle a_{lm} a^*_{l'm'} \rangle \propto \delta_{ll'} \delta_{mm'}$
$\langle a^*_{lm} a_{lm} \rangle = C_l$ are measured; usually shown $D_l = \frac{l(l+1)}{2\pi} C_l$
larger l $\Longleftrightarrow$ smaller angular scales, shorter wavelengths
NB: One Universe, one realization of an ensemble $\Longrightarrow$ cosmic variance $\Delta C_l / C_l \simeq 1/\sqrt{2l}$

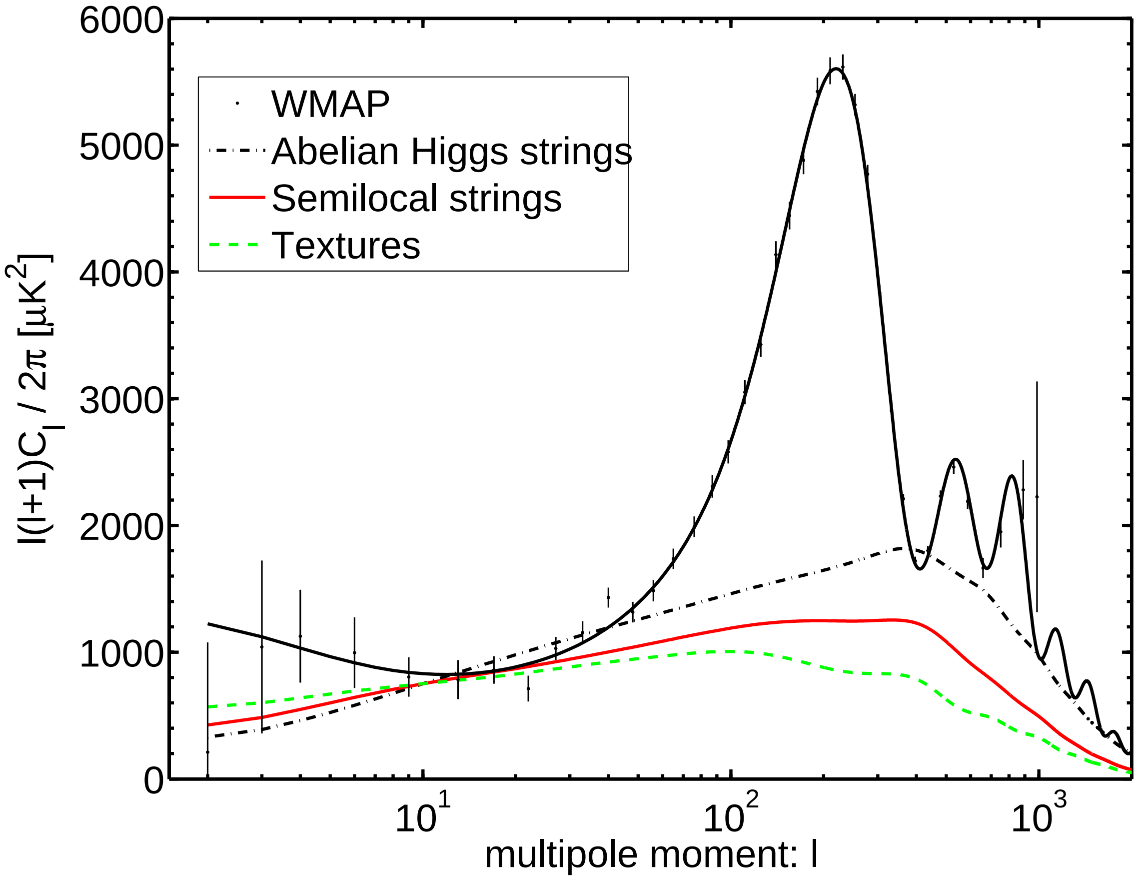
- Physics:
  - Primordial perturbations
  - Development of sound waves in cosmic plasma from early hot stage to recombination
    $\Longrightarrow$ composition of cosmic plasma
  - Propagation of photons after recombination
    $\Longrightarrow$ expansion history of the Universe

CMB angular spectrum



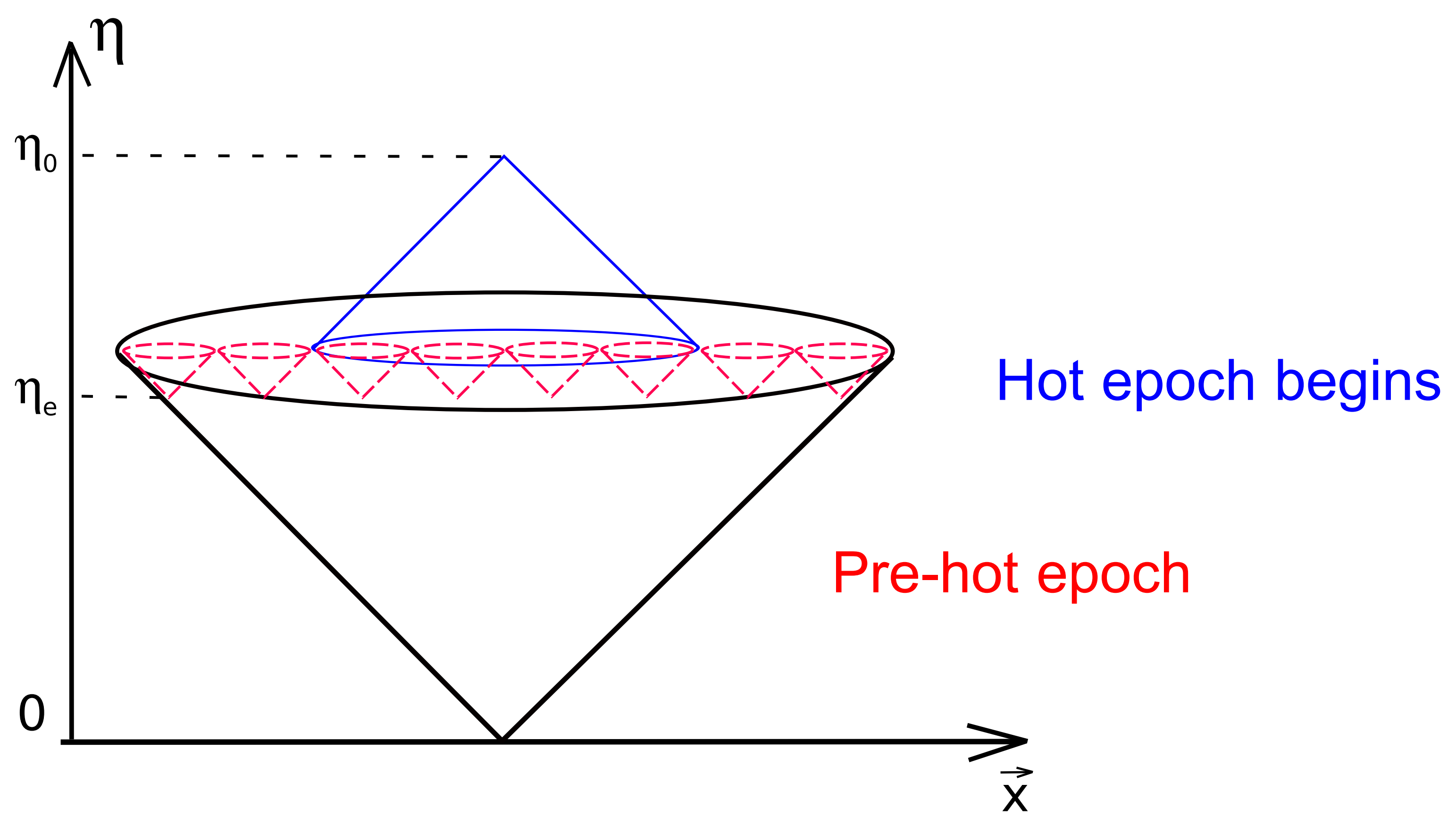
Furthermore, there are perturbations which were superhorizon at the time of photon last scattering (low multipoles, $l \lesssim 50$)

These properties would not be present if perturbations were generated at hot epoch in causal manner: phase ψ would be random function of k, no oscillations in CMB angular spectrum.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long (in conformal time) and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.



Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt}, \qquad H \approx \text{const}$$

- Initially Planck-size region expands to entire visible Universe in $t \sim 100\ H^{-1} \Longrightarrow$ for $t \gg 100\ H^{-1}$ the Universe is VERY large
- Perturbations subhorizon early at inflation:

$$\lambda(t) = 2\pi \frac{a(t)}{k} \ll H^{-1}$$

since $a(t) \propto \mathrm{e}^{Ht}$ and $H \approx$ const;
wavelengths gets redshifted, the Hubble parameter stays constant

Alternatives to inflation:

Contraction — Bounce — Expansion,

Start up from static state ("Genesis")

Difficult, but not impossible.

Other suggestive observational facts about density perturbations
(valid within certain error bars!)

- Perturbations in overall density, not in composition (jargon: “adiabatic”)

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: “isocurvature” or “entropy”) $\Longrightarrow$ wrong prediction for CMB angular spectrum $\Longleftrightarrow$ strong constraints from Planck.

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation.

- Primordial perturbations are Gaussian.

Gaussian random field $\delta(\mathbf{k})$: correlators obey Wick's theorem,

$$\begin{aligned} \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0 \\ \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle\cdot\langle\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle \\ &+ \text{ permutations of momenta} \end{aligned}$$

- $\langle\delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.
- strong hint on the origin:
  enhanced vacuum fluctuations of free quantum field
  Free quantum field

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{kx}} \left(f_{\mathbf{k}}^{(+)}(t) a_{\mathbf{k}}^{\dagger} + e^{i\mathbf{kx}} f_{\mathbf{k}}^{(-)}(t) a_{\mathbf{k}} \right)$$

  In vacuo $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$

  Enhanced perturbations: large $f_{\mathbf{k}}^{(\pm)}$. But in any case, Wick's theorem valid

- Inflation does the job very well: vacuum fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

  Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

  Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

- Enhancement of vacuum fluctuations is less automatic in alternative scenarios

- Non-Gaussianity: big issue
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum (3-point function; vanishes for Gaussian field)

$$\langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$$

    Shape of $G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$ different in different models $\Longrightarrow$ potential discriminator.

  - In some models bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.

Non-Gaussianity has not been detected yet
strong constraints from Planck

Primordial power spectrum is nearly flat

Homogeneity and anisotropy of Gaussian random field:

$$\langle \frac{\delta\rho}{\rho}(\vec{k}) \frac{\delta\rho}{\rho}(\vec{k}')\rangle = \frac{1}{4\pi k^3} \mathscr{P}(k) \delta(\vec{k}+\vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle \left(\frac{\delta\rho}{\rho}(\vec{x})\right)^2 \rangle = \int_0^\infty \frac{dk}{k}\, \mathscr{P}(k)$$

Flat spectrum: $\mathscr{P}$ is independent of k

Harrison' 70; Zeldovich' 72, Peebles,Yu' 70

Parametrization

$$\mathscr{P}(k) = A \left(\frac{k}{k_*}\right)^{n_s - 1}$$

$A =$ amplitude, $(n_s - 1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow n_s = 1$.
Experiment: $n_s = 0.96 \pm 0.01$ (WMAP, Planck, ...)

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Relevant symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

- Alternative: conformal symmetry

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.
$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10.

Exploratory stage: toy models + general arguments so far.

- Tensor modes = primordial gravitational waves

Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models
but not alternatives to inflation

Smoking gun for inflation

BICEP-2 saga

Power spectra of tensor (gravity waves) and scalar perturbations (per log interval of momenta=wave numbers)

$$\mathscr{P}_T = \frac{16}{\pi}\frac{H_{infl}^2}{M_{Pl}^2} = \frac{128}{3}\frac{\rho_{infl}}{M_{Pl}^4}, \quad \mathscr{P}_s = 2.5 \cdot 10^{-9}$$

Notation: tensor-to-scalar ratio

$$r = \frac{\mathscr{P}_T}{\mathscr{P}_s}$$

Scalar spectral index

$$\mathscr{P}_s(k) = \mathscr{P}_s(k_*) \cdot \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Predictions of inflationary models

Assume power-law inflaton potential $V(\phi) = g\phi^n$. Then

$$r = \frac{4n}{N_e} \qquad n_s - 1 = -\frac{n+2}{2N_e}$$

$$N_e = \ln \frac{a_e}{a_\times} = 50 - 60$$

$a_e =$ scale factor at the end of inflation

$a_\times =$ scale factor at the time when our visible Universe exits the horizon at inflation.

Tensor perturbations = gravity waves

Metric perturbations

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$h_{ij} = h_{ij}(\vec{x}, t)$, $h^i_i = \partial_i h^i_j = 0$, spin 2.

Gravity waves: effects on CMB

- Temperature anisotropy (in addition to effect of scalar perturbations)

V.R., Sazhin, Veryaskin' 1982; Fabbri, Pollock' 83

WMAP, Planck

NB: gravity wave amplitudes are time-independent when superhorizon and decay as $h_{ij} \propto a^{-1}(t)$ in subhorizon regime.

Strongest contribution to δT at large angles

$\Delta\theta \gtrsim 2^o$, $l \lesssim 50$, Present wavelengths ~ 1 Gpc

- Polarization

Basko, Polnarev' 1980; Polnarev' 1985; Sazhin, Benitez' 1995

especially B-mode

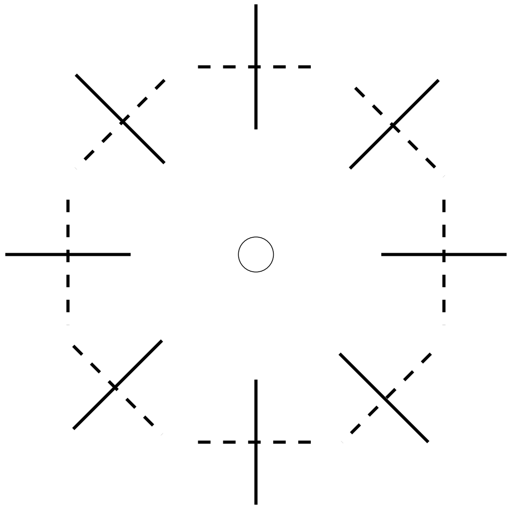
Kamionkowski, Kosowski, Stebbins' 1997; Seljak, Zaldarriaga' 1997

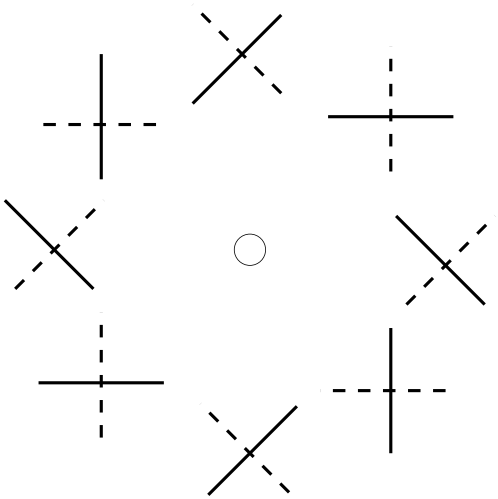
Weak signal, degree of polarization $P(l) \propto l$ at $l \lesssim 50$ and decays with l at $l > 50$.

Amplitude at $r = 0.2$:

$P(l \sim 30) \sim 3 \cdot 10^{-8} \Longrightarrow P \cdot T \sim 0.1\ \mu$K

Linear polariation: E- and B-modes





E-mode, parity even

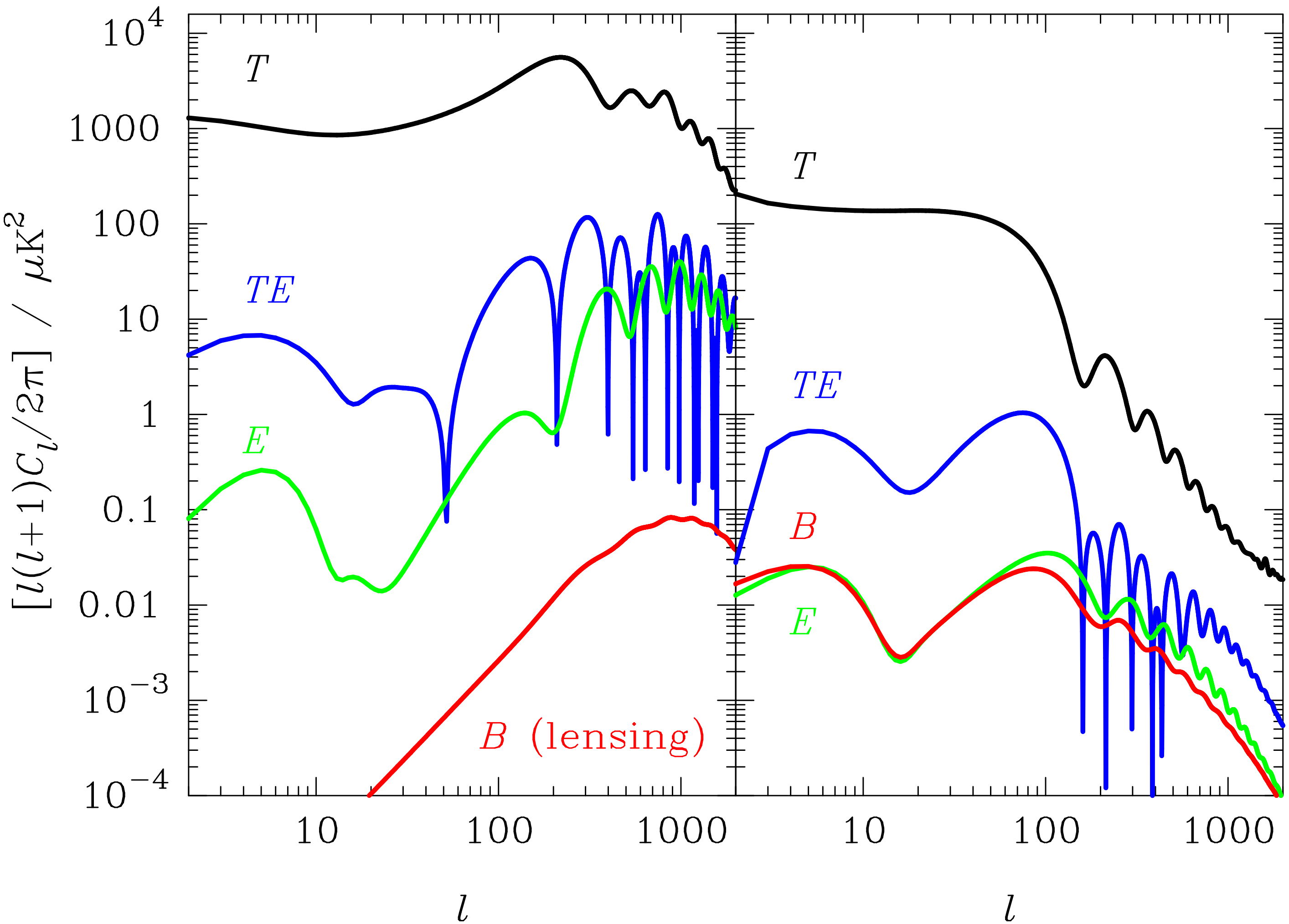
B-mode, parity odd

From both scalar
and tensor perturbations

From tensors only

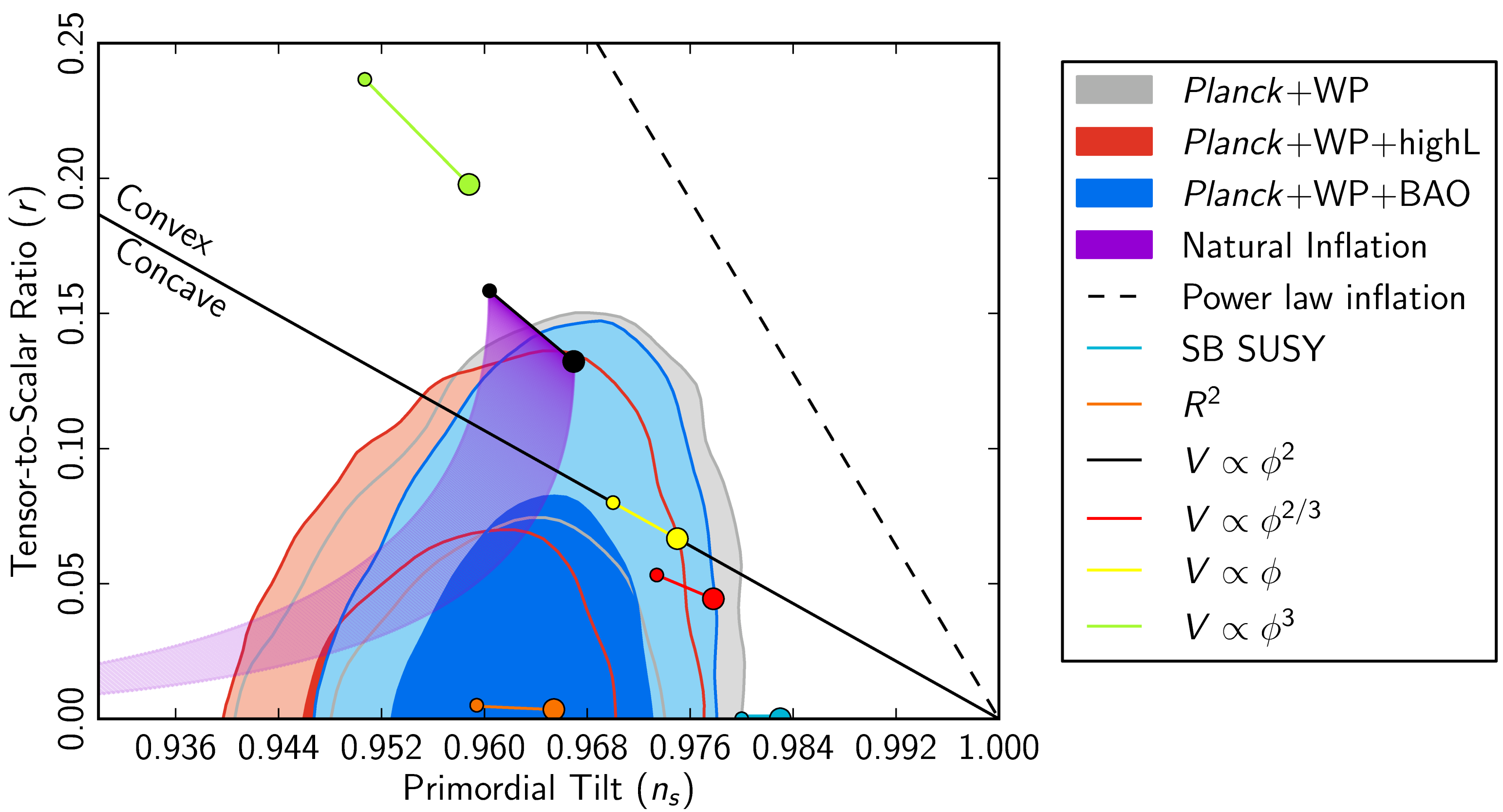
(+ lensing by structures
at relatively small angular scales)

Effects of scalars (left) and tensors (right)



Planck-2013 + everybody else

Scalar spectral index vs. power of tensors

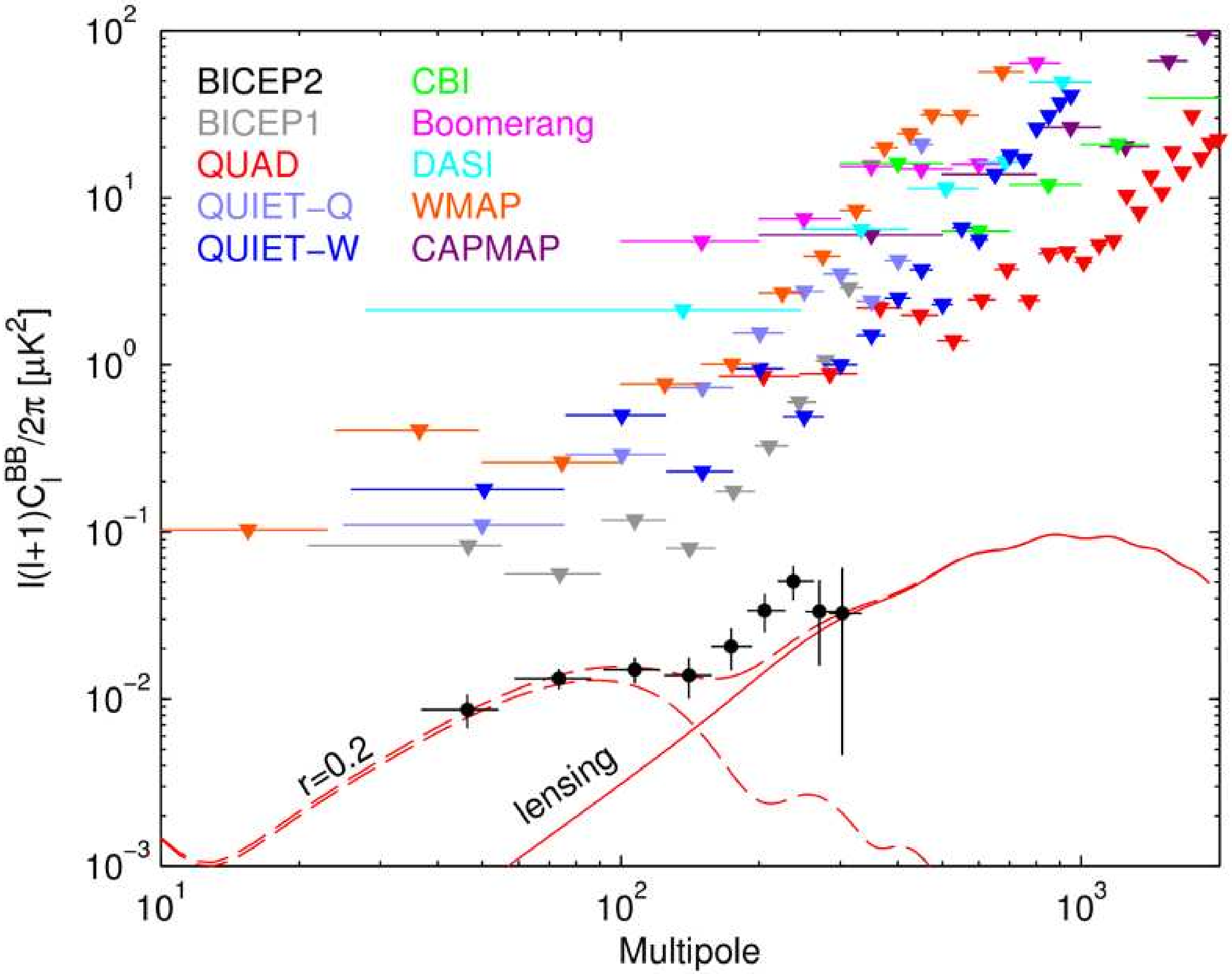


BICEP-2 at South pole

- 590 days of data taking
- Sky region of 390 square degrees towards Galactic pole
- One frequency 150 GHz
- March 2013: claim of discovery of CMB polarization generated by relic gravity waves

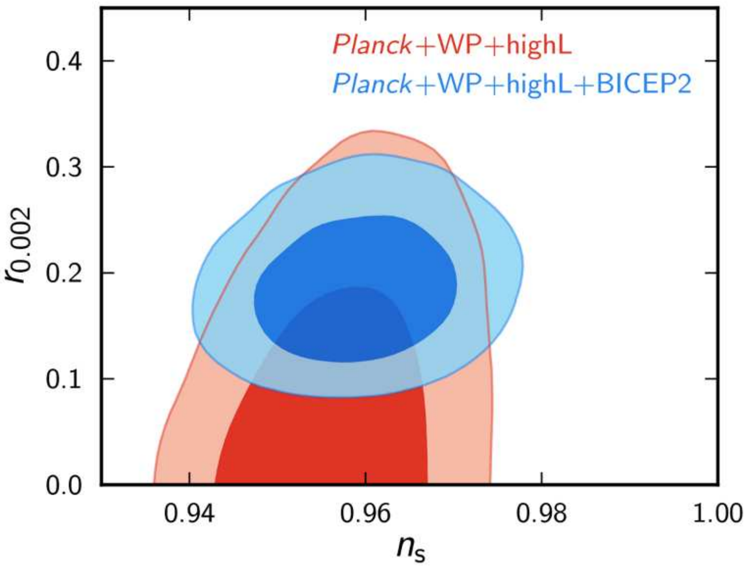
BICEP-2 result

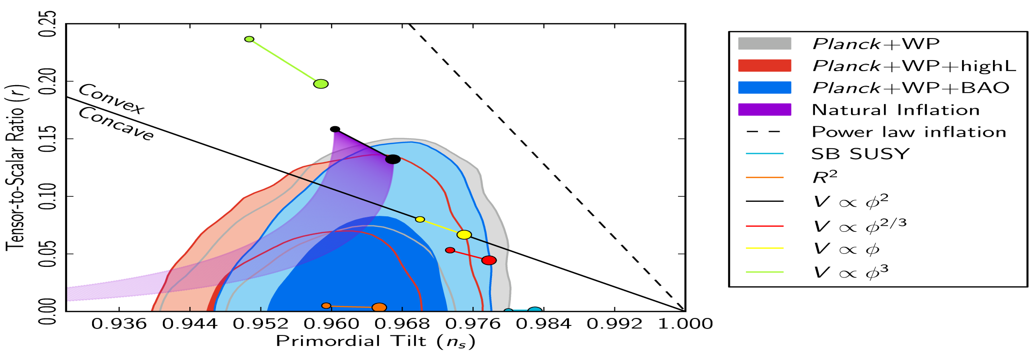
- $30 < l < 150, \quad r = 0.2^{+0.07}_{-0.05}, \quad r \neq 0 \; > 5\sigma$



Tension between BICEP-2 and Planck:

- $r = 0.2$ is large: 10% contribution to δT at low multipoles $l \lesssim 30$.





BICEP-2 and Planck with $dn_s/d\ \ln k = -0.02$ (very large!)
Inflation: $dn_s/d\ \ln k \approx -0.001$

Planck + others

Were this the discovery, then

- Proof of inflation
- $\rho_{infl}^{1/4} = 2 \cdot 10^{16}$ GeV
- Experimental proof of linearized quantum gravity (no wonder!)

In future:

Tensor spectral index $\Longrightarrow$ consistency relation in single field inflation

$$n_T = -\frac{r}{8}$$

Signal is there.

Are there relic gravity waves???

Dangerous “foreground”: polarized dust in our Galaxy, $r \sim 0.1\ \mu$m

Oriented by Galactic magnetic field, emits polarized radiation (way to study magnetic fields in our Galaxy)

Dominates completely at high frequencies

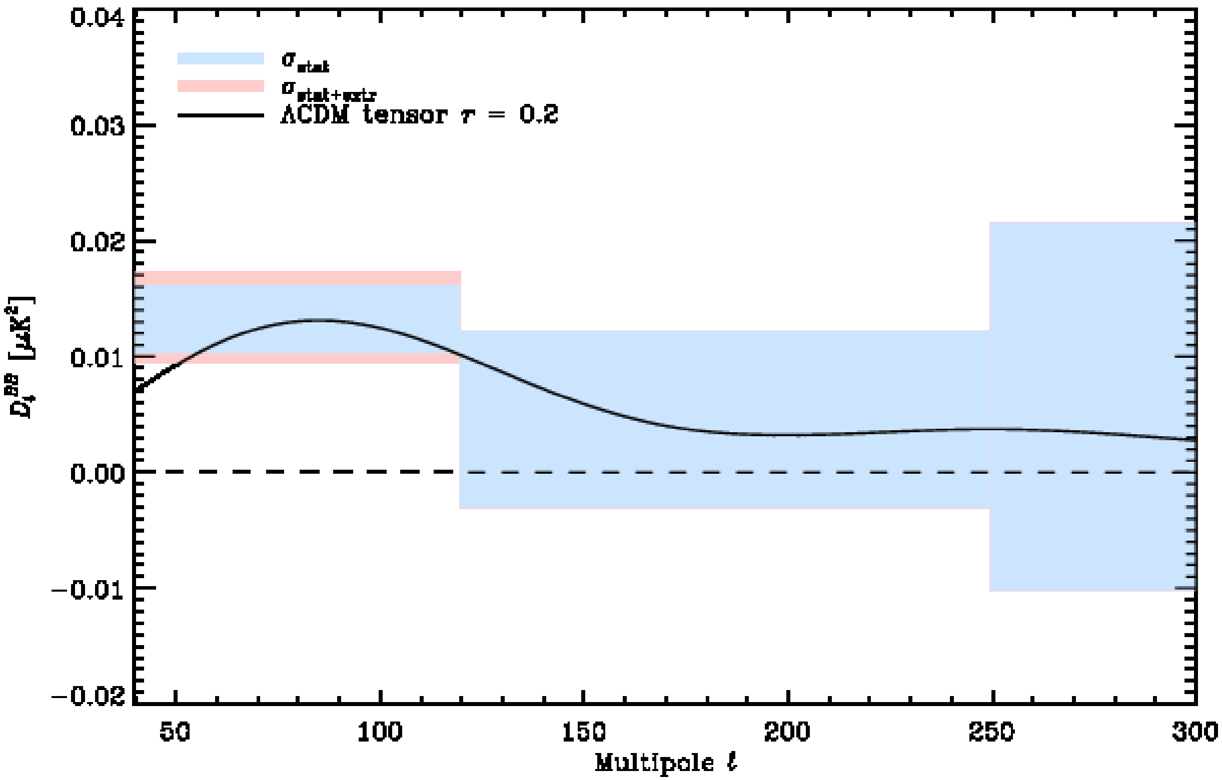
Poorly known until very recently

Prejudice: negligible at Galactic polar regions.

And what’s the reality?

Planck, September 2014: analyzed dust contribution to polarization

Planck-2014



Extrapolation of dust contribution from 353 GHz to 150 GHz (shaded regions)

Solid line: expected gravity wave signal at $r = 0.2$

NB: Same patch of the sky as used by BICEP-2

Smells like dust, looks like dust, tastes like dust...

Discovery postponed – too bad!

Hard task for experimentalists: extract signal from relic gravity waves from dust foreground

Especially if $r < 0.1$

To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceeded by some other epoch, at which these perturbations were generated.

- Inflation is consistent with all data. But there are competitors: the data may rather be viewed as pointing towards early conformal epoch of the cosmological evolution.

More options:

Matter bounce,

Negative exponential potential,

Lifshitz scalar, . . .

- Only very basic things are known for the time being.

Good chance for future

- Detection of B-mode of CMB polarization generated by primordial gravity waves $\Longrightarrow$ simple inflation
  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflaton
- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ contrived inflation, or something entirely different.
  - Shape of non-Gaussianity $\Longrightarrow$ choice between various alternatives
- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.
  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model
- Admixture of entropy (isocurvature) perturbations $\Longrightarrow$ generaion of dark matter and/or baryon asymmetry before the hot epoch

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

Good chance to learn
what preceeded the hot Big Bang epoch

Barring the possibility that Nature is dull

Appendices

Calculation of WIMP mass density

- Expansion at radiation domination
  - Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

($M_{Pl} = G^{-1/2} = 10^{19}$ GeV)

  - Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M_{Pl}^*}$$

- Number density of X-particles in equilibrium at $T < M_X$: Maxwell–Boltzmann

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\frac{\sqrt{M_X^2+p^2}}{T}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

- Mean free time wrt annihilation: travel distance $\tau_{ann} v$, meet one X particle to annihilate with in volume $\sigma \tau_{ann} v \Longrightarrow$

$$\sigma \tau_{ann} v n_X = 1 \quad \Longrightarrow \quad \tau_{ann} = \frac{1}{n_X \langle \sigma v \rangle}$$

- Freeze-out: $\tau_{ann}^{-1}(T_f) \sim H(T_f) \Longrightarrow n_X(T_f)\langle \sigma v \rangle \sim T_f^2 / M_{Pl}^* \Longrightarrow$

$$T_f \simeq \frac{M_X}{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\Longleftrightarrow T_f \sim M_X / 30$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)

- Number density at freeze-out

$$n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*}$$

- Number-to-entropy ratio at freeze-out and later on

$$\frac{n_X(T_f)}{s(T_f)} = \# \frac{n_X(T_f)}{g_* T_f^3} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

where $\# = 45/(2\pi^2)$.

- Mass-to-enropy ratio

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Most relevant parameter: annihilation cross section $\sigma_0 \equiv \langle \sigma v \rangle$ at freeze-out

Changing geers

Estimating axion mass density

Rolling down starts when $m_a(T) \sim H(T)$: before that time scale of rolling m_a^{-1} is larger than the cosmological time scale $\sim H^{-1}$.

After initial rolling, θ oscillates about minimum $\theta = 0$.

Homogeneous oscillating field $=$ condensate $=$ collection of quanta with zero spatial momentum. Just what we need for cold dark matter!

Estimate for present mass density:

Energy density at beginning of rolling

$$V(\theta_0, T) = m_a^2(T) a_0^2 = m_a^2(T) f_{PQ}^2 \theta_0^2$$

Number density of quanta at that time

$$n_a(T) = V(\theta_0, T)/m_a(T) = m_a(T) f_{PQ}^2 \theta_0^2$$

Recall $m_a(T) \sim H(T) = T^2/M_{Pl}^*$ $\Longrightarrow$ number-to-entropy

$$\frac{n_a}{s} = \# \frac{H(T) f_{PQ}^2 \theta_0^2}{g_* T^3} = \# \frac{f_{PQ}^2 \theta_0^2}{\sqrt{g_*} M_{Pl} T}$$

with $T = T_{QCD} \sim 200$ MeV and $\# \sim 1$.

Present mass-to-entropy

$$\frac{\rho_a}{s} = m_a^{(T=0)} \cdot \frac{n_a}{s} = \# \frac{m_a^{(T=0)} f_{PQ}^2}{\sqrt{g_*} M_{Pl} T_{QCD}} \theta_0^2$$

Recall $m_a f_{PQ}^2 \propto m_a^{-1}$: the lighter axions, the more dark matter.

$\rho_{DM}/s \sim 4 \cdot 10^{-10}$ GeV is obtained for $m_a = 10^{-5} - 10^{-6}$ eV
(for $\theta_0 = \pi/2 - 0.1$).

Axions of mass $1 - 10$ μeV are good cold dark matter candidates.

NB: Misalignment is not the only possible production mechanism.

Changing geers

Wave equation in expanding Universe

Prototype example: wave equation in expanding Universe
(not exactly the same as equation for sound waves, but captures main properties).

Massless scalar field ϕ in FLRW spacetime: action

$$S = \frac{1}{2} \int d^4x \sqrt{-g}\, g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$g_{\mu\nu} = (1, -a^2, -a^2, -a^2)$: spacetime metric;
$g^{\mu\nu} = (1, -a^{-2}, -a^{-2}, -a^{-2})$: its inverse;
$g = \text{det}\,(g_{\mu\nu}) = a^6$: its determinant
($d^4x\sqrt{-g}$: invariant 4-volume element).

$$S = \frac{1}{2} \int d^3x dt \; a^3(t) \left(\dot{\phi}^2 - \frac{1}{a^2} \vec{\partial}\phi \cdot \vec{\partial}\phi \right)$$

Field equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\Delta\phi = 0$$

NB. $\dot{a}/a = H$: Hubble parameter.

Fourier decomposition in 3d space

$$\phi(\vec{x},t) = \int d^3k \; \mathrm{e}^{i\vec{k}\vec{x}} \phi_{\vec{k}}(t)$$

NB. $\vec{k}$: coordinate momentum, constant in time.
Physical momentum $q(t) = k/a(t)$ gets redshifted.

Wave equation in momentum space:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{k^2}{a^2(t)}\phi = 0$$

- Redshift effect: frequency $\omega(t) = k/a(t)$.
- Hubble friction: the second term.

As promised, evoltion is different for $k/a > H$ (subhorizon regime) and $k/a < H$ (superhorizon regime).

Subhorion regime (late times): damped oscillations

$$\phi_{\vec{k}}(t) = \frac{A_{\vec{k}}}{a(t)} \cos\left(\int_0^t \frac{k}{a(t)}\, dt + \psi\right), \qquad \psi = \text{ arbitrary phase}$$

NB. Subhorizon sound waves in baryon-photon plasma:
– Amplitude of $\delta\rho/\rho$ does not decrease
– Sound velocity v_s different from 1 ($v_s \approx 1/\sqrt{3}$).
All the rest is the same

Solution to wave equation in superhorizon regime (early times) at radiation domination, $H = 1/(2t)$:

$$\phi = \text{const} \qquad \text{and} \qquad \phi = \frac{\text{const}}{t^{3/2}}$$

Constant and decaying modes.

NB: decaying mode is sometimes called growing, it grows as $t \to 0$.

Same story for density perturbations.

Changing geers

CMB anisotropies, BAO and recent Universe

Standard ruler at recombination: sound horizon.

Angle at which it is seen today depends on geometry of space, dark energy density and to some extent other cosmological parameters

Fig.

Together with other data used to measure the Universe

Baryon acoustic oscillations.

Baryon perturbations at recombination:

$$\frac{\delta\rho_B}{\rho_B}(k) \propto \cos(kr_s)$$

Suddenly freeze out, stay constant in time after recombination.

Oscillations in k of matter density perturbations, $\delta\rho_{DM} + \delta\rho_B$.

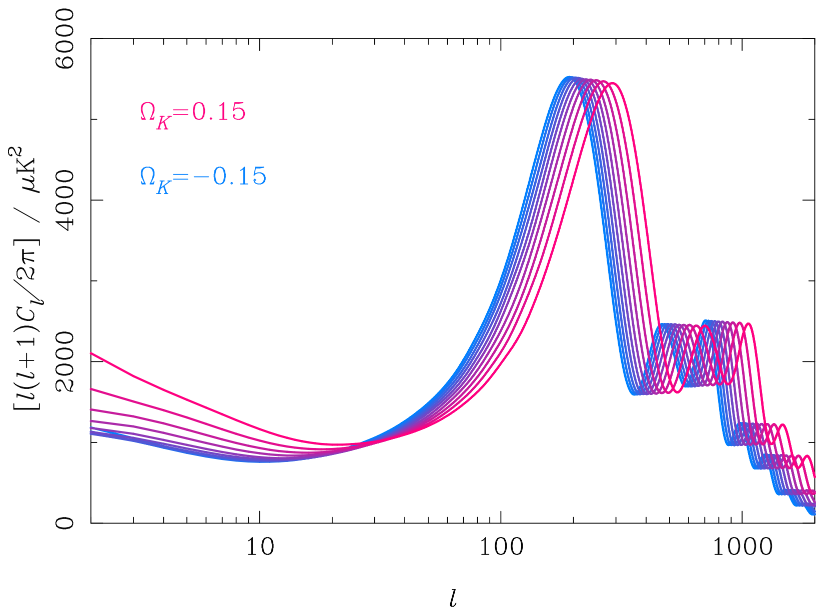
cf. Sakharov oscillations

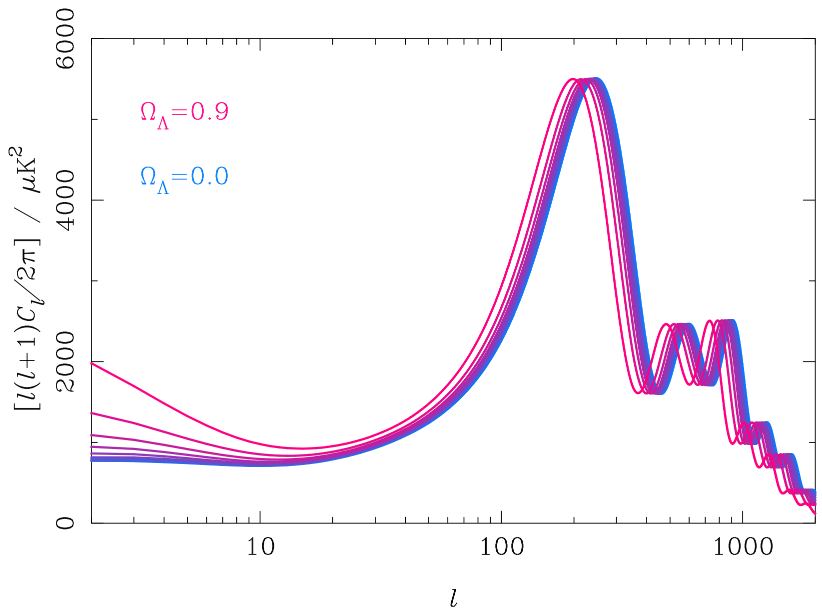
Observed in power spectrum of galaxy distribution.

Fig.

Standard ruler at low redshifts

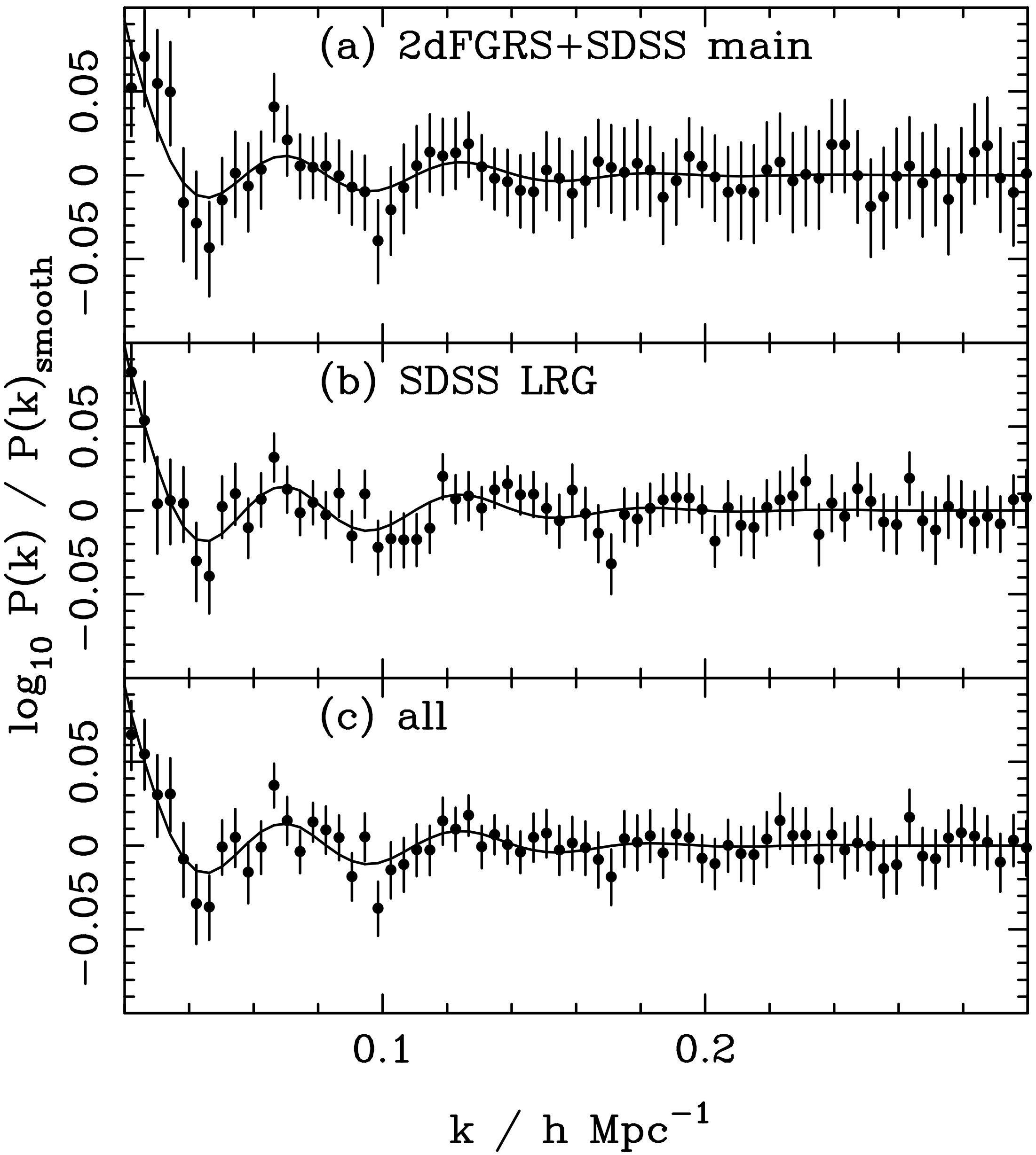
Effect of curvature (left) and Λ



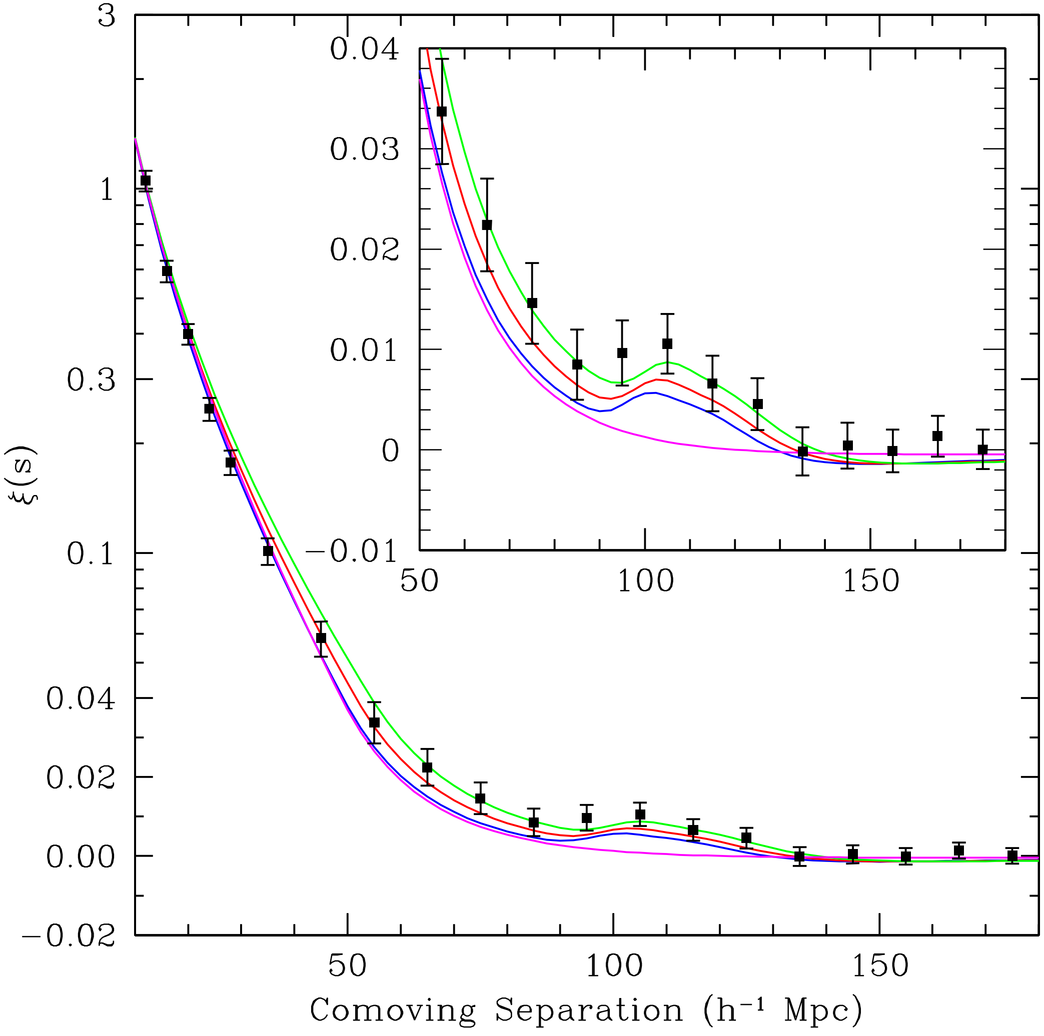


$\Omega_K = \pm 1/(RH_0)^2$, relative contribution of spatial curvature to Friedmann equation; $R =$ radius of spatial curvature; negative sign: 3-sphere.

BAO in power spectrum



BAO in correlation function



Changing geers

Dark energy

- Homogeneously distributed over the Universe, does not clump into galaxies, galaxy clusters.
- Determines the expansion rate at late times $\Longrightarrow$ Relation between distance and redshift. Expansion of the Universe accelerates.

Fig.

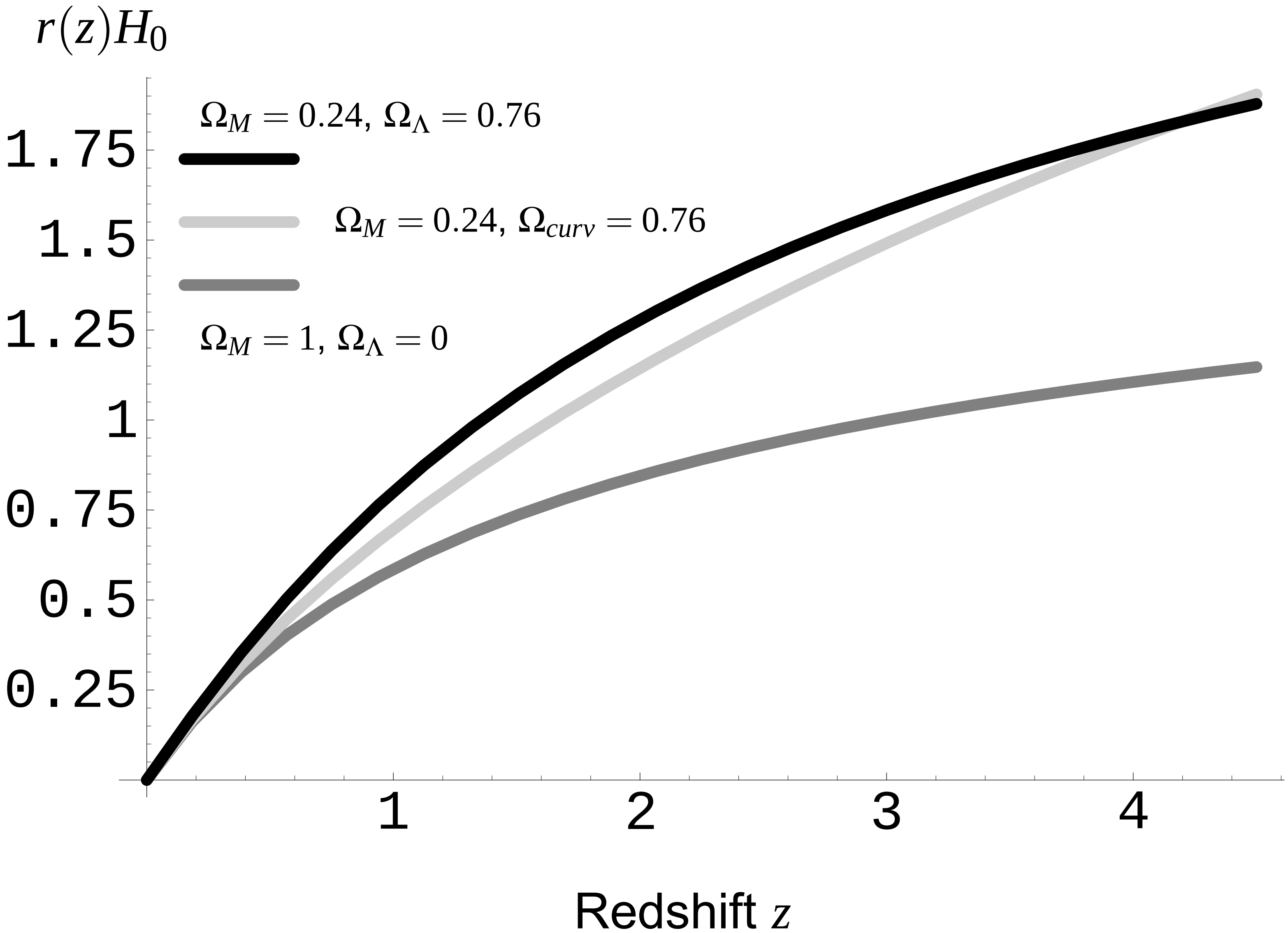
- Measure redshifts (“easy”) and distances by using standard candles, objects whose absolute luminocity is known.

Supernovae 1a

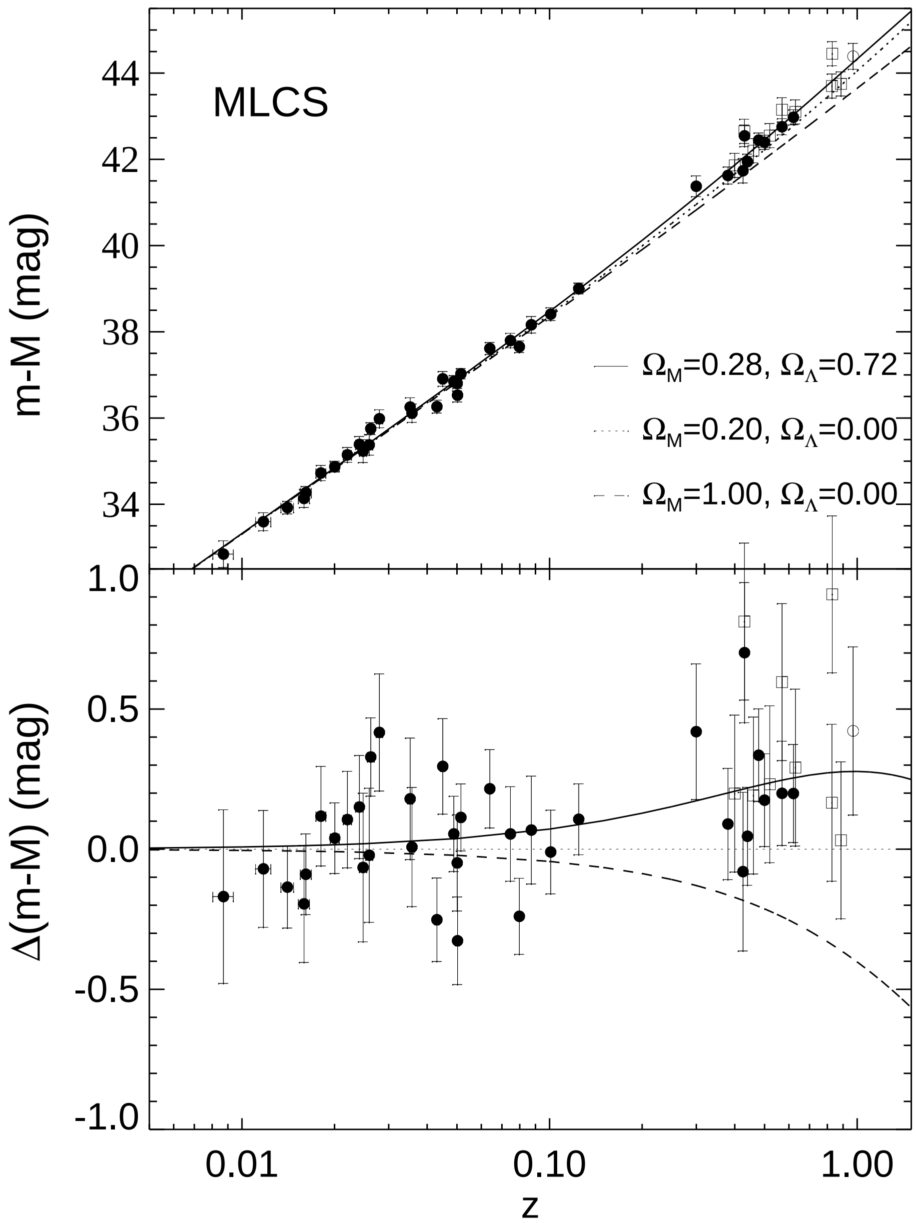
Figs.

- Other, independent measurements of ρ_Λ: cluster abundance at various z, CMB anisotropies combined with standard ruler at small redshift (baryon acoustic oscillations, BAO), etc.

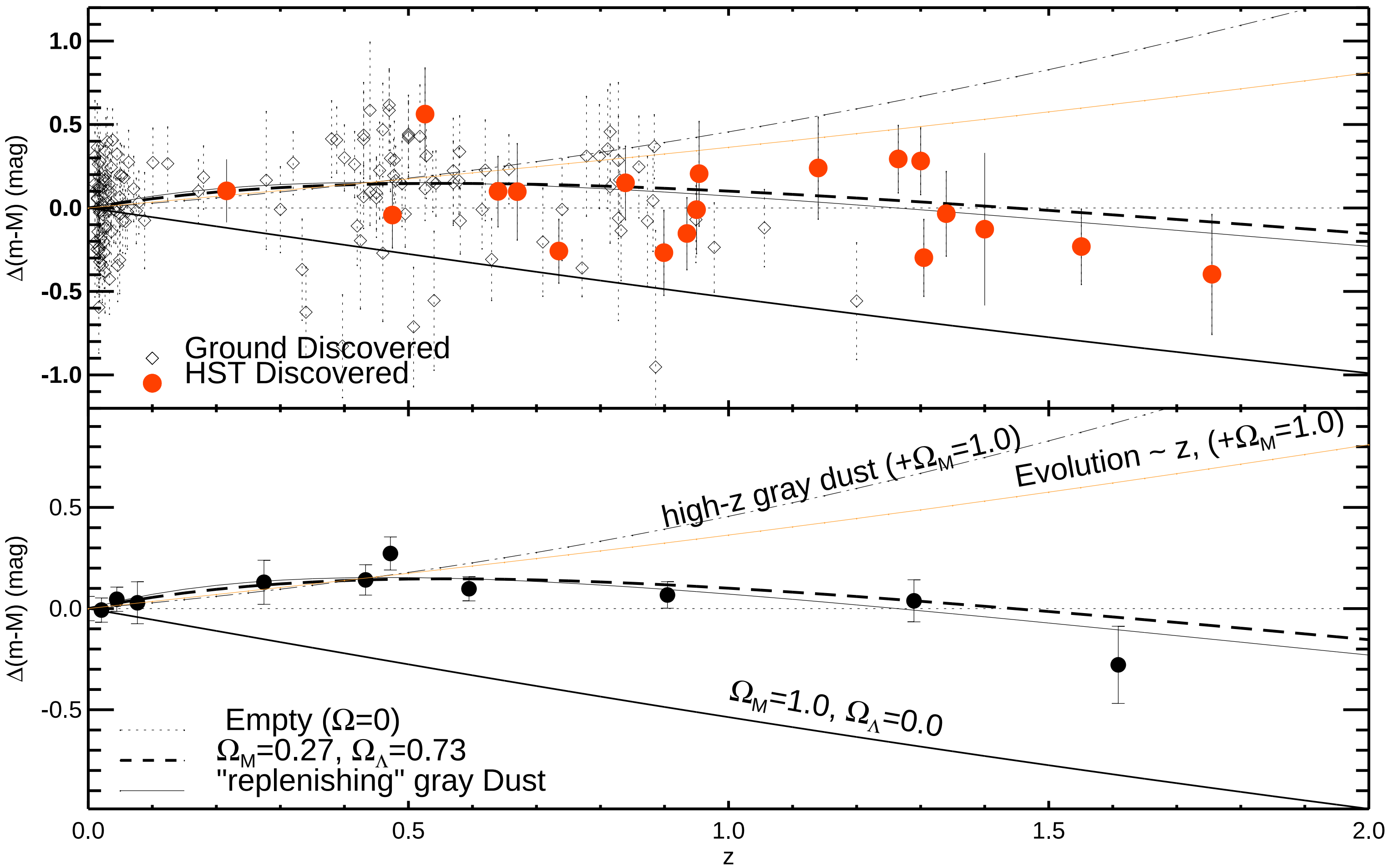
Distance-redshift for different models



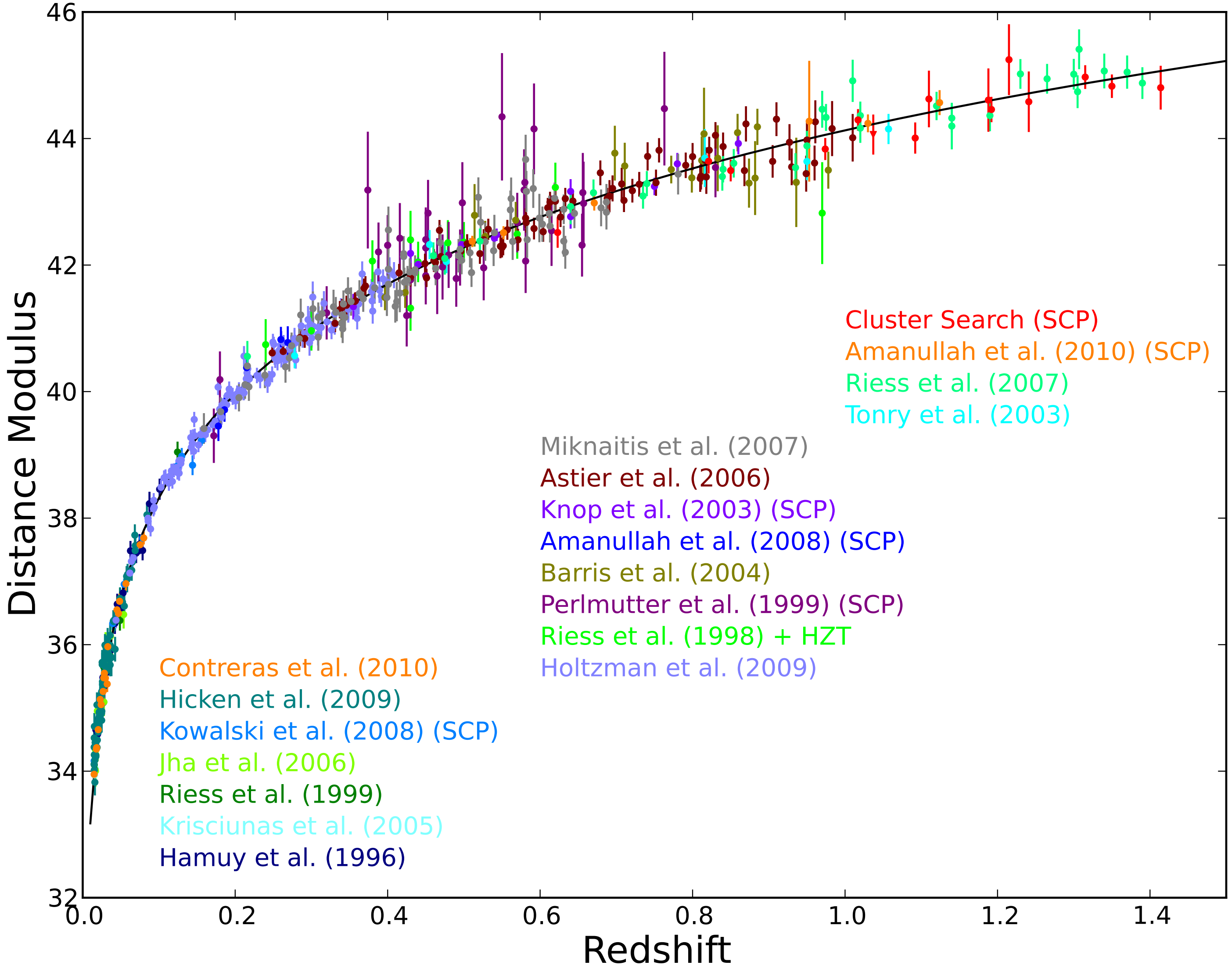
First SNe-1a data, 1998



Newer SNe-1a data

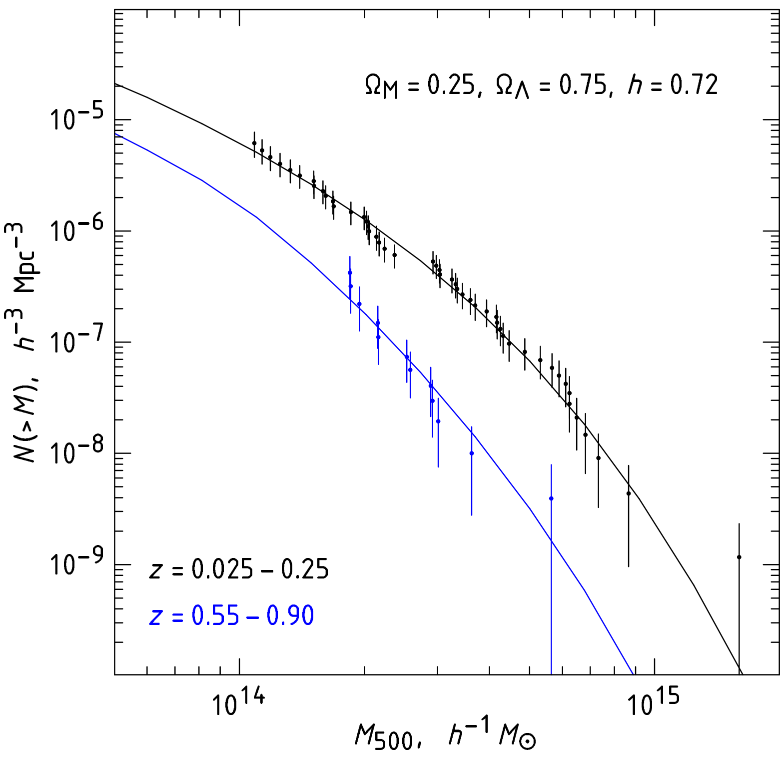


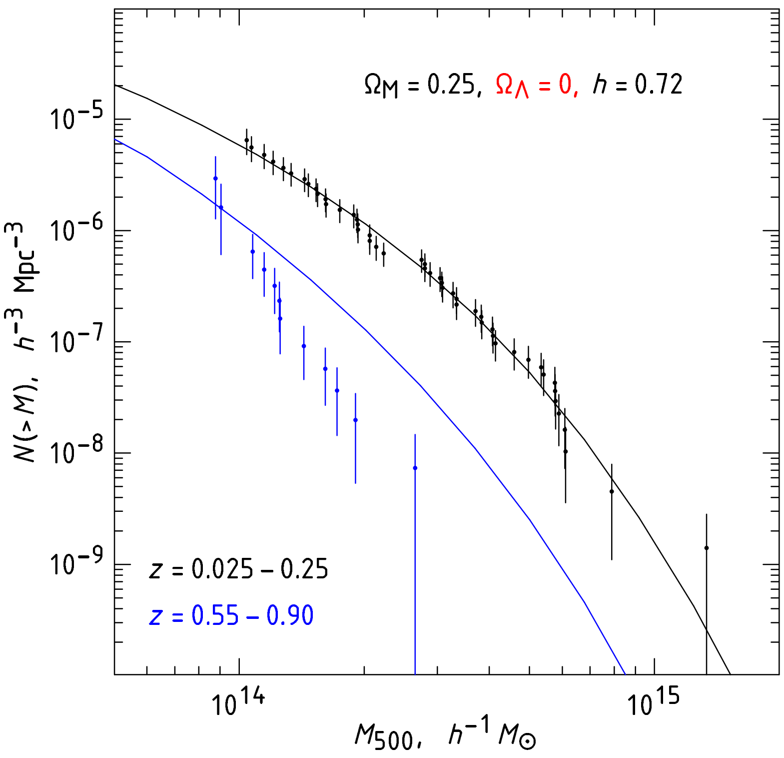
Recent data



Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

Cluster abundance





$\Omega_\Lambda = 0.75$

$\Omega_\Lambda = 0$, curvature domination

Who is dark energy?

- Vacuum

  By Lorentz-invariance

  $$T^{vac}_{\mu\nu} = \text{const}\cdot\eta_{\mu\nu} \quad \text{Minkowski} \quad \Longrightarrow \quad T^{vac}_{\mu\nu} = \text{const}\cdot g_{\mu\nu}$$

  $\text{const} = \rho_\Lambda$, fundamental constant of Nature.

  $\rho_\Lambda = (2\cdot 10^{-3}\ \text{eV})^4$: ridiculously small.

  No such scales in fundamental physics.

  Problem for any interpretation of dark energy

  Equivalent description: cosmological constant

  $$S = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}R - \Lambda\int d^4x\,\sqrt{-g}$$

  $\Lambda \equiv \rho_\Lambda = \text{const}$ = cosmological constant.

- “Old” cosmological constant problem: Why Λ is zero?
- “New” cosmological constant problem: Why Λ is very small but non-zero?

Dark energy need not be vacuum energy = cosmological constant.

- Definition of energy density and pressure:

$$T_{\mu\nu} = (\rho, p, p, p)$$

  Hence, for vacuum $p = -\rho$.

- Parametrize: $p_{DE} = w\rho_{DE} \implies w_{vac} = -1$

w determines evolution of dark energy density:

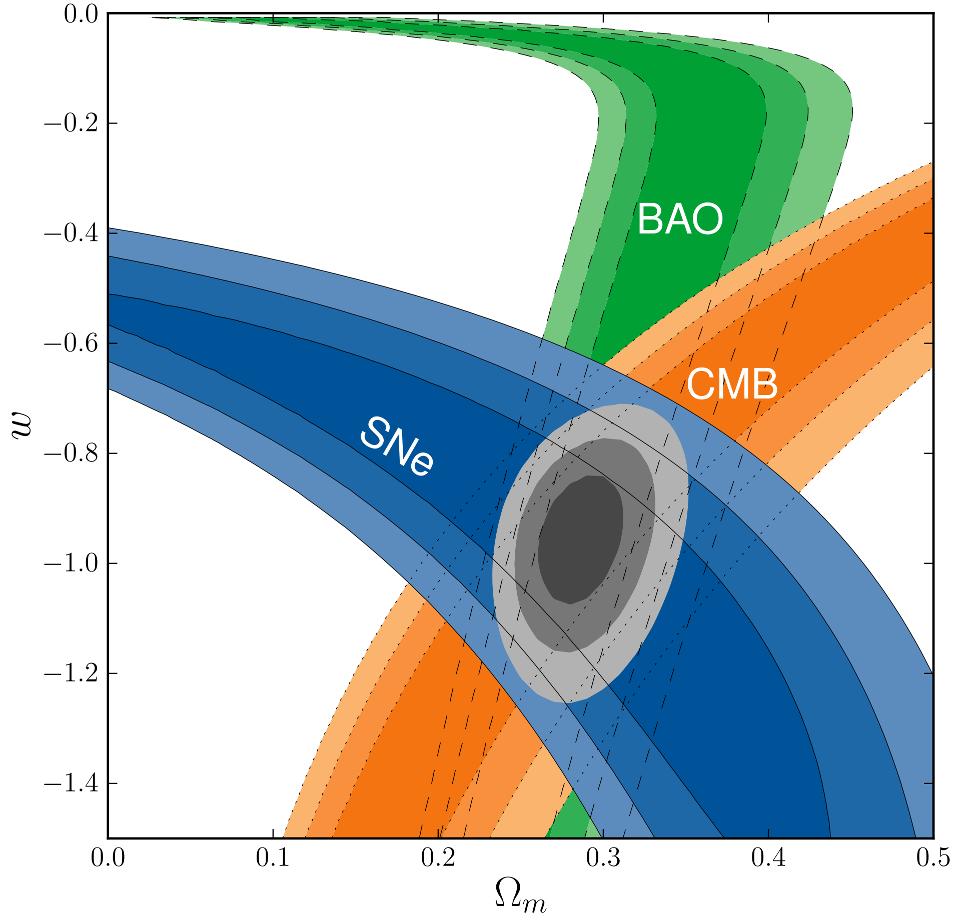
$$dE = -pdV \implies d(\rho a^3) = -pd(a^3) \implies \frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(p+\rho)$$

$$\frac{\dot{\rho}_\Lambda}{\rho_\Lambda} = -3(w+1)\frac{\dot{a}}{a}$$

Options:

- Vacuum: $w = -1$, ρ_Λ constant in time.
- Quintessence, “usual” field (modulo energy scale): $w > -1$, ρ_Λ decays in time.
- Phantom: $w < -1$, ρ_Λ grows in time; typically has instabilities
- General relativity modified at cosmological scales. Effective dark energy depends on time.

Present situation



Changing geers

Anthropic principle/Environmentalism

Cosmology may be telling us something different — and unpleasant

- Friendly fine-tunings
  - Cosmological constant $\sim (10^{-3}\ eV)^4$

    Just right for galaxies to get formed
  - Primordial density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-5}$

    Just right to form stars
    but not supermassive galaxies w/o planets
  - Dark matter sufficient to produce structure

Also

  - Light quark masses and α_{EM}

    Just right for $m_n > m_p$
    but stable nuclei
  - Many more...

Is the electroweak scale a friendly fine-tuning?

Anthropic principle/environmentalism

"Our location in the Universe
is neccessarily priviledged to
the extent of being compatible
with our existence as observers"

Brandon Carter'1974

Fig

Recent support from "string landscape"

We exist where couplings/masses are right

Problem: never know which parameters are environmental and which derive from underlying physics

Disappointing, but may be true

May gain support from the LHC, if not enough new physics

横山　操「グランド キャニオン」1961年