What is the correct definition of entropy for general relativistic field theory?

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International Conference on Particle Physics and Cosmology dedicated to Prof. Rubakov memory

Shuichi Yokoyama

Ritsumeikan University

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Entropy

Entropy

→ ”en” + “tropy” [Clausius, 1865]

“energy” “τροπή”(Greek)

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Rudolf Julius Emmanuel Clausius
Germany, 1822-1888

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→ Entropy allows us to describe the laws of thermodynamics most concisely.

I. (Energy conservation)

$$TdS = dU + pdV$$

II. (Monotonic increase of entropy)

$$dS \geq 0$$

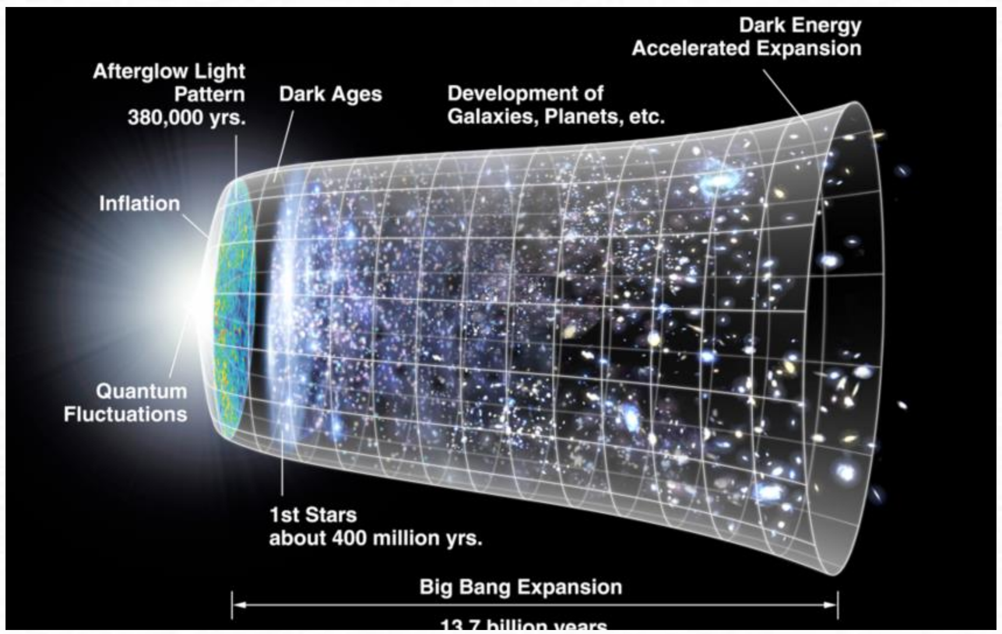
III. (Nernst-Planck’s theorem)

$$\lim_{T \to 0} S = 0$$

These are basic tools to study thermodynamic equilibrium system!

Local thermodynamic equilibrium (LTE)

There are systems in which thermodynamic equilibrium is achieved locally.



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Isotropic homogeneous expanding universe

Astronomical bodies

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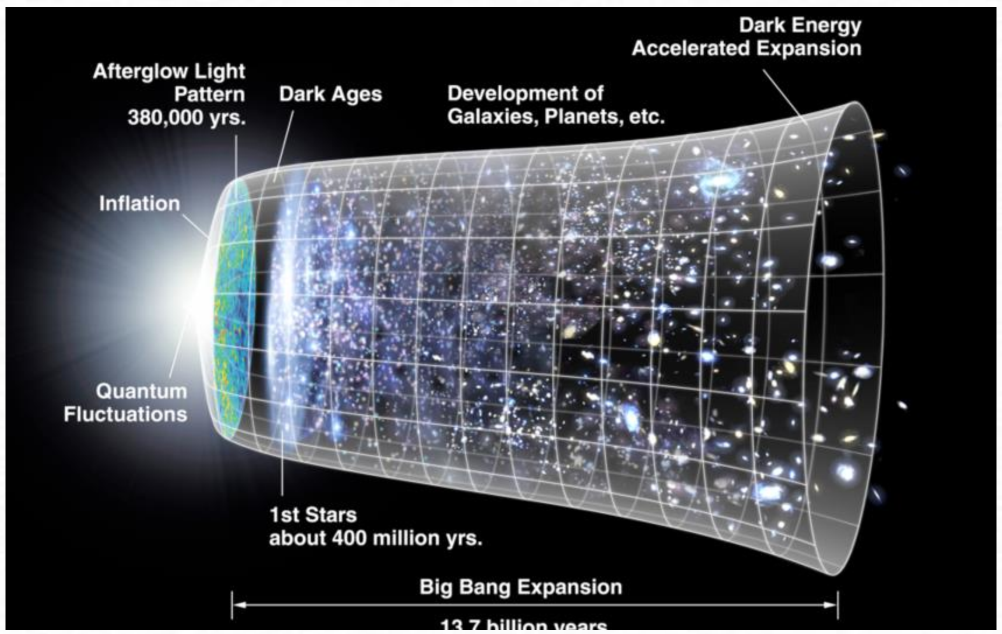

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Isotropic homogeneous expanding universe

Astronomical bodies

A precise analysis of these systems needs General Relativity.

Q1 What is the definition of entropy for a system in **curved spacetime**?

Q2 How are laws of thermodynamics modified in **curved spacetime**?

What is difficult in curved spacetime?

What is difficult in curved spacetime?

The definition of (total) energy for field theory on flat spacetime

$$E = \int_{R^3} d^3x\, T^{00}(t, \vec{x}) \qquad g_{\mu\nu}(x) = \eta_{\mu\nu} = \begin{pmatrix} -1 & \vec{0} \\ \vec{0} & 1_3 \end{pmatrix}$$

$$\partial_\mu T^{\mu\nu} = 0$$ → Energy is conserved (time independent).

What is difficult in curved spacetime?

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⬇

<u>Q.</u> What is the definition of energy on curved spacetime?

$$E = ???\qquad g_{\mu\nu}(x) \neq \eta_{\mu\nu} = \begin{pmatrix} -1 & \vec{0} \\ \vec{0} & 1_3 \end{pmatrix}$$

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$$E = ???\qquad g_{\mu\nu}(x) \neq \eta_{\mu\nu} = \begin{pmatrix} -1 & \vec{0} \\ \vec{0} & 1_3 \end{pmatrix}$$

→ The continuity equation changes into the 'covariant' conservation equation.

$$\nabla_\mu T^{\mu\nu} = 0 \quad \leftrightarrow \quad \partial_\mu T^{\mu\nu} = -\Gamma^{\mu}_{\mu\sigma} T^{\sigma\nu} - \Gamma^{\nu}_{\mu\sigma} T^{\mu\sigma}$$

What is the correct guiding principle to define energy?

What is difficult in curved spacetime?

The definition of (total) energy for field theory on flat spacetime

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What is the correct guiding principle to define energy?

There is a long history on this issue and remain several proposals.

"pseud-tensor"

[Einstein '16]
[Landau-Lifshitz '47, '75]

"quasi-local energy"

[ADM '62] [Bondi '62] [Brown-York '92] [Hawking-Horowitz '95]
[Horowitz-Mayers '98] [Balasubramanian-Kraus '98][Ashtekar-Das '98]…

"Komar mass"

[Komar '62]

Plan

✓ 1. Introduction

2. Proposals

3. Applications to LTE

4. Summary

Our proposal of definition of energy

[Aoki-Onogi-SY '20]

$$E = \int_{\Sigma_t} d^3\vec{x}\,\sqrt{|g|}\,T^0{}_\mu(t,\vec{x})\,n^\mu(t,\vec{x})$$

n^μ	**Time evolution vector field**
Σ_t	**Time slice at an arbitrary time x^0=t**
g	**Determinant of the metric in the total spacetime**

Comments

- This expression was written in the old textbook of Fock.

The quantity $$I = \int T^{\mu 0}\varphi_\mu \sqrt{(-g)}\cdot dx_1\,dx_2\,dx_3 \qquad (49.07)$$
will be constant, i.e. will be independent of x_0, the coordinate that has the character of time, if the vector φ_μ satisfies the equations
$$\nabla_\nu \varphi_\mu + \nabla_\mu \varphi_\nu = 0 \qquad (49.08)$$

[V. Fock, *The Theory of Space, Time and Gravitation* 1959]
Cf. [Trautman 2002]

©Wikipedia

Vladmir Aleksandrovich Fock
Soviet, 1898-1974

- This is **manifestly invariant under general coordinate transformation**.

- This **reduces to the original definition in the flat limit**.

$$E \xrightarrow{\;g_{\mu\nu}(x)\to\eta_{\mu\nu}\;} E = \int_{R^3} d^3x\,T^{00}(t,\vec{x})$$

- This **reproduces the masses of well-known black holes**.

Extension to a general charge

$$Q[v] = \int_{\Sigma_t} d^{d-1}\vec{x}\,\sqrt{|g|}\,T^0{}_\mu(t,\vec{x})\,v^\mu(t,\vec{x})$$

[Aoki-Onogi-SY '20]

$v^\mu = n^\mu$ Time evolution ⇒ $Q[n] = E$ **Energy**

$v^\mu = \delta^\mu_{(i)}$ Translation for i-th direction ⇒ $Q[\delta_{(i)}] = P^i$ **Momentum**

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[Aoki-Onogi-SY '20]

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This charge **conserves** when v=ξ is a **Killing vector field.**

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

→ Q[ξ] is **a Neother charge.**

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Q. Is there any case for Q[v] to conserve unless v is a Killing vector?

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<u>Q.</u> Is there any case for Q[v] to conserve unless v is a Killing vector?

<u>A.</u> **YES** if EM tensor is covariantly conserved $\nabla_\mu T^{\mu\nu} = 0$

and ∃ a vector field to satisfy $T^\mu_\nu \nabla_\mu \zeta^\nu = 0$

$\rightarrow \partial_\nu s^\nu = 0$ where $s^\nu = \sqrt{|g|}\,T^\nu{}_\mu \zeta^\mu$

A wider class of conserved charges including Neother charge!

Cf. [Kodama '80]

A new conserved charge

$$Q[\zeta] = \int_{\Sigma_t} d^{d-1}\vec{x}\,\sqrt{|g|}\,T^0{}_\mu(t,\vec{x})\zeta^\mu(t,\vec{x}) \qquad T^\mu{}_\nu \nabla_\mu \zeta^\nu = 0$$

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Q. **Is there any physical meaning of the new conserved charge?**

Claim

$Q[\zeta]$: entropy, $\quad s^\nu = \sqrt{|g|}\,T^\nu{}_\mu \zeta^\mu$: entropy current

by finding the vector field ζ suitably.

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Intuitive argument

Theory of gravity is fundamental and reversible. Entropy must be conserved.

(If this interpretation is not correct, then what is the physical meaning of this conserved quantity?)

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Evidence

This interpretation leads to the **local Euler's relation** and the **1st law of thermodynamics** for several well-known gravitational systems.

Plan

- ✓ 1. Introduction
- ✓ 2. Proposals
- 3. Applications to LTE
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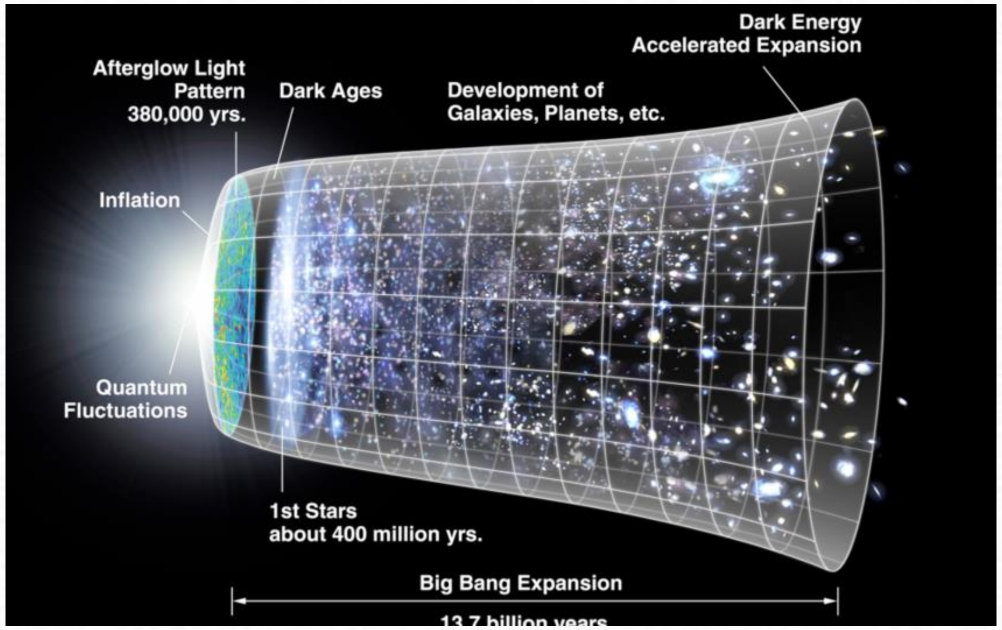
Application 1: FLRW model

[Aoki-Onogi-SY Int.J.Mod.Phys.A 36 (2021) 29, 2150201]



Isotropic homogeneous expanding universe

FLRW model

1 dynamical variable

Homogeneous & isotropic system $ds^2 = -(dt)^2 + a(t)^2 \, \tilde{g}_{ij} dx^i dx^j$

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LTE → Perfect fluid $$T^{\mu}{}_{\nu} = (\rho + P) u^{\mu} u_{\nu} + P \delta^{\mu}_{\nu}$$

Einstein eq: $$\rho = \frac{1}{8\pi G_N}\left(\frac{(d-1)(d-2)}{2}\frac{k+\dot{a}^2}{a^2} - \Lambda\right) \quad P = \frac{1}{8\pi G_N}\left((2-d)\left(\frac{\ddot{a}}{a} + \frac{d-3}{2}\frac{k+\dot{a}^2}{a^2}\right) + \Lambda\right)$$

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$\zeta^{\nu} = -\beta u^{\nu}$

$$T^{\mu}{}_{\nu} \nabla_{\mu} \zeta^{\nu} = 0 \;\Rightarrow\; \rho u^{\mu} \nabla_{\mu} \beta = P \theta \beta \;\Rightarrow\; \beta = \beta_0\, e^{-\int_{t_0}^{t} dt \left(\frac{PK}{\rho}\right)}$$

Comoving frame

Expansion

$$\rightarrow \theta = d \times H$$

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Expansion $\rightarrow \theta = d \times H$

entropy density: $s = \sqrt{|g|} T^{0}{}_{\mu} \zeta^{\mu} = \sqrt{\tilde{g}} a^{d-1} \rho \beta = u\beta$ $\beta = 1/T$ ➡ $Ts = u$

$s \coloneqq s^0$

$u := \rho v$ internal energy density

$v := \sqrt{\tilde{g}} a^{d-1}$ Volume element

Local Euler's relation

FLRW model

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Direct calculation

➡ $T\frac{ds}{dt} = \frac{du}{dt} + P\frac{dv}{dt}$ **1st law of thermodynamics**

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Expansion (θ) $\to \theta = d \times H$

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Local Euler's relation

Direct calculation

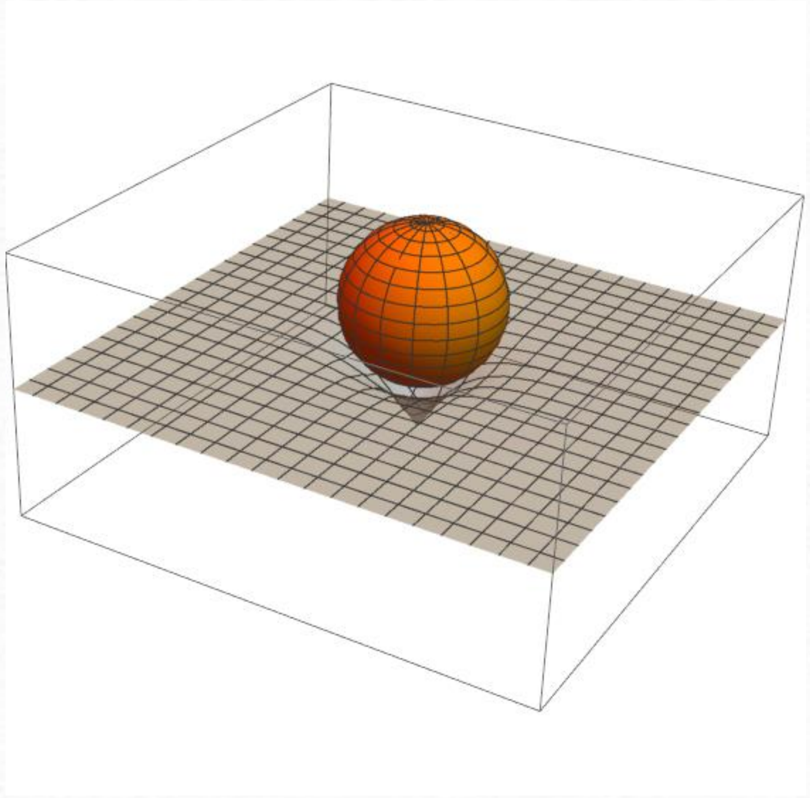
⇒ $T\frac{ds}{dt} = \frac{du}{dt} + P\frac{dv}{dt}$ **1st law of thermodynamics**

Comments

- Energy does not conserve, but entropy does conserves. $T\frac{ds}{dt} = \frac{du}{dt} + P\frac{dv}{dt} = 0$
- The "Big-bang nature" of the universe is inevitable and easily seen.
- These properties hold regardless of any equation of state. Cf. [Kolb-Turner]

Application 2: Spherically symmetric hydrostatic equilibrium

[SY arXiv:2304.06196]



Spherically symmetric hydrostatic equilibrium

2 dynamical variables

Spherically symmetric system

$$ds^2 = -f(dt)^2 + h(dr)^2 + r^2\,\tilde{g}_{ij}dx^i dx^j$$

Spherically symmetric hydrostatic equilibrium

2 dynamical variables

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LTE → Perfect fluid $$T^{\mu}{}_{\nu} = (\rho + p)u^{\mu}u_{\nu} + p\delta^{\mu}_{\nu}$$

Comoving frame

Einstein eq ⇒ $$p' = -\frac{p+\rho}{2}(\log f)', \quad (\log h)' = \frac{2rh}{d-2}(8\pi G_N\rho + \Lambda) - \frac{(d-3)(h-1)}{r}, \quad (\log f)' = \frac{2rh}{d-2}(8\pi G_N\rho - \Lambda) + \frac{(d-3)(h-1)}{r}$$

⇒ **TOV equation** [Oppenheimer-Volkov '39]

Spherically symmetric hydrostatic equilibrium

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$$\beta = \frac{1}{T} := \beta_0\sqrt{f}$$

➡ $$s = \sqrt{|g|}T^{0}{}_{\mu}\zeta^{\mu} = \sqrt{f}v\rho\zeta = \sqrt{f}\beta_0(u + vp)$$ ➡ $$Ts = u + pv$$

$$u^t \to 1/\sqrt{f} \quad v := \sqrt{\tilde{g}}hr^{d-1}$$

Local Euler's relation

Spherically symmetric hydrostatic equilibrium

2 dynamical variables

Spherically symmetric system $$ds^2 = -f(dt)^2 + h(dr)^2 + r^2\, \tilde{g}_{ij} dx^i dx^j$$

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Local Euler's relation

Direct calculation ➡ $$T \frac{ds}{dr} = \frac{du}{dr} + p \frac{dv}{dr}$$

1st law of thermodynamics

Spherically symmetric hydrostatic equilibrium

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$$➡ \quad s = \sqrt{|g|}T^0{}_\mu\zeta^\mu = \sqrt{f}v\rho\zeta = \sqrt{f}\beta_0(u + vp) \quad ➡ \quad Ts = u + pv$$

$u^t \to 1/\sqrt{f} \quad v := \sqrt{\tilde{g}h}r^{d-1}$

Local Euler's relation

Direct calculation

$$➡ \quad T\frac{ds}{dr} = \frac{du}{dr} + p\frac{dv}{dr}$$

1st law of thermodynamics

Comments

- Entropy density is a constant. $T\dfrac{ds}{dr} = \dfrac{du}{dr} + p\dfrac{dv}{dr} = 0$
- The local temperature $T \propto 1/\sqrt{f}$ is exactly coincident with that derived by Tolman. [Tolman '30] Cf. [MTW][Zel'dovich]
- These hold non-perturbatively in the Newton constant.

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Summary

・**A definition of charges whose form was introduced by Fock** in the past was **proposed** as the **precise** one for general relativistic field theory **on curved spacetime**.

・There was found **a new conserved charge different from the Noether one** for GR field theory with energy-momentum coveriantly conserved.

・ The newly found conserved charge was proposed as **entropy**.

・ The proposed interpretation leads to the **local Euler's relation** and the **1st law of thermodynamics** exactly holding in several well-known gravitational system such as FLRW model and a spherically symmetric hydrostatic equilibrium one.

・ For **FLRW model** for the isotropic homogeneous universe, **the energy does not conserve, but the entropy conserves**.

・ For the case of **LTE with spherical symmetry**, **the local temperature** satisfying the laws of thermodynamics is exactly coincident with the **Tolman temperature**.

Future work

- Application to BHs? [Aoki-Onogi-SY ’21]

- Application to astronomical bodies? [SY arXiv:2306.16647]

- Application to non-equilibrium hydrodynamic systems on curved spacetime?

- How about the 2nd and 3rd laws of thermodynamics for such systems?

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Thank you!