First order phase transitions within holographic approach in application to baryon asymmetry problem

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Electroweak baryogenesis (Motivation)

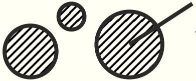
Baryon asymmetry cannot be explained within SM — “**baryon asymmetry problem**”

It implies Sakharov conditions:

- Baryon number violation
  satisfied within non-perturbative SM
  (with sphalerons)
- C, CP violation
  (particles – anti-particles)
  appears in SM, but effect is too small
- CPT violation
  (thermodynamic equilibrium breaking)
  prevent the got asymmetry from ‘washing’

termodynamical equilibrium breaking $\Leftarrow$ first order phase transition

unbroken symmetry $\langle\phi\rangle = 0$



broken symmetry $\langle\phi\rangle \neq 0$

Electroweak Baryogenesis within SM

$$\mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y} \to \mathsf{U}(1)_\mathsf{em}$$

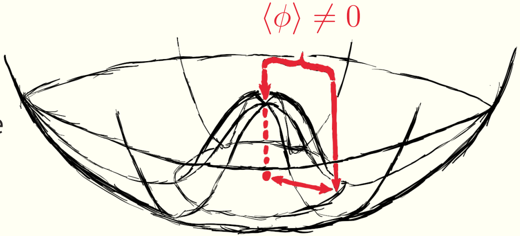
$m_\mathsf{Higgs} > 70$ GeV $\Rightarrow$ NO first order phase transition

Composite Higgs model

$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm CH} + \mathcal{L}_{\rm Int.}$, $\mathcal{L}_{\rm CH}$ – strongly coupled with $\mathcal{G}$ inner symmetry

$\left(\mathcal{G}\ \begin{matrix}\text{invariant}\\ \text{vacuum}\end{matrix}\right) \xrightarrow[\text{breaking}]{\text{spontaneous}} \left(\mathcal{H}\ \begin{matrix}\text{invariane}\\ \text{vacuum}\end{matrix}\right) \Rightarrow$ Goldstone bosons $\ni$ Higgs boson phase transition

Effective potential $V_{\rm eff}$ of broken phase

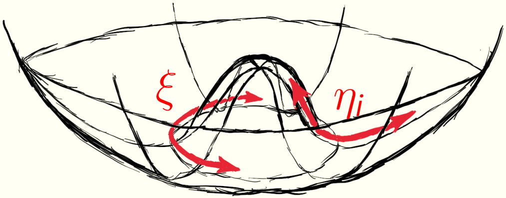


order parameter : $\left(\mathcal{G}\ \begin{matrix}\text{invariant}\\ \text{vacuum}\end{matrix}\right) \Leftrightarrow \langle\phi\rangle = 0$ $\quad\left(\mathcal{H}\ \begin{matrix}\text{invariane}\\ \text{vacuum}\end{matrix}\right) \Leftrightarrow \langle\phi\rangle \neq 0$

Minimal Composite Higgs model

minimal model $\mathcal{G} = SO(5) \times U(1)_Y, \mathcal{H} = SO(4) \times U(1)_Y \supset SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$

minimal $\Leftarrow$ the coset must contain Higgs doublet $\phi \in SU(2)_\mathbb{C} \subset S^4_\mathbb{R} \cong SO(5)/SO(4)$



$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^\top \left[\begin{pmatrix} 0_{4\times4} & 0 \\ 0 & \varsigma \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \xrightarrow[\text{low energy}]{\mathrm{SO}(5)\to\mathrm{SO}(4)} \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & \varsigma \end{pmatrix} \Rightarrow \text{symmetry breaking}$$

Σ_{IJ} is a condensate of the $SO(5)$-inn.sym. fundamental fields Ψ;
ξ is NB-bosons, η is “radial” fluctuations, ς is background field

Effective field theory

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S-J\cdot\phi} =: e^{W[J]}$$

$$\Gamma[\langle\phi\rangle] = W[J] - \frac{\delta W[J]}{\delta J}\cdot J = \int_X d^d x \Big(\underbrace{K_{\text{eff}}[\partial\langle\phi\rangle]}_{=0 \text{ if } \langle\phi\rangle=\text{const}} + V_{\text{eff}}[\langle\phi\rangle] \Big)$$ – effective action

Effective potential: $V_{\text{eff}} = \frac{1}{\text{Vol}_4}\Gamma$

Equation of motion (EoM): $\frac{\delta\Gamma}{\delta\langle\phi\rangle} = J \overset{\langle\phi\rangle=\text{const}}{=\!=\!=} \frac{\delta V_{\text{eff}}}{\delta\langle\phi\rangle} \overset{J=0}{=\!=} 0$ gives extrema condition

AdS/CFT

In a narrow sense: non-perturbative method for correlators calculations;
In general: conformal field theory $\mathcal{Z}_{\text{CFT}} \sim$ field theory in AdS $\mathcal{Z}_{\text{AdS}}$

$$\mathcal{Z}[J] \xlongequal[\text{correspondence}]{\text{AdS/CFT}} \mathcal{Z}_{\text{AdS}} \underset{\text{approximation}}{\overset{\text{quasiclassical}}{\approx}} e^{-S_{\text{AdS}}}\Big|_{\partial\text{AdS}}$$

— **quasiclassical non-perturbative**

strongly coupled $\lambda_{\text{CFT}} \gg 1$ with $\lambda_{\text{CFT}} \sim \frac{1}{\lambda_{\text{AdS}}}$
gives $\lambda_{\text{AdS}} \ll 1$ weakly coupled field theory

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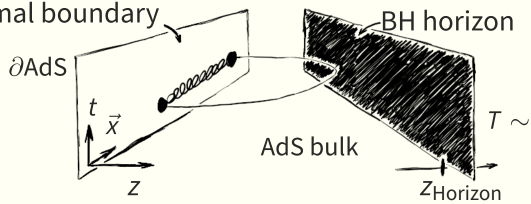
$$\mathcal{Z}_{\text{CFT}}[J] = \int \mathcal{D}[\text{*fields*}] \; \exp(-S - \mathcal{O} \cdot J) \xlongequal[\text{correspondence}]{\text{AdS/CFT}} \mathcal{Z}_{\text{AdS}}\Big|_{\partial\text{AdS}}$$

Fields of CFT are unknown, action is unknown,
but we know something (symmetries) about the sources J and the operators $\mathcal{O}$

$$\mathcal{O} \in \mathbb{C}[\phi, \partial\phi, \partial^2\phi, \ldots]$$

Holographic correlators

$$\frac{\delta S_{\text{AdS}}}{\delta \phi} = 0 \Rightarrow \text{ solution of the equation of motion: } \psi(x,z) \xrightarrow[\partial\text{AdS}]{z\to 0} z^{d-\Delta}\,\psi_0(x) + z^{\Delta}\,\psi_1(x)$$



$T \sim \frac{1}{z_0} \equiv$ gravity at the horizon

$$\mathcal{Z}[J] = \int \mathcal{D}[\ldots] e^{-S-\mathcal{O}\cdot J} \overset{\text{AdS/CFT}}{=\!=} \exp\left(-S_{\text{AdS}}[\psi]\big|_{z=0}\right), \quad J \overset{\text{AdS/CFT}}{=\!=} \psi_0(x), \quad \langle \mathcal{O} \rangle \overset{\text{AdS/CFT}}{=\!=} \psi_1(x)$$

EoM solutions $\xRightarrow[\text{asymptotic}]{\partial\text{AdS}}$ boundary part $S_{\partial\text{AdS}}$ $\xRightarrow[\text{approach}]{\text{quasiclassical}}$ CFT generating function

$$G_n = \langle \mathcal{O} \ldots \mathcal{O} \rangle = \left(\frac{\delta}{\delta J}\right)^n \log \mathcal{Z}[J]\Big|_{J=0} = -\left(\frac{\delta}{\delta \psi_0}\right)^n S_{\text{AdS}}[\psi] = -\underbrace{\left(\frac{\delta^n S^{\text{bulk}}_{\text{AdS}}}{\delta \psi_0^n}\right)}_{=0 \text{ due to EoM}} - \left(\frac{\delta^n S^{\text{border}}_{\partial\text{AdS}}}{\delta \psi_0^n}\right)$$

Holographic potential

Extrema condition: $V_{\text{eff}}\big|_{\text{extrema}} = V_{\text{eff}}\big|_{J=0} \Leftrightarrow G_0$;
AdS/CFT: $G_0 \Leftarrow$ boundary term of dual theory $S_{\partial\text{AdS}}$

$$\text{Vol}_X\, V_{\text{eff}}\big|_{\text{extrema}} = G_0 = W[J=0] \overset{\text{AdS/CFT}}{=\!=} S_{\text{AdS}}\Big|_{\partial\text{AdS}}^{\psi_0=0}$$

Extrema condition & duality: $J = \psi_0 = 0$; duality: $\langle\phi\rangle = \psi_1$

$$\frac{\delta V_{\text{eff}}}{\delta\langle\phi\rangle} \overset{\text{AdS/CFT}}{=\!=} \frac{\delta}{\delta\psi_1}\left(S[\psi]\big|_{\partial\text{AdS}}\right)\Big|_{\psi_0=0} = 0 \qquad \left(\begin{matrix}\text{with assumption}\\ \langle\phi\rangle=\text{const}\end{matrix}\right)$$

gives vacuum expectation values: $\left\{\langle\phi\rangle_{\text{min 1}}, \langle\phi\rangle_{\text{min 2}}, \ldots\right\}$ — possible vacuums

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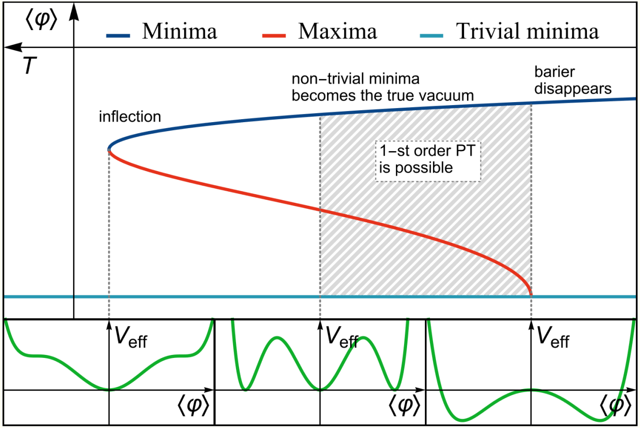
Extrema positions and values $\left\{\left(\langle\phi\rangle_{\text{min i}}, V_{\text{eff}}[\langle\phi\rangle_{\text{min i}}]\right)\right\} \Rightarrow$ phase transitions

Phase transition



$$\left.\frac{\partial V_{\text{eff}}}{\partial\langle\varphi\rangle}\right|_{\langle\varphi\rangle_0} = 0 \quad\Rightarrow\quad \left(\langle\varphi\rangle_0,\, V_{\text{eff}}\big|_{\langle\varphi\rangle_0}\right)$$

Effective potential extremal values and the positions allow one to judge about PT:

- trivial minimum (vacuum) only $\Rightarrow$ there is no PT;
- **non-trivial true vacuum with the potential barrier $\Rightarrow$ 1-st PT**;
- non-trivial true vacuum without a potential barrier $\Rightarrow$ there is no PT.

The extrema of the effective quantum potential V_{eff}: T is the plasma temperature, $\langle\varphi\rangle$ is the vacuum expectation.

Unscaled schematic illustration! Data in real scale are at the backup slides.

Temperature estimations

Experimental restrictions $\Leftrightarrow$ mass of the **lightest predicted** particle.

$$\Sigma_{IJ} = \xi^\top \left[\begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \varsigma \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \underbrace{\overset{\text{AdS/CFT}}{}}_{\text{dual to}} X_{IJ} \to \begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \chi \end{pmatrix}$$

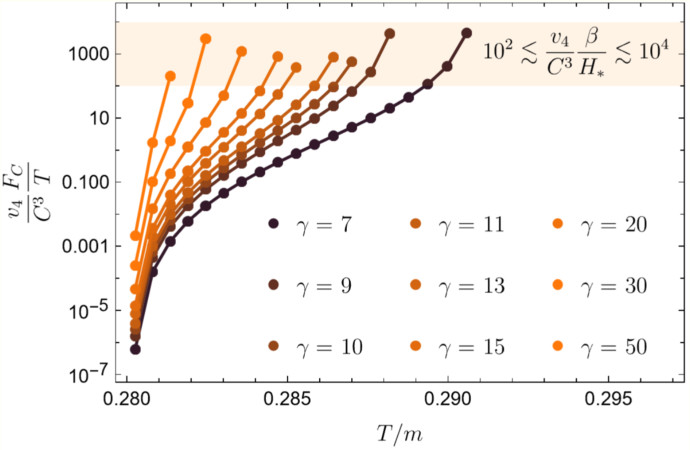
$m_\eta \sim m_{\delta\chi}$ fluctuation mass $\sim$ slope of the "hat".

$$\chi(z) \to \chi(z) + \delta\chi(t, \vec{x}, z) \quad \Rightarrow \quad \text{EoM}_z[\chi] \to \text{EoM}_{t,\vec{x},z}[\chi + \delta\chi] \xRightarrow{\partial\text{AdS}} \boxed{2m^2 = \phi_2}$$

$$T = \frac{1}{\pi}\frac{1}{z_\text{H}} \quad \Rightarrow \quad T = \frac{m}{\pi}\sqrt{\frac{2}{\phi_2}}, \quad z_\text{H}^2 = \frac{\phi_2}{2m^2}$$

Bubble free energy

Free energy of a bubble: $F[V_{\text{eff}}] \underset{\text{approximation}}{\overset{\text{thin walls}}{=\!=}} 4\pi R^2 \mu - \frac{3\pi}{4} R^3 \left(\mathcal{F}_{\text{out}} - \mathcal{F}_{\text{in}}\right)$



$F_C \overset{\text{def}}{=} F(R_C)$: if $R > R_C$, bubbles grow and **PT occurs**.

$$\frac{\beta}{H_*} \sim \frac{F_C}{T} + \mathcal{O}(T)$$

$1/\beta \sim$ appear $\to$ collide time

$1/H_* \sim$ universe expansion

$v_4 \sim 10^{-1} \ll 1, \quad C \sim 1$

$$10^3 \gtrsim \frac{\beta}{H_*} \gtrsim 10^5$$

$0.28 \gtrsim \frac{T(\gamma)}{m} < \frac{T(\gamma_{\text{min}})}{m}$

$T \sim \frac{1}{\gamma} < \frac{1}{\gamma_{\text{min}}}$ – suggestion, $\quad \gamma_{\text{min}} \ll 1$

Gravitational Waves

The spectrum of the gravitational waves can be estimated as
(within the approach of **relativistic** velocity of the bubble walls $v_w \sim 1$)

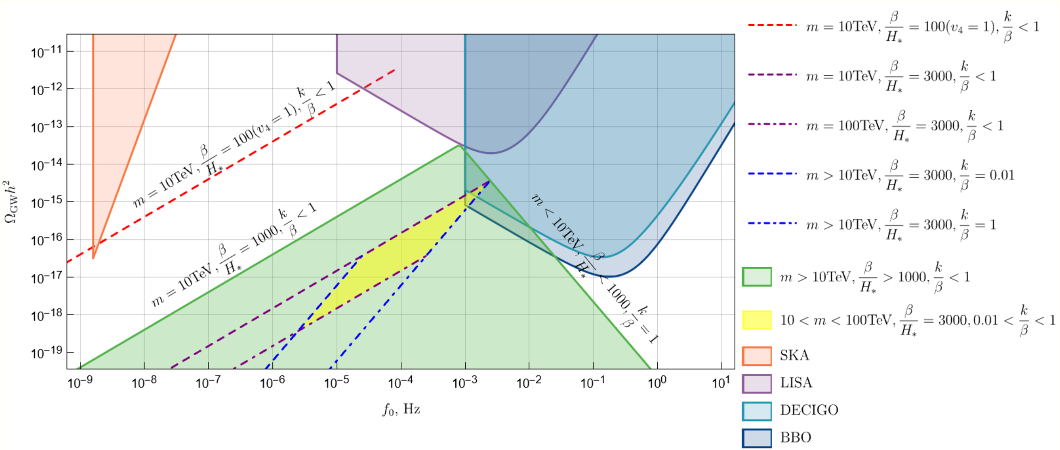
$$\Omega_{\mathsf{GW}} h^2 = 1.67 \cdot 10^{-5} \kappa \Delta \left(\frac{\beta}{H_*}\right)^{-2} \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

Only scalar waves! Sound waves and turbulence are not included!
We **estimate** only scalar waves produced during initial collisions.

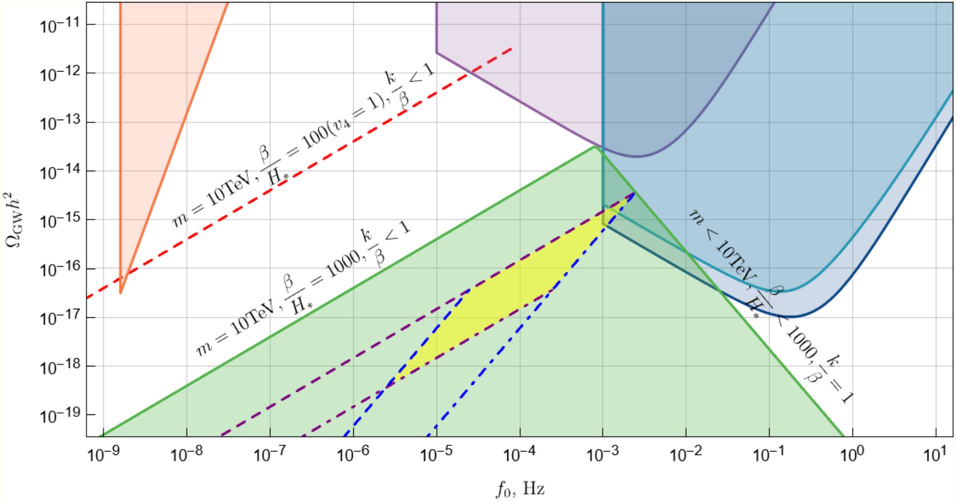
$$f_0 = 1.65 \cdot 10^{-5} \mathrm{Hz} \cdot \frac{f_*}{\beta} \frac{\beta}{H_*} \frac{T}{0.1 \mathrm{TeV}} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \ \mathrm{Hz}$$

$(\Omega_{\mathsf{GW}} h^2, f_0)$-curve is the estimation GW amplitude (peak value).
It does not contain the spectral shape $S(f_0)$ (in this case $S(f_0 = f_0^{\mathsf{peak}}) = 1$).

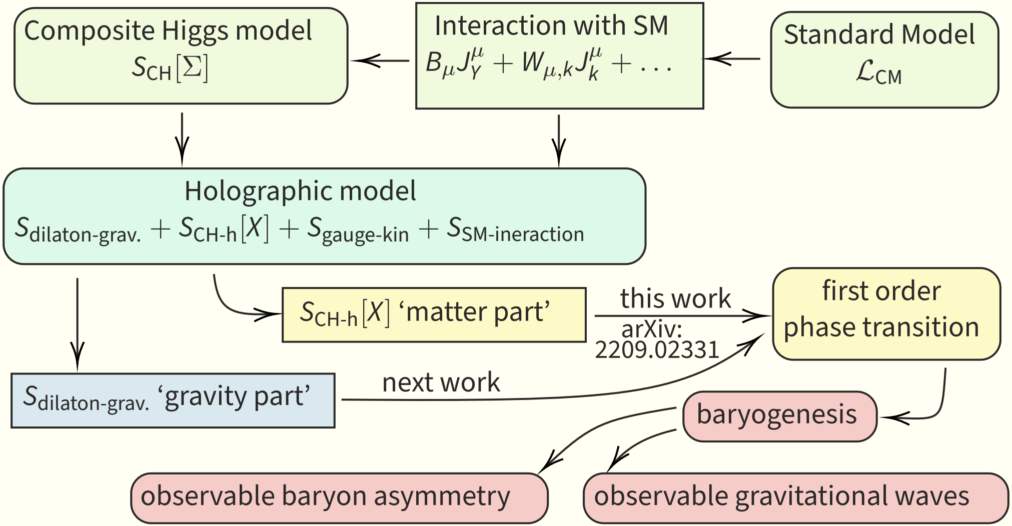
Observations



Observations (without legend)



Conclusion



Composite Higgs model
$S_{\text{CH}}[\Sigma]$
Interaction with SM
$B_\mu J_Y^\mu + W_{\mu,k} J_k^\mu + \dots$
Standard Model
$\mathcal{L}_{\text{CM}}$
Holographic model
$S_{\text{dilaton-grav.}} + S_{\text{CH-h}}[X] + S_{\text{gauge-kin}} + S_{\text{SM-ineraction}}$
$S_{\text{CH-h}}[X]$ ‘matter part’
this work
arXiv:
2209.02331
first order
phase transition
$S_{\text{dilaton-grav.}}$ ‘gravity part’
next work
baryogenesis
observable baryon asymmetry
observable gravitational waves

Holographic model

$\mathcal{L}_{\text{CH}}$ – strongly coupled $\Rightarrow$ consider $N \gg 1$ $\Rightarrow$ $\mathcal{Z}_{\text{CH}}[J] = \mathcal{Z}_{\text{AdS}}[J]$

The dual theory: $\mathcal{Z}_{\text{AdS}}[J] \sim \exp\Big(-S_{\text{AdS}}[J]\Big)$ is weakly coupled $\Rightarrow$ quasiclassical limit

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The asymptotic behavior near the conformal border ∂AdS of the dual theory fields **defines the sources** of the CH operators (i.e. the **correlator functions**)

$$X_{IJ} \overset{z\to 0}{\sim} \frac{\sqrt{N}}{2\pi} J_{IJ} z + \frac{2\pi}{\sqrt{N}} \Sigma_{IJ} z^3 + \dots \qquad X_{IJ} : \text{AdS}_5 \overset{\text{dual}}{\Longleftrightarrow} \Sigma_{IJ} : \mathbb{R}^{1,3}$$

Holography is the **duality** between **strongly coupled** theory on the border and **weakly coupled** (quasiclassical) bulk theory.

$F = -T \log \mathcal{Z}_{\text{CH}} \sim T S_{\text{AdS}} \propto \text{Vol}_4 \cdot \mathcal{F}$ In homogeneous case ($\chi = \chi(z)$): $\mathcal{F} \propto V_{\text{eff}}[\chi]$

Action of the holographic model

$$S_{\text{tot}} = S_{\text{grav}+\phi} + S_{\text{X}} + S_{\text{A}} + S_{\text{SM}} + S, \quad S_{\text{A}} = -\frac{1}{g_5^2}\int d^5x\sqrt{|g|}e^{\phi}g^{ac}g^{bd}F_{ab}F_{cd}$$

$$S_{\text{grav}+\phi} = \frac{1}{l_P^3}\int d^5x\sqrt{|g|}e^{2\phi}\Big[-R+2|\Lambda|-4g^{ab}\partial_a\phi\partial_b\phi - V_\phi(\phi)\Big], \quad a,b=0,\dots 4$$

$$S_{\text{int}} = \epsilon^4\int_{z=\epsilon} d^4x\sqrt{|g^{(4)}|}\left[c_Y B_\mu \operatorname{Tr}\left(T_Y A^\mu\right) + c_W W_{k,\mu}\operatorname{Tr}\left(T_k A^\mu\right) + \mathcal{L}_\psi\right]$$

$$S_{\text{X}} = \frac{1}{k_s}\int d^5x\sqrt{|g|}e^{\phi}\left[\frac{1}{2}g^{ab}\operatorname{Tr}\left(\nabla_a X^T\nabla_b X\right) - V_X(X)\right], \quad \nabla_a X = \partial_a X + [A_a, X], \quad A_a = 0$$

$$V_X(X) = \operatorname{Tr}\left(-\frac{3}{2L^2}X^TX - \frac{\alpha}{4}(X^TX)^2 + L^2\frac{\beta}{6}(X^TX)^3 + O(X^8)\right)$$

$$L\cdot X_{IJ} \sim \frac{\sqrt{N}}{2\pi}J_{IJ}\tilde{z} + \frac{2\pi}{\sqrt{N}}\Sigma_{IJ}\tilde{z}^3 + \dots$$

Geometry

$$S_{\mathrm{grav}+\phi} = \frac{1}{l_P^3}\int d^5x\sqrt{|g|}e^{2\phi}\Big[-R + 2|\Lambda| - 4g^{ab}\partial_a\phi\partial_b\phi - V_\phi(\phi)\Big], \quad a,b = 0,\dots 4$$

$$ds^2 = \frac{L^2}{\tilde{z}^2}A(\tilde{z})^2\left(f(\tilde{z})d\tau^2 + \frac{d\tilde{z}^2}{f(\tilde{z})} + d\vec{x}^2\right), \quad \phi = \phi(\tilde{z})$$

$$f = 1 - \frac{\tilde{z}^4}{z_H^4}, \quad \phi = \tilde{\phi}_2\tilde{z}^2, \quad z_H = \frac{1}{\pi T}.$$

“Extrema” curves

$$\frac{\delta S_\chi}{\delta \chi} = 0 \Rightarrow \chi \xrightarrow{z\to 0} Jz + \left(\sigma - \left(\frac{3}{2}J^3 + \phi_2 J\right)\log z\right) z^3 + o(z^5)$$ — give the sourses for CFT operators

Knowing the *extrema* of the effective potential and its *values* at these points, we can judge abut the phase transition

$$V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_\chi \Big|_{\partial \text{AdS}} \Rightarrow \text{from EoM for effective action}: \ \text{Vol}_4 \frac{\delta V_{\text{eff}}}{\delta \langle \varphi \rangle} = J \Rightarrow \text{extrema condition is absence of sources} \Rightarrow J = 0$$

$$\overbrace{\chi \xrightarrow{z\to 0} \sigma z^3 + o(z^5)}^{\text{“extreme” solutions}} \ \text{must give} \ \overbrace{\frac{\delta V_{\text{eff}}}{\delta \langle \varphi \rangle}}^{\text{extrema}} = 0 \Rightarrow$$ a new condition for ϕ_2 and $\langle \varphi \rangle$

$$T \sim \frac{1}{\sqrt{\phi_2}}, \quad \frac{\delta V_{\text{eff}}}{\delta \langle \varphi \rangle} = 0 = \frac{\delta}{\delta \sigma} S_\chi \left[\chi_{\text{Sol.}}(z; J, \sigma) \right] \Big|_{J=0} \quad \Rightarrow \quad \{\sigma_1, \ldots, \sigma_n\}$$ – extrema

σ is (source) dual to $\langle \varphi \rangle$, vacuum average of the effective theory

Nucleation ratio

The next step is to consider
Baryogenesis generates enough asymmetry (enough efficient) if there is one bubble per Hubble volume

$$\underbrace{\text{Nucleation Ratio}: \quad A T^4 e^{-\frac{F_C}{T}}}_{\text{Bubbles produced per time}\times\text{space volume}} \sim \underbrace{H^4(T) = \left(\frac{T^2}{M_{\text{Pl}}}\right)^4 \text{ -- Expansion of the Univerce}}_{1/(\text{Hubble time}\times\text{volume})}$$

$F = F[\langle\varphi\rangle, R]$ – Free energy of the bubble; R is the radius of the bubble

Hubble horizon (time, volume, radius) — speed of receding object behind it is greater than the speed of light (Don't confuse with cosmological horizon)

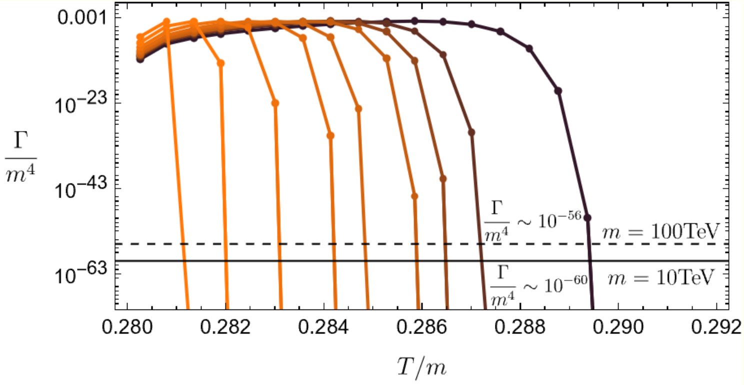
Bubble appears with a certain size. It defines with "micro-physics".
If its radius is grater, then critical one $\left.\frac{\partial F}{\partial R}\right|_{R_C \stackrel{\text{def}}{=} R}$, the bubble grow. Otherwise, it bursts.

It gives $F_C \stackrel{\text{def}}{=} F(R_C)$ and defines nucleation ratio and "viability of the model".

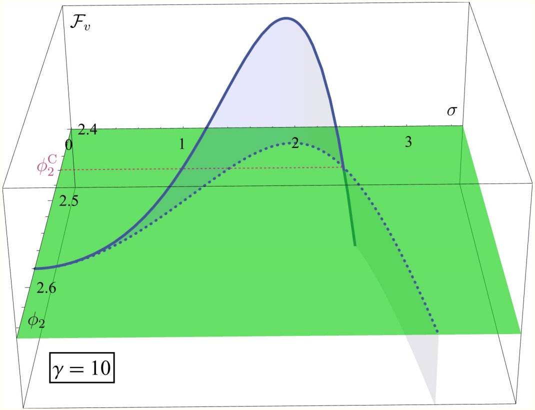
Estimations of the nucleation ratio

$\Gamma \sim H_*^4, \quad \frac{\Gamma}{m^4} \sim \frac{H_*^4}{m^4} \propto \frac{m^4}{M_{\rm Pl}};$

F_C is defined with an error, so $e^{\frac{F_C}{T}}$ has large error



Free energy density



The free energy density of the non-trivial solution (blue line) crosses the free energy density of the trivial solution (green field) at the poitnt where 1-st PT becomes possible.

$\mathcal{F}_V$ is free energy density of the CM model, σ is the source for vacuum expectation value $\langle\varphi\rangle \sim \sigma$, ϕ_2 is the temperature parameter $T \propto \frac{1}{\sqrt{\phi_2}}$.

Effective potential isn't "Tuned"

NO, it's just ill-defined

$$V_\chi = a_2\chi^2 + a_4\chi^4 + a_6\chi^6, \quad a_2 < 0,\ a_4 < 0,\ a_6 > 0 \quad \textbf{no barrier}$$

$$V_{\text{eff}} = b_2\langle\varphi\rangle^2 + b_4\langle\varphi\rangle^4 + b_6\langle\varphi\rangle^6, \quad b_2 > 0,\ b_4 < 0,\ b_6 > 0 \quad \textbf{there's a barrier}$$

in details:

- $V_{\text{eff}} = V_{\text{eff}}[\langle\varphi\rangle]$ describes a quantum objects at the border. V_χ is a dual classical potential in the bulk.
- $V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_{\text{AdS}}\Big|_{\partial\text{AdS}}$ includes the solutions of the EoM $\frac{\delta S_\chi}{\delta\chi=0}$ in bulk. In other words, V_{eff} includes physics of AdS

“Symmetries” of the dual theory potential

$V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6$ is the expantion of a more general theory

Suggestions:

- The potential V_χ always has true vacuum with $E_{\min}$ ($V_\chi \xrightarrow{\chi\to\pm\infty} \infty$). So we may use any even power χ^n instead of the last term χ^6.
- The expansion of V_χ has certain sign of the second term $\lambda > 0$ (the first one m^2 chosen for the theory to be conformal in AdS).
- Higher orders of the expansion don't give new minima at the considered temperatures.

The certain parametrization has been chosen with respect to the “symmetries”

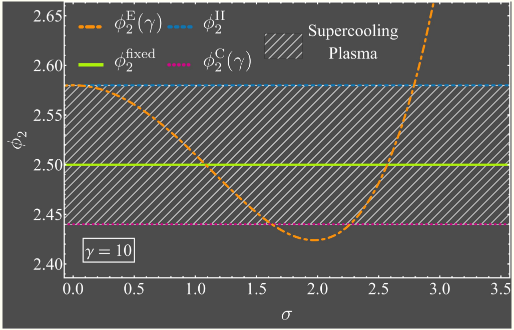
“Scale invariace”, defining the coefficents : $L \to L'$; $\chi \to \sqrt{\lambda}\chi$;

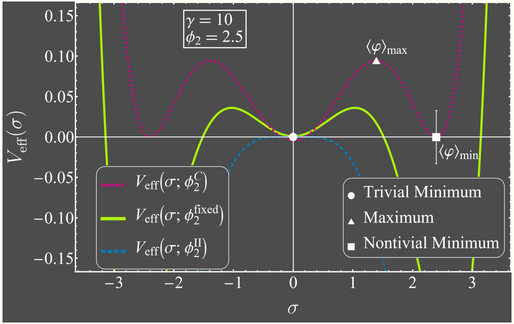
Conformality near the AdS border (“correct” conformal weights) : $\Delta_- = 1$, $\Delta_+ = 3$ $\Rightarrow$ $m^2 = -\frac{D}{3L^2}$

D is for *the Large D limit*. But its usage doesn't give any results.
(to keep interaction constants finite at $D \to \infty$)

Extrema curve on natural scale

“Extrema” curve of the effective potential V_{eff} **in real scale** with the 1st order PT “temperature” range (left picture).

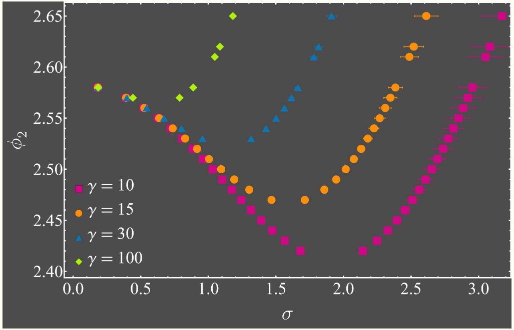


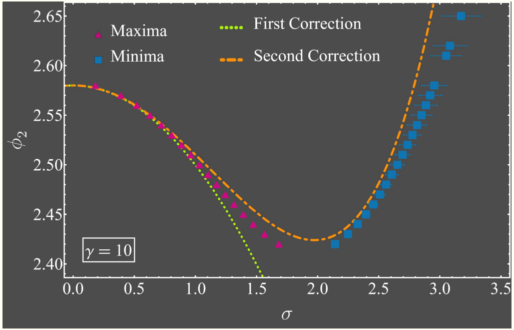


The approximation of the $V_{\text{eff}} = a_0 + a_2\sigma^2 + a_4\sigma^4 + a_6\sigma^6$ with the points $\left(\sigma_{\max}, V_{\max}(\sigma_{\max})\right)$ and $\left(\sigma_{\min}, V_{\min}(\sigma_{\min})\right)$ (right picture).

Analytical and numerical solution

The “extrema” curves defines the positions $\sigma \sim \langle\varphi\rangle$ of the effective potential extrema as functions of the parameter γ and temperature ϕ_2 ($T \sim \frac{1}{\sqrt{\phi_2}}$)





The dotted lines are the numerical solutions.

The dashed lines are the perturbation solution with expantion by λ coupling constant.

SM - CH model interactions

$$F \; \underset{\text{approximation}}{\overset{\text{thin walls}}{=\!=}} \; 4\pi R^2\mu - \frac{3\pi}{4}R^3\left(\mathcal{F}_{\rm out} - \mathcal{F}_{\rm in}\right)$$ — **physical units are required**

- Fix the Parameters (Interaction with Standard Model – bulk gauge fields)
- Physical Units (Infrared Regularization and finite temperature – “radial” heavy fluctuations)

$$W^{\alpha}_{\mu} J^{\alpha\,\mu}_{L} + B_{\mu} J^{\mu}_{Y} \quad \Leftrightarrow \quad J^{\mu} \sim A^{M}$$ – bulk $\mathcal{G}$ gauge field

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The physical values can be estimated without gauge field:

$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^{\top}\left[\begin{pmatrix} 0_{4\times4} & 0 \\ 0 & X \end{pmatrix} + \eta_i \tilde{T}_i\right]\xi \quad \Leftrightarrow \quad \frac{1}{T} \propto \sqrt{\phi_2} \sim \mu_{\rm IR} \sim m_{\eta} \gtrsim 10\ \text{TeV}$$

$m_{\eta} \quad \Leftarrow \quad X \to X + \delta X$ – correction of the background field $\quad \Rightarrow \quad \eta$ – pNG boson

CH gauge field

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm CH} + \underbrace{B_\mu \,\mathrm{Tr}\left(T_Y \hat{J}^\mu\right) + W_{k,\mu}\,\mathrm{Tr}\left(T_k \hat{J}^\mu\right) + \sum_r \bar{\psi}_r \mathcal{O}_r + \mathrm{h.c.}}_{=\mathcal{L}_{\rm interactions}}$$

$$SO(5)\times U(1) : A_M = A_M^K T^K + A_{M,Y} T_Y$$

$$SO(5) \to SO(4) : \quad \underbrace{A_M^K T^K}_{\in SO(5)} \to \underbrace{A_M^a T^a}_{\in SO(4)} + \underbrace{A_M^i T^i}_{\in SO(5)/SO(4)}$$

$$SO(4) \cong SU(2)\times SU(2) : \quad A_M^K T^K = \underbrace{A_M^{k,L} T_L^k}_{\in SU(2)_L} + \underbrace{A_M^{k,R} T_R^k}_{\in SU(2)_R}$$

conserved currents: $\hat{J}_\mu \overset{\text{dual}}{\Longleftrightarrow} A_\mu(t,x,z)\Big|_{z=0}^{\partial \mathrm{AdS}}$, holographic gauge: $A_z = 0$

$\mathcal{O} \overset{\text{dual}}{\Longleftrightarrow} \mathcal{J}(A, \Psi, \phi, \ldots)$ — composite operators of the CH fields