Shower formation constraints on cubic Lorentz Invariance Violation parameters in quantum electrodynamics

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LIV: Dispersion relations and Effective Field Theory

- Motivation: how to produce the theories with the traces of the Planck scale.
- Kinematical approach – **modified dispersion relation**:

$$E^2 = m^2 + p^2 (1 \pm \eta_0) \pm \frac{p^3}{E_{\mathrm{LIV},1}} \pm \frac{p^4}{E^2_{\mathrm{LIV},2}} \pm \dots \quad (1)$$

  Kinematical effects:
  - time delays,
  - birefringence,
  - threshold modifications (decays, ...)
- Dynamical approach EFT Lagrangian – dynamical effects:
  - (Non-threshold) Modification of cross-sections,
    Example: Bethe-Heitler process $\gamma N \to N e^+ e^-$
    (the 1st interaction in γ-induced air shower).

Current experimental limits on LIV parameters

e^-/γ	Test of QG	Sub($-$) or super($+$) luminal	$\lvert\xi_0\rvert(\lvert\eta_0\rvert)$	Limits $E^{(1)}_{\rm LIV}$ (eV)	$E^{(2)}_{\rm LIV}$ (eV)	Source	Ref.
e^-	Synch.	both	2×10^{-20}	10^{33}	2×10^{25}	CRAB	[1340, 1341, 1361]
e^-	VC	$(+)$	10^{-20}	10^{31}	10^{23}	CRAB	[1338, 1344, 1393]
γ	PD	$(+)$	7.1×10^{-19}	1.7×10^{33}	1.4×10^{24}	LH. J2032+4102	[1167]
γ	PD	$(+)$	1.3×10^{-17}	2.2×10^{31}	8×10^{22}	MultiSrc	[1356]
γ	PD	$(+)$	1.8×10^{-17}	1.4×10^{31}	5.8×10^{22}	eHWCJ1825-134	[1356]
γ	PD	$(+)$	2.2×10^{-17}	9.9×10^{30}	4.7×10^{22}	eHWCJ1907+063	[1356]
γ	3γ	$(+)$	-	-	2.5×10^{25}	LH. J2032+4102	[1167]
γ	3γ	$(+)$	-	-	1.2×10^{24}	eHWC J1825-134	[1356]
γ	3γ	$(+)$	-	-	1.0×10^{24}	eHWC J1907+063	[1356]
γ	3γ	$(+)$	-	-	4.1×10^{23}	CRAB	[1355]
γ	AS	$(-)$	-	-	1.7×10^{22}	diffuse (Tibet)	[1168]
γ	AS	$(-)$	-	-	6.8×10^{21}	LH. J1908+0621	[1168]
γ	AS	$(-)$	-	-	1.4×10^{21}	CRAB	[1355]
γ	AS	$(-)$	-	-	9.7×10^{20}	CRAB	[1355]
γ	AS	$(-)$	-	-	2.1×10^{20}	CRAB	[1361]
γ	PP	$(-)$	-	1.2×10^{29}	2.4×10^{21}	MultiSrc (6)	[1394]
γ	PP	$(-)$	2×10^{-16}	2.6×10^{28}	7.8×10^{20}	Mrk 501	[1348, 1395]
γ	PP	$(-)$	-	1.9×10^{28}	3.1×10^{20}	MultiSrc (32)	[1359]

A. Addazi et al. (2022)

Myers-Pospelov EFT = QED with cubic LIV

LI is broken by external fixed timelike vector $n_\mu = (1, 0, 0, 0)$.
EFT (CPT-odd!): the only LIV dim 5 operators to the Lagrangian.

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_\gamma + \mathcal{L}_e, \tag{2}$$

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{3}$$

$$\mathcal{L}_\gamma = \frac{\xi}{M_{\text{Pl}}} n^\mu F_{\mu\nu} n \cdot \partial \left(n_\sigma \tilde{F}^{\sigma\nu} \right) \leftarrow \text{break the DR for photons,} \tag{4}$$

$$\mathcal{L}_e = \frac{1}{M_{\text{Pl}}} \bar{\psi}(n \cdot \gamma) \left(\eta_L (1 - \gamma_5) + \eta_R (1 + \gamma_5) \right) (n \cdot \partial)^2 \psi \tag{5}$$

$\leftarrow$ break the DR for electrons (will not be considered).

Myers, Pospelov, Phys. Rev. Lett. (2003)

Myers-Pospelov model - Dispersion relations

Left- and right- polarized photons:

$$\varepsilon^{\mu}_{(L)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \qquad \varepsilon^{\mu}_{(R)} = \frac{1}{\sqrt{2}}(0, 1, i, 0). \tag{6}$$

Different signs in the dispersion relation for different polarizations:

$$E^2_{(L)} = k^2_{(L)} + \frac{2\xi}{M_{\text{Pl}}} k^3_{(L)} \qquad \text{– superluminal,} \tag{7}$$

$$E^2_{(R)} = k^2_{(R)} - \frac{2\xi}{M_{\text{Pl}}} k^3_{(R)} \qquad \text{– subluminal.} \tag{8}$$

Left- and Right- chiral electrons:

$$E^2_{(.)} = m^2 + p^2_{(.)} + 2\eta_{(.)} \frac{p^3_{(.)}}{M_{\text{Pl}}}, \qquad (.) = (L) \text{ or } (R) \tag{9}$$

Myers-Pospelov model: Kinematical constraints on $E_{\mathrm{LIV},1}$ for photons

H.E.S.S. 2011 and Fermi 2009

Time delays

AGN: $E_{\mathrm{LIV},1} > 2 \cdot 10^{18}$ GeV
GRB: $E_{\mathrm{LIV},1} > 1.5 \cdot 10^{19}$ GeV

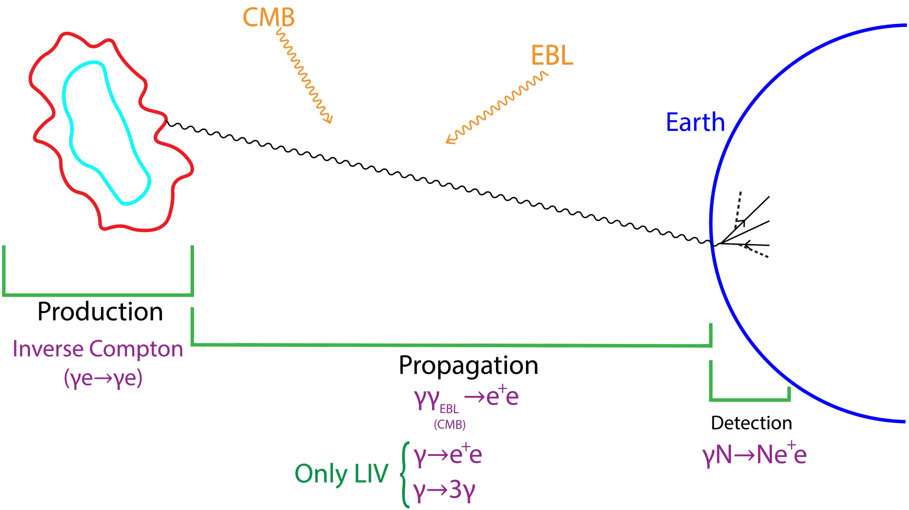
Gotz et al, 2013 and Galaverni et al, 2015

Birefringence (n=1 only)

GRBs: $\xi < 3.4 \cdot 10^{-16} \leftrightarrow E_{\mathrm{LIV},1} > 1.8 \cdot 10^{34}$ GeV
Combined: $\xi < 8.6 \cdot 10^{-17} \leftrightarrow E_{\mathrm{LIV},1} > 7.1 \cdot 10^{34}$ GeV

Extremely strong limits from birefringence. However, independent constraints from other processes may be also interesting.

The fate of VHE (TeV-PeV) photon & crucial reactions



QED processes crucial for super- and subluminal photons

Appear in case of superluminal LIV ($E^2 = k^2 + \frac{k^{n+2}}{E^n_{\mathrm{LIV},n}}$):

- Photon decay $\gamma \to e^+e^-$,
- Photon splitting $\gamma \to 3\gamma$.

Both processes suppress the photon flux.

Modified in case of subluminal LIV ($E^2 = k^2 - \frac{k^{n+2}}{E^n_{\mathrm{LIV},n}}$):

- Pair production on background photons, $\gamma\gamma_b \to e^+e^-$. It responsible for suppression of the extragalactic photon flux in LI case. In subluminal LIV the process suppressed $\to$ the photon flux may be enhanced.
- Pair production in Coulomb field of a nuclei $\gamma N \to N\, e^+e^-$ (Bethe-Heitler process) in subluminal LIV the process suppressed $\to$ the observed photon flux suppressed.

VHE (TeV - PeV energies) photons

Assumption: both polarizations produced in the source (additional analysis is needed!)

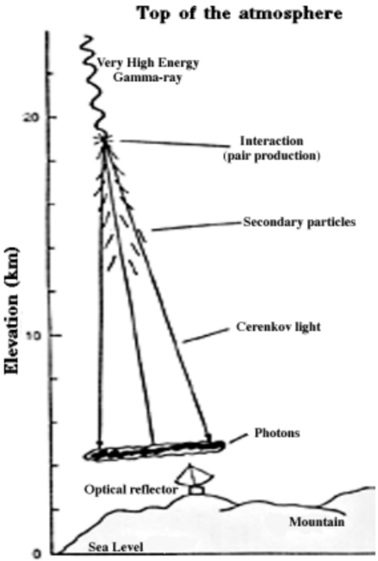
$$E^2_{(L)} = k^2_{(L)} + \frac{2\xi}{M_{Pl}} k^3_{(L)} \text{ – superluminal case,} \tag{10}$$

$$E^2_{(R)} = k^2_{(R)} - \frac{2\xi}{M_{Pl}} k^3_{(R)} \text{ – subluminal case.} \tag{11}$$

If some photon-like events detected (polarization is unknown):

- No decay/splitting at these energies,
- No observational suppression of shower formation.

Atmosphere shower formation: sensitivity to LIV



- The first interaction in the atmosphere is pair production in the Coulomb field of a nucleus (Bethe, Heitler, 1934).
- The most energetic interaction → the most sensitive to LIV. Suppressed in case of subluminal LV.
- Subsequent interactions — less energetic, no change in LIV case in the leading order.

Bethe-Heitler process and sensitivity to LIV

Cross-section in LI case with screening (Bethe, Heitler, 1934):

$$\sigma_{\mathrm{BH}}^{\mathrm{LI}} = \frac{28 Z^2 \alpha^3}{9 m_e^2} \left(\log \frac{183}{Z^{1/3}} - \frac{1}{42} \right) . \qquad (12)$$

In case of **subluminal** LIV the cross-section gets suppressed (idea: Vankov, Stanev, 2002).
Calculation for LIV ($n = 2$) – Rubtsov, Satunin, Sibiryakov, 2012.
In the limit $E_\gamma^3 \gg m_e E_{LIV,1}^2$, the cross-section reads ($n = 1$, R-polarization),

$$\boxed{\sigma_{\mathrm{BH}}^{\mathrm{LV}} \simeq \sigma_{\mathrm{BH}}^{\mathrm{LI}} \cdot 1.7 \frac{m_e^2 E_{\mathrm{LIV},1}}{E_\gamma^3} \cdot \log \frac{E_\gamma^3}{2 m_e^2 E_{\mathrm{LIV},1}} .} \qquad (13)$$

The cross-section decreases with energy as $E_\gamma^{-3} \log E_\gamma$ (fixed $E_{\mathrm{LIV},1}$).

Photon-induced shower formation: LI vs. LIV cases

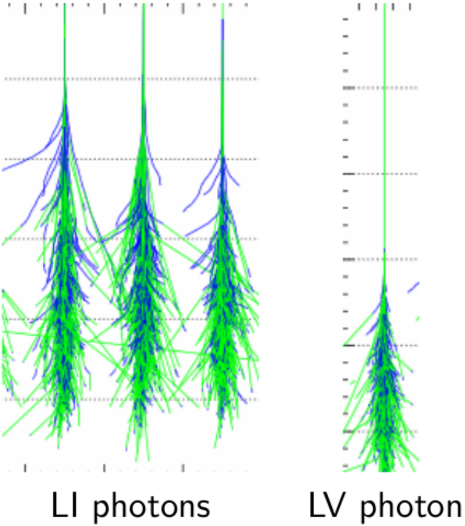
LI case:

- First interaction $\langle X_0 \rangle = m_{at}/\sigma_{BH} \approx 57$ g cm^{-2}.
- Shower maximum: $X_{max} = X_0 + \Delta X$, $\langle X_{max} \rangle \approx 320$ g cm^{-2}.

LIV case:

- X_0 increases.
- ΔX does not change (in the leading order).

Photon-induced showers become deeper and may avoid detection!



Shower formation

$$\langle X_0\rangle_{\mathrm{LIV}} = \frac{\sigma_{\mathrm{BH}}^{\mathrm{LI}}}{\sigma_{\mathrm{BH}}^{\mathrm{LIV}}} \langle X_0\rangle_{\mathrm{LI}}\,,\ \langle X_0\rangle_{\mathrm{LIV}} = m_{\mathrm{at}}/\sigma_{\mathrm{BH}}^{\mathrm{LIV}} \tag{14}$$

The probability for a photon to produce pair in the atmosphere reads,

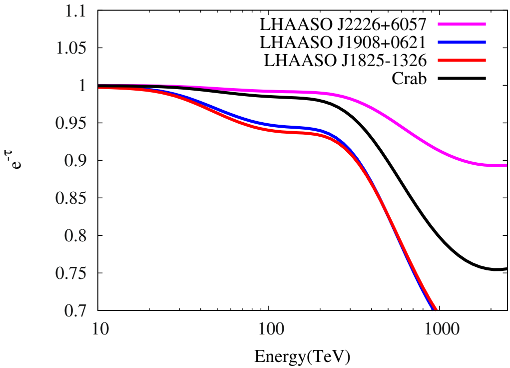
$$P = \int_0^{X_{\mathrm{atm}}} \mathrm{d}X_0\, \frac{\mathrm{e}^{-X_0/\langle X_0\rangle_{\mathrm{LIV}}}}{\langle X_0\rangle_{\mathrm{LIV}}} = 1 - \mathrm{e}^{-X_{\mathrm{atm}}/\langle X_0\rangle_{\mathrm{LIV}}}\,. \tag{15}$$

The detected photon flux gets reduced,

$$\left(\frac{\mathrm{d}\Phi}{\mathrm{d}E}\right)_{\mathrm{LIV}} = P \times \left.\frac{\mathrm{d}\Phi}{\mathrm{d}E}\right|_{\mathrm{source}}\,. \tag{16}$$

Attenuation of galactic γ-ray flux due to pair production on CMB

Mean free path for 1 PeV photon is $\sim$ 10 kpc – galactic scales!



LHAASO coll. Nature, 2021

Sub-PeV γ-ray flux: Shower formation vs pair production on CMB

Subluminal LIV shifts the threshold of p.p. from CMB peak to EBL where it is almost negligible:

$$\left(\frac{\mathrm{d}\Phi}{\mathrm{d}E}\right)_{\mathrm{LIV}} = \frac{P_{\mathrm{sh.form}}(E_\gamma, E_{\mathrm{LIV},1})}{\mathrm{e}^{-\tau(L_{\mathrm{source}}, E_\gamma)}} \times \left.\frac{\mathrm{d}\Phi}{\mathrm{d}E}\right|_{\mathrm{source}} . \qquad (17)$$

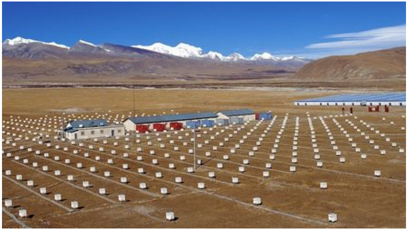
More details in application to n=2 case – Satunin, EPJC (2021).

The modified threshold for pair production in soft photon background:

$$\epsilon_{\mathrm{th}} = \frac{m^2}{\omega_b} \mp \frac{1}{4}\frac{k^2}{E_{\mathrm{LIV}}}, \qquad (18)$$

'+' is for subluminal case, '-' is for superluminal case.

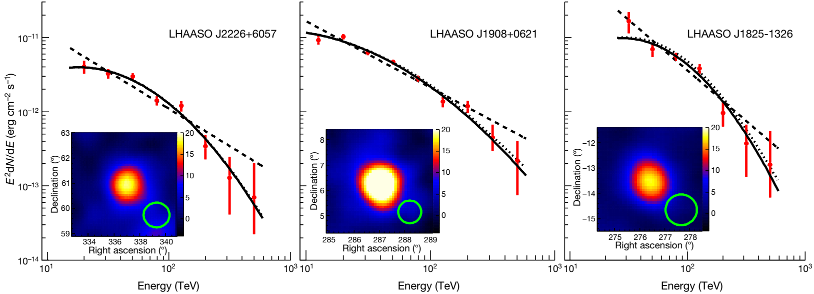
Experimental data



- Tibet ASγ — diffuse γ-rays from the Galactic Disk. Maximal photon energy is 0.8 PeV (Tibet ASγ, PRL 2021).
- LHAASO – observation of 12 galactic sources in > 100 TeV. Maximal photon energy is 1.4 PeV (LHAASO, Nature, 2021).
- LHAASO – Crab Nebula spectrum up to PeV. Maximal photon energy is 1.1 PeV (LHAASO, Science, 2021).

LHAASO

- 12 sources (Pevatrons) with energy 100+ TeV discovered.
- Energy spectra for 3 sources:



LHAASO, Nature, 2021

We test the hypothesis of LIV shower suppression assuming the most conservative power-law flux with experimental data points.

Shower formation limits on subluminal $E_{\mathrm{LIV},1}$

Source	L, kpc	$E_{\mathrm{LIV},1}$, 10^{20} GeV
Crab Nebula	2	0.5
J2226+6057	0.8	1.5
J1908+0621	2.37	2.1

Table: The 95% CL constraints on LIV mass scale from 3 sub-PeV sources observed by LHAASO.

Conclusions

- New constraints on $E_{\mathrm{LIV},1}$!
- Obtained shower formation constraints are many orders of magnitude weaker than the birefringence limits but independent and comparable with other limits.

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Thank you!

Appendix: the equation of motion

LV term for A^μ:

$$\mathcal{L}_\gamma = \frac{\xi}{M_{\rm Pl}} n^\mu F_{\mu\nu} n \cdot \partial \left(n_\sigma \tilde{F}^{\sigma\nu} \right) \tag{19}$$

$$\implies 0 = \delta S_\gamma = \frac{\xi}{M_{\rm Pl}} \int \mathrm{d}^4 x \, \delta A_\tau \left(-n_\sigma \epsilon^{\sigma\tau\rho\nu} \left(n \cdot \partial \right)^2 F_{\rho\nu} \right) \tag{20}$$

In the Lorentz gauge, QED gives the following term:

$$\Box A^\mu = 0. \tag{21}$$

Equation of motion for A^μ:

$$\Box A^\tau = \frac{\xi}{M_{\rm Pl}} n_\sigma \epsilon^{\sigma\tau\rho\nu} \left(n \cdot \partial \right)^2 F_{\rho\nu}. \tag{22}$$

Appendix: the dispersion relation

Let consider a photon propagating along the z-axis: $k_\mu = (\omega, 0, 0, k)$. Then we obtain from equation of motion the dispersion relation in the limit of high at high energies (thus the electron mass m can be neglected):

$$\left(\omega^2 - k^2 \pm \frac{2\xi}{M_{\text{Pl}}} k^3\right)(\varepsilon_x \pm i\varepsilon_y) = 0. \tag{23}$$

And dispersion relation reads,

$$\omega^2 = k^2 \pm \frac{2\xi}{M_{\text{Pl}}} k^3 \equiv k^2 - \frac{k^3}{E_{\text{LIV}}}. \tag{24}$$

Appendix: the details of pair production

We consider the external field to be the Coulomb field generated by a nucleus with a charge of magnitude Z,

$$A^{\mu} = (Ze/r, 0, 0, 0)\,. \tag{25}$$

The external field of a virtual photon from the Coulomb field of a nucleus is written as

$$A_0^{\text{ext}}(x) = Ze \int \frac{\mathrm{d}^4 q}{(2\pi)^3}\, \delta(q^0)\, e^{-iqx} \frac{1}{|\mathbf{q}|^2},\;\; A_i^{\text{ext}}(x) = 0, \tag{26}$$

where $\mathbf{q}$ is 3-momentum of the virtual photon of a nucleus.

Appendix: the details of the cross section

The differential cross section for $\gamma\gamma^* \to e^- e^+$ is written as

$$\mathrm{d}\sigma = 2\pi\,\delta(\omega - E_1 - E_2)\,\frac{1}{2\omega}\frac{1}{2}|i\mathcal{M}|^2\frac{\mathrm{d}^3 p_1}{2E_1(2\pi)^3}\frac{\mathrm{d}^3 p_2}{2E_2(2\pi)^3}. \tag{27}$$

The total cross section of the pair production in Lorentz invariant case is giving by Bethe-Heitler formula:

$$\sigma_{\mathrm{BH}}^{\mathrm{LI}} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2} \times \begin{cases} \log\frac{2\omega}{m} - \frac{109}{42} \text{ (no screening)}, \\ \log\frac{183}{Z^{1/3}} - \frac{1}{42} \text{ (complete screening)}. \end{cases} \tag{28}$$

Appendix: constructing gauge fields with $s = 0, 1/2$

The adding terms for the different gauge theories are the following (5-dimension operators):

- Scalar field: $\mathcal{L} = i\frac{\kappa}{M_{\mathrm{Pl}}}\left(n\cdot\partial\right)^3\Phi$,
- Spin 1/2: $\mathcal{L} = \frac{1}{M_{\mathrm{Pl}}}\bar{\Psi}\left(\eta_1 \not{n} + \eta_2 \not{n}\gamma_5\right)\left(n\cdot\partial\right)^2\Psi$.

Myers, Pospelov, Phys. Rev. Lett. (2003)