Colour structure of three-reggeon cuts in QCD

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Contents

Introduction

One of remarkable properties of QCD is the Reggeization of all its elementary particles (quarks and gluons) in perturbation theory,
which is very important for theoretical description of high energy processes.
The gluon Reggeization is especially important because it determines the high energy behaviour of non-decreasing with energy cross sections. In particular, it appears to be the basis of the BFKL (Balitskii-Fadin-Kuraev-Lipatov) equation, which was first derived in non-Abelian theories with spontaneously broken symmetry
F. V.S., Kuraev E.A., Lipatov L.N., 1975
and whose applicability in QCD was then shown
Balitsky I.I., Lipatov L.N., 1978.
The equation was derived using unitarity and analyticity.

In each order of perturbation theory dominant (having the largest $\ln s$ degrees) are amplitudes with gluon quantum numbers and negative signatures in cross-channels. They determine the s-channel discontinuities of amplitudes with the same and all other possible quantum numbers.
Both in the leading logarithmic approximation (LLA) and in the next-to-leading one (NLLA) the amplitudes used in the unitarity relations are determined by the Regge pole contributions and have a simple factorized form (pole Regge form). Due to this, the Reggeization provides a simple derivation of the BFKL equation in the LLA and in the NLLA.
But the Regge pole contributions are not sufficient in the NNLLA.
The deviations can be explained by the three-Reggeon cuts.

Introduction

There is a big difference between the reggeon cuts in QCD and in the earlier (before appearence of QCD) theory of complex angular momenta.

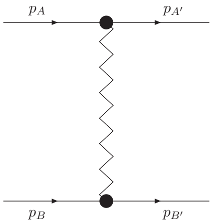
The original reason for this difference is the different concept of reggeons. In the previous theory, reggeons were represented by the sum of ladder diagrams, while in QCD the primary Reggeon is a Reggeized gluon.

An important question is the colour structure of reggeon cuts.

This question is simple in the two-reggeon case, because in the product of two adjoint representations there is only one representation of given dimension and parity. But it becomes quite non-trivial in the case of three-reggeon cuts.

Gluon Reggeization

For elastic scattering processes $A + B \to A' + B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel and negative signature (symmetry with respect to $s \leftrightarrow u$) is written as



$$\mathcal{A}_{AB}^{A'B'} = \Gamma^{c}_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma^{c}_{B'B} \; ;$$

Gluon Rggeization

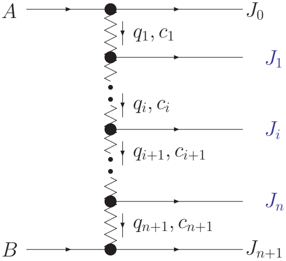
$\Gamma^{c}_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices ("c" are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.

The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well.

MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.

The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

MRK amplitudes can be presented by the picture



and their real parts have a simple factorized form

$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.

Two-Reggeon cuts

It is well known that there is no consistent theory in which all singularities in *j* plane (plane of complex angular momenta) are poles.

Regge poles in the *j* plane generate Regge cuts.

In QCD we know only one Regge pole – the Reggeized gluon with the trajectory $j(t) = 1 + \omega(t)$ and negative signature.

In in amplitudes with different from gluon quantum numbers in the *t*-channel Regge cuts appear already in the LLA.

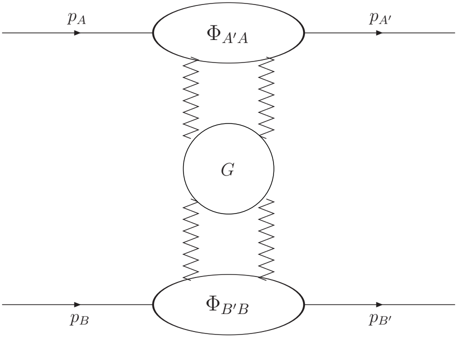
In particular, the BFLK Pomeron is a two-Reggeon cut.

It appears that in amplitudes with positive signature, where the real parts of the leading logarithmic terms cancel out, so that remaining piece is pure imagine in the LLA.

From the unitarity relation, using the pole Regge form of elastic and MRK amplitudes, we obtain that the *s* channel imaginary parts of elastic amplitudes are presented in the form:

Two-Reggeon cuts

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}$$



where Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$ $B \to B'$

G– Green's function for two interacting Reggeized gluons,

Two-Reggeon cuts

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.

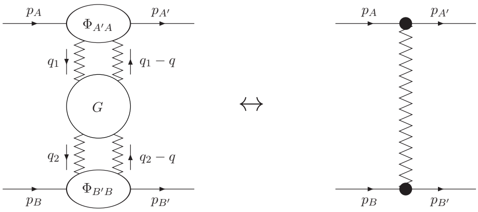
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.

The kernel is universal (process independent).

Thus, in the BFKL approach, two-Redgeon cuts appear. In particular, the BFKL pomeron is a fixed branch point

$$\omega_P = \frac{4N_c\alpha_s}{\pi}\ln 2.$$

It may seem that the contribution under consideration also in the channel with gluon quantum numbers and a negative signature leads to the appearance of contributions other than the Regge pole. It turns out, however, that this is not the case thanks to the bootstrap relations.
These relations are quite simple in the elastic case:



There is an infinite number of the bootstrap relations for production amplitudes in the MRK, and they provide the pole Regge form in the NLLA.

Violation of the factorization

Validity of the pole Regge form is proved now in all orders of perturbation theory in the coupling constant g both in the LLA and in the NLLA.
The pole Regge form is violated in the NNLLA.
The first observation of the violation was done
Del Duca V., Glover E.W.N., 2001
at consideration of the high-energy limit of the two-loop amplitudes for gg, gq and qq scattering. The discrepancy appears in non-logarithmic terms.
If the pole Regge form would be correct in the NNLLA, they should satisfy the factorization condition. However, it is not the case.
Using the infrared factorization techniques, consideration of the terms responsible for breaking of the pole Regge form in amplitudes of elastic scattering in QCD was performed by
Del Duca V., Falcioni G., Magnea L., Vernazza L., 2013-2015.

Violation of the factorization

In particular, the non-logarithmic terms not satisfying the factorization condition at two-loops were recovered and single-logarithmic terms at three loops violating the pole Regge form were found.

It was natural to explain the observed violation by Regge cut contributions. As it was already said, Regge cuts appear in amplitudes with positive signature already in the LLA.

But in amplitudes with negative signature Regge cuts must be at least three-Reggeon ones and can appear only in the NNLLA.

Violation of the factorization

The first explanation was done in
F. V.S., 2016; F. V.S., Lipatov L.N., 2017,
In the two-loop approximation the cut contribution comes from diagrams with three t-channel gluons, which represent Reggeons in this approximation. For separation of pole and cut contribution, difference in their energy dependence was used in the three-loop approximation.
Another explanation was done by
Caron-Huot S., Gardi E., Vernazza L., 2017
It is based on representation of scattering amplitudes by wilson lines and using the shock wave approximation. Unfortunately, the approaches used and the results given in these papers are different, although both approach explain the observed violation.
In particular, in the last approach the Reggeon-cut mixing is introduced, which is not needed in the first one.

Three-Reggeon cuts

There is a big difference between the reggeon cuts in QCD and in the earlier (before appearence of QCD) theory of complex angular momenta. The original reason for this difference is the different concept of Reggeons. In the previous theory, reggeons were represented by the sum of ladder diagrams, while in QCD the primary Reggeon is a Reggeized gluon.
In particular, this difference makes invalid the assertion
J.C. Polkinghorne, 1963
S. Mandelstam, 1963
on the fallacy of the proposal
Amati D., Stanghellini A., Fubini S.
to form a cut by contributions of plane diagrams and the statement that only nonplanar diadrams do contribute to the cut.
It is clearly demonstrated by the BFKL Pomeron.

Three-Reggeon cuts

Thus, there are two approaches to calculation of three-Reggeon cut contributions. Our approach is based on consideration of Feynman diagrams (we will call it diagrammatic approach). Another approach is based on representation of amplitud by Wilson lines and shock wave formalism (we will call it Wilson line approach).

The main difference between them is the consideration of the color structure of the cuts. The momentum space parts of the cut contribution are calculated in similar way.

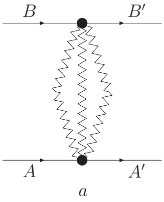
In the two and three loop approximation they were found in
F. V.S., 2016; F. V.S., Lipatov L.N., 2017,
Caron-Huot S., Gardi E., Vernazza L., 2017
and in four loops in
F. V.S., 2019, 2020
G. Falcioni, G. Gardi, N. Maher, C. Milloy, L. Vernazza, 2022

Three-Reggeon cuts

In the leading (two loop) approximation cut contributions are represented by the left diagram. In both approaches the Reggeon propagator is

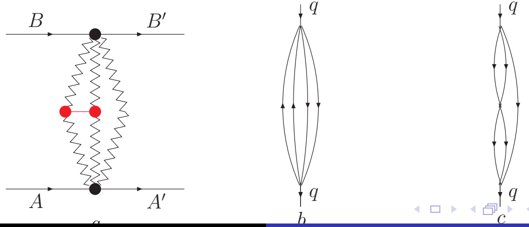
$$\frac{1}{\mathbf{k}_\perp^2},$$

so that momentum part of the diagram is presented by the right diagram.



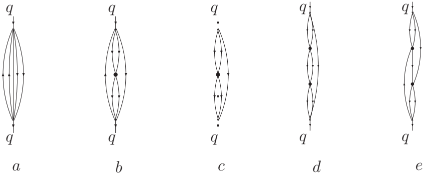
Three-Reggeon cuts

In higher approximations cut contributions are calculated supposing pair interaction between Reggeons describing by the BFKL kernel. In fact, such interaction was asumed many years in the BKP equation
Bartels J.,Kwiecinski J., Praszalowicz M., 1980
for the odderon (colouless state of three Reggeized gluons with posidive signture, wich differs from Pomeron by C-parity). This assumption looks rather natural, although, as I know, its strict proof does not exist, as well as its check in perturbation theory. Therefore, in the three-loop approximation the cut contribution is presented by the diagram a on the left



Three-Reggeon cuts

and its momentum pars by two diagrams, b and c, on the right. Conribution of these diagrams were found in
Accordingly, in the four-loop approximation the cut contribution is presented by two upper diagramms and its momentum parts by five diagrams below.



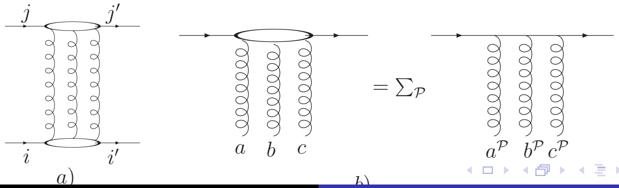
Colour structure of the cuts

A ctucial question is the colour structure of reggeon cuts. This question is simple in the two-reggeon case, because in the product of two adjoint representations there is only one representation of given dimension and parity. But it becomes quite non-trivial in the case of of three-reggeon cuts.

The Reggeon cut contributions are obtained as the sums of their momentum parts with the colour coefficients. The colour coefficients are defined by their leading order values.

Due to the signature conservation the cut with negative signature must be the three-Reggeon one.

Since our Reggeon is the Reggeized gluon, the three-Reggeon cut first contribute to amplitudes corresponding to the diagrams



Colour structure of the cuts

The difference between diagrammatic and Wilson line approaches is that in the first one colour structures of whole diagrams are considered, while in the second particle-cut interaction vertices are introduced.

Note that In contrast to the Reggeon, which contributes only to amplitudes with the adjoint representation of the colour group (colour octet in QCD) in the t-channel, the cut can contribute to various representations.

Possible representations for quark-quark and quark-gluon scattering are only singlet (**1**) and octet (**8**), whereas for the gluon-gluon scattering there are also **10**, **10*** and **27**.

It occurs that the difference between two approaches concern only to the adjoint representation, where Reggeon-cut is possible.

Colour structure of the cuts

It turns out that for the representations R different from the Reggeized gluon one the colour coefficients do not depend on σ, so that momentum dependent factors for them summed up to the eikonal amplitude. It is very important because it garantee gauge invariance

It is not so in the Reggeized gluon channel in the Wilson line approach.

The vertex of particle-cut interaction for this channel introduced in

Caron-Huot S., Gardi E., Vernazza L., 2017

has the form

$$V^{(c)}_{a'a} = \mathcal{T}^{c}_{a'a} \frac{1}{3!(N_c^2-1)T_i T_j} \operatorname{Tr} \sum_{\sigma} \left(\mathcal{T}^{c_1^\sigma} \mathcal{T}^{c_2^\sigma} \mathcal{T}^{c_3^\sigma} \mathcal{T}^{c} \right) ,$$

But their introduction contradicts to the limit of large number of colours and gauge invariance.

Colour structure of the cuts

Recently, in the Wilson line approach, the contributions of the three-reggeon cuts to the elastic scattering amplitudes have been calculated in four loops
G. Falcioni, G. Gardi, N. Maher, C. Milloy, L. Vernazza2022
To avoid contradiction with the planar N = 4 SYM, the authors proposed a scheme for separating the contribution of the pole and the cut to the NLLA in all orders of perturbation theory, based on the assertion that the diagrams for Regge cuts are nonplanar.
As was already discussed, this statement, which comes from the old theory of complex angular momenta, is inapplicable in QCD. In addition, such a separation of the pole and cut contributions is not gauge invariant, which is unacceptable.

Vertex of particle-cut interaction

The question arises:
Is it possible to introduce particle-cut interaction vertices in the colour space without contradictions with large N_c limit and gauge invariance?
Supposing symmetry of the vertex in the momentum space, it has to have the form

$$V^A(cut)_{i\to abc} = a_A\left[\delta_{ab}\delta_{ic} + \delta_{bc}\delta_{ia} + \delta_{ac}\delta_{ib}\right]$$
$$+N_c b_A\left[d_{abl}d_{icl} + d_{bcl}d_{ial} + d_{acl}d_{ibl}\right]$$

Let

$$V^A(total)_{i\to abc} = V^A(R)_{i\to abc} + V(cut)^A_{i\to abc}$$

Due ro symmetry $V(cut)$

$$V^A(cut)i \to abc = V^A(cut)_{i\to a^\sigma b^\sigma c^\sigma}$$

Vertex of particle-cut interaction

we have

$$V^A(cut)_{i\to a^\sigma b^\sigma c^\sigma} V(R)^B_{i\to abc} = V(cut)^A_{i\to abc} V(R)^B_{i\to abc}$$

In the two-loop aproximation there is not Reggeon-cut mixing. Therefore it should be

$$V^A(cut)_{i\to abc}\Big[V^B i \to abc - V^B(cut)_{i\to abc}\Big] = 0$$

Direct calculation gives equations

$$(N_c^2+1)a_G(2-3a_G)+(N_c^2-4)(N_c^2-8)b_G(\frac{1}{2}-6b_G)$$

$$+\frac{N_c^2-4}{2}(a_G+8b_G-24a_Gb_G)=0\ ,$$

Vertex of particle-cut interaction

$$(N_c^2+1)a_Q(\frac{1}{4N_c}-3a_G)$$

$$+(N_c^2-4)(N_c^2-8)b_Q(\frac{1}{4N_c}-6b_Q)+\frac{N_c^2-4}{4N_c}(a_Q+2b_Q-48N_c a_Q b_Q)=0\,,$$

$$(N_c^2+1)(2a_Q+\frac{a_G}{4N_c}-6a_G a_Q)$$

$$+(N_c^2-4)(N_c^2-8)(\frac{b_Q}{2}+\frac{b_G}{4N_c}-12b_Q b_G)$$

$$+\frac{N_c^2-4}{4N_c}(2a_Q N_c+a_G+4b_Q N_c 2b_Q-48N_c(a_Q b_G+a_G b_Q)=0\,.$$

These equations have solution (that is not trivial), but it gives the vertex introduced in

Caron-Huot S., Gardi E., Vernazza L., 2017

which is not acceptable.

What does it mean?

Summary

- The pole Regge form of amplitudes with gluon quantum numbers in cross channels and negative signature, which is the basis of the BFKL equation, is violated in the NNLLA.
- The observed violation can be explained by the cut contributions.
- There are two different explanations of the violation.
- The main difference between the approaches is the colour structure of the three-Reggeon cut contributions.
- In the diagrammatic approach it is determined from direct calculations of Feynman diagrams.
- In the Wilson line approach vertices of particle-cut interaction are introduced.
- But it contradicts to large N_c limit and gauge invariance.
- Separation of the cut contributions as contributions of nonplanar diagrams seems not having serious ground.
- It seems that using particle -Reggeon vertices in the colour space is not possible in the case of three-Reggeon cut.