Thermal False Vacuum Decay around Black Holes

Andrey Shkerin

Perimeter Institute for Theoretical Physics

with V. Briaud, S. Sibiryakov

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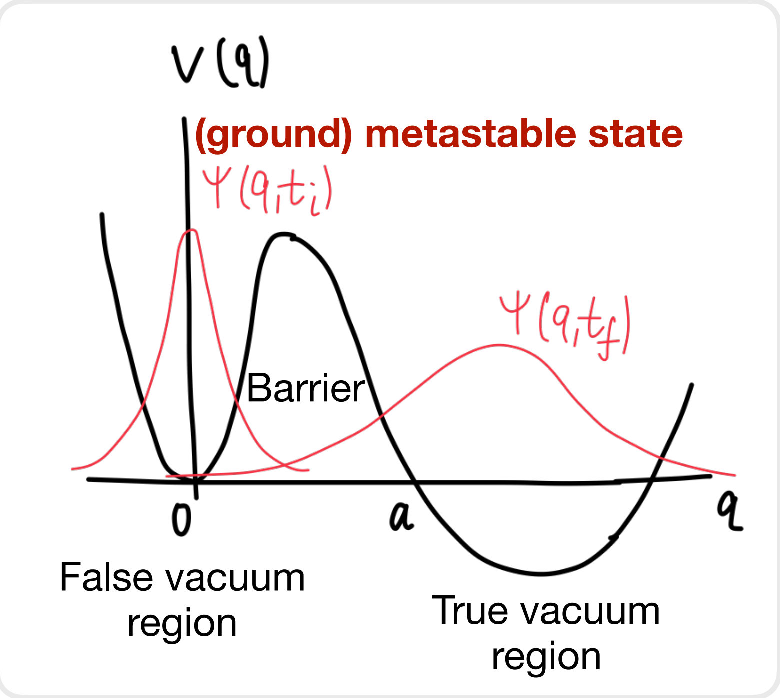
Decay of metastable state

- Consider the quantum-mechanical system with the Hamiltonian $H = \frac{p^2}{2m} + V(q)$ and the “tunneling” potential.

- Probability of survival of the metastable state is $P \sim e^{-\Gamma t}$

WKB: $\Gamma \sim e^{-B}$ — decay rate

$$B = 2\int_0^a \sqrt{2mV(q)}\, dq$$ — suppression exponent



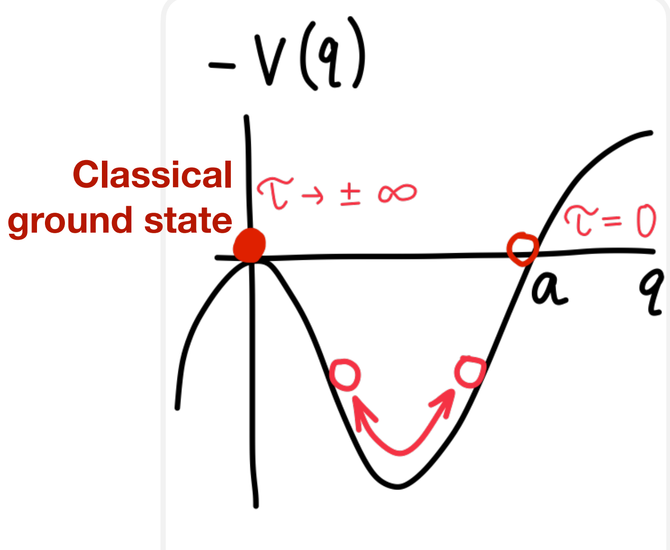
- One can write in terms of the **bounce** trajectory: $B = S_E[q_B]$

Euclidean action: $$S_E = \int d\tau \left(\frac{m}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right)$$

Classical equation of motion: $$m\frac{d^2q}{d\tau^2} = \frac{\partial V}{\partial q} \equiv -\frac{\partial(-V)}{\partial q}$$

(False) vacuum boundary conditions: $q_B(\pm\infty) = 0$,

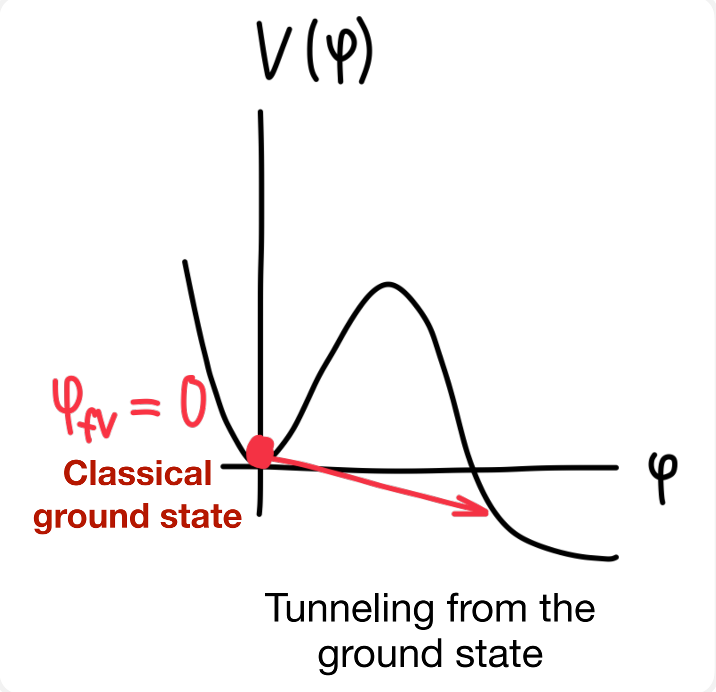
Turning point: $\dot{q}_B(0) = 0$



The classical particle moves along the bounce trajectory

Decay of metastable state in field theory

- Consider the scalar field theory with the Lagrangian $\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ (in flat space) and the tunneling (configuration-space) potential



Tunneling from the ground state

WKB: $\Gamma \sim e^{-B}$

Coleman 77; Callan, Coleman 77

$$B = S_E[\varphi_B]$$

$$S_E = \frac{1}{g^2}\int d\vec{x}\, d\tau \left(\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial\vec{x}}\right)^2 + V(\varphi)\right)$$

$g \ll 1$ — coupling constant

$$\partial_\mu \partial^\mu \varphi - V'(\varphi) = 0$$

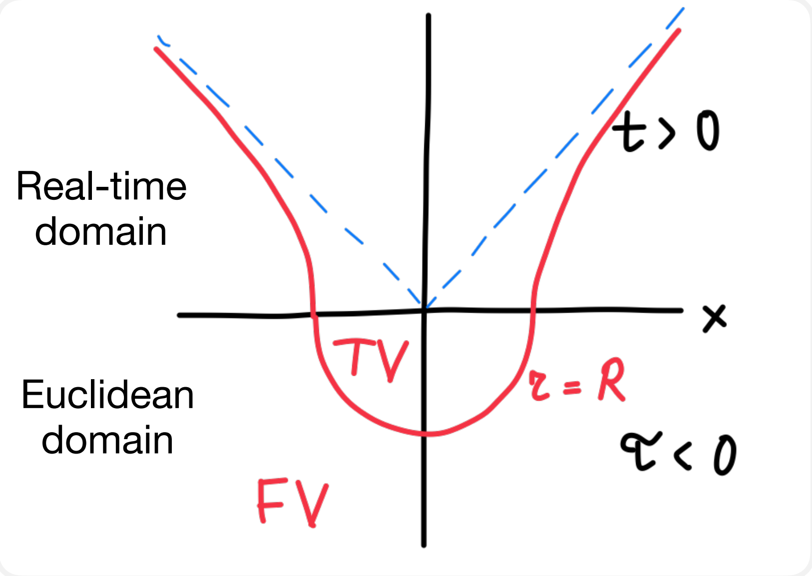
- The vacuum bounce is spherically symmetric in *d+1* dimensions,

Coleman, Glacer, Martin 78;
Blum, Honda, Sato, Takimoto, Tobioka 16

$$\varrho = \sqrt{\tau^2 + \vec{x}^2}$$

- Vacuum boundary conditions: $\varphi_B(\varrho \to \infty) \to 0$

Turning point: $\dot{\varphi}_B(\tau = 0, \vec{x}) = 0$



Decay of metastable state at finite temperature

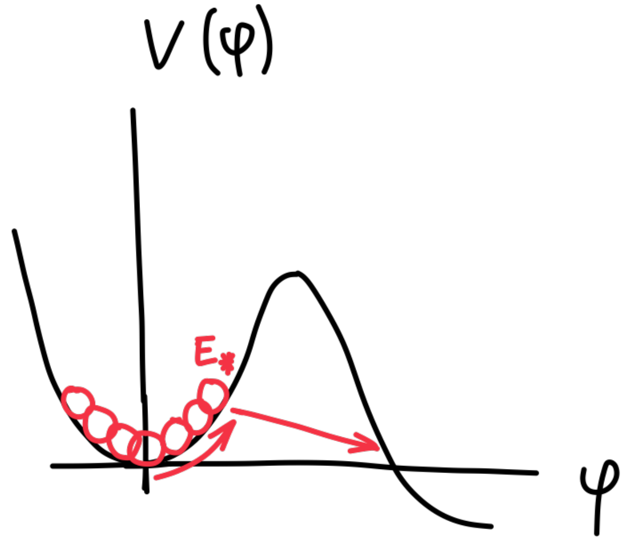
- Consider the scalar field theory with the Lagrangian $\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ and tunneling from the **thermally-populated** initial state

As usual (at not too high temperatures), $\Gamma \sim e^{-B}$

$$B = S_E[\varphi_B]$$

$$\partial_\mu \partial^\mu \varphi - V'(\varphi) = 0$$

Boundary conditions for the thermal bounce?



Tunneling from an excited state

- Thermal partition function implies **periodic** boundary conditions

Thermal averaging: $\Gamma \sim \int dE\, e^{-\frac{E}{T}}\, e^{-S_E[\varphi_{B,E}]} \sim e^{-\frac{E_*}{T} - S_E[\varphi_{B,E^*}]} = e^{-B}$

$$\longrightarrow \quad \varphi_B(\tau + 1/T, \vec{x}) = \varphi_B(\tau, \vec{x})$$

Linde 82;
Brown, Weinberg 07

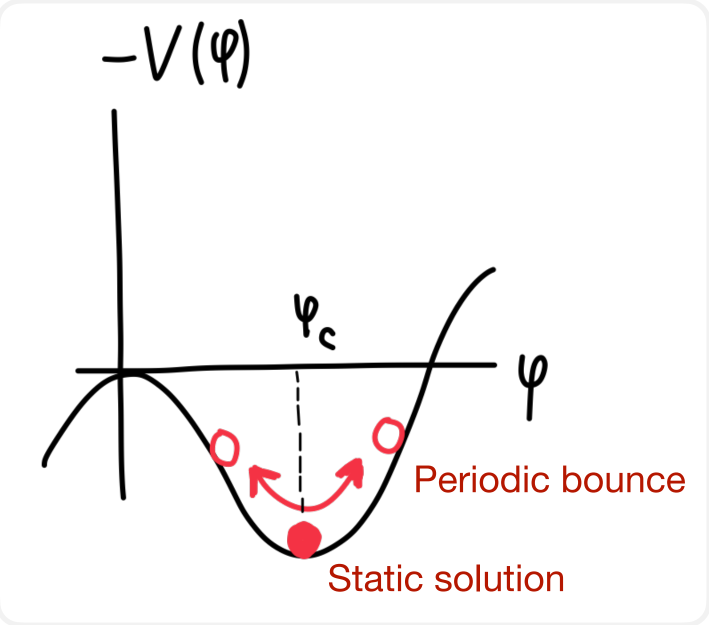
Decay of metastable state via thermal activation

- At large T, one expects the decay to occur via classical thermal jumps of the field over the barrier.

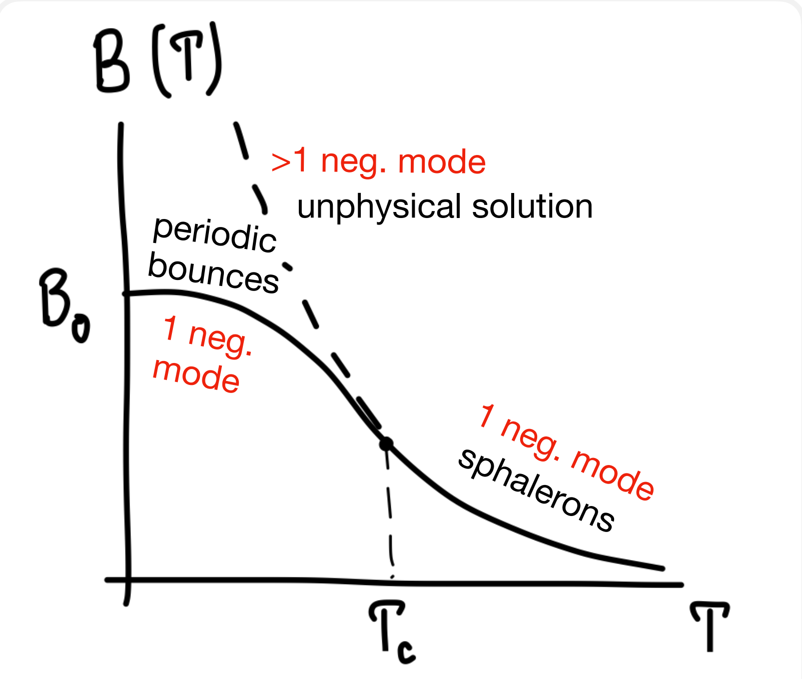
  In the WKB, this is described by the static solution — **sphaleron**.

Klinkhamer, Manton 84

$$\Gamma \sim e^{-\frac{E_{sph}}{T}} \qquad B = \frac{1}{T} E_{sph} \qquad (T \gtrsim T_c)$$



- Periodic bounces dominate at low T — tunneling
  Sphaleron dominates at large T — thermal jumps



Exponential suppression of vacuum decay as a function of temperature - typical plot

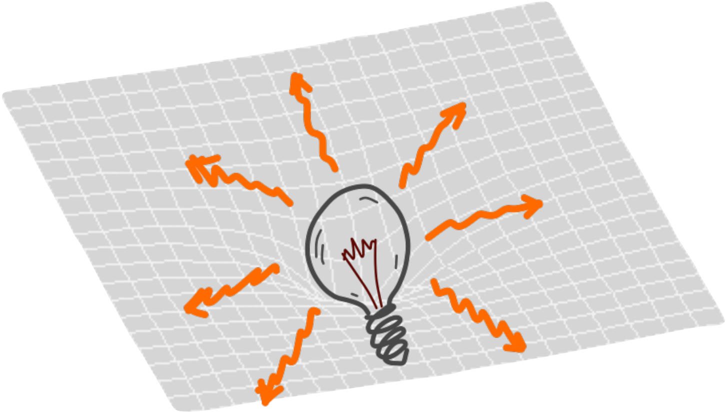
- Phase transition driven by classical fluctuations can be studied in real-time lattice simulations

Grigoriev, Rubakov 87

Grigoriev, Rubakov, Shaposhnikov 88, 89

Black holes and thermal vacuum decay

How is the above picture of thermal vacuum decay modified in the presence of black holes?



radiating BH

BH features:

- It’s a simple gravitational impurity — **curved geometry**

  S. Chandrasekhar:

  > “The black holes are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time… They are the simplest objects as well.”

- It’s a simple source of (almost) thermal radiation — **quantum vacuum**

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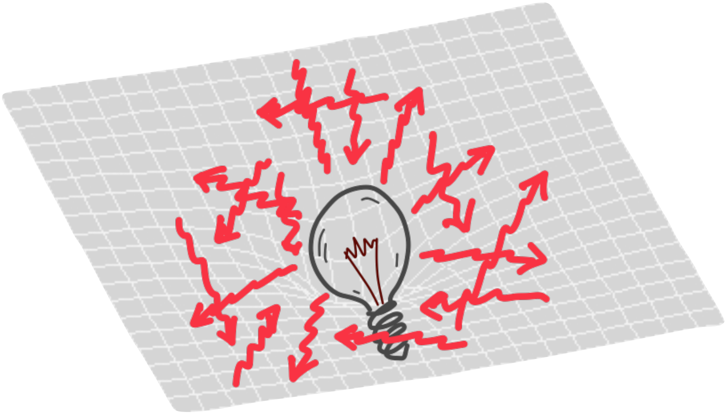

BH in equilibrium with the environment

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- It’s a simple source of (almost) thermal radiation — **quantum vacuum**
- In thermal equilibrium, vacuum decay can be treated in the Euclidean time approach

  This is not true in the (non-equilibrium) case of BH emitting radiation in vacuum

- Previous studies reveal **the puzzle:**

  the minimal-action O(3)-symmetric configurations were found to be time-independent, regardless the BH temperature

  It seems that no periodic bounces exist around at least certain classes of BHs.

  Physical understanding? Proof?

  We’ll discuss scalar fields in 2d/4d BH backgrounds and **neglect gravitational back-reaction.**

Rindler Valley

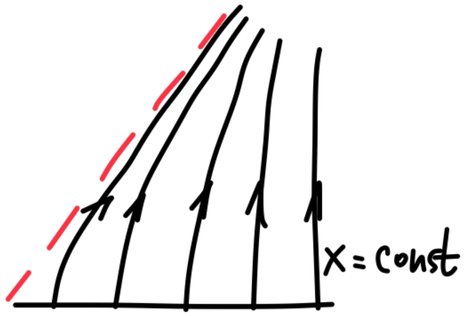
- Consider uniformly accelerating observers in flat spacetime in 2d:

$$ds^2 = dT^2 + dX^2$$

$$T = \frac{1}{\lambda} e^{\lambda x} \sin \lambda \tau \ , \quad X = \frac{1}{\lambda} e^{\lambda x} \cos \lambda \tau$$

$$ds^2 = \Omega^2(x) \left(d\tau^2 + dx^2 \right) , \quad \Omega^2(x) = e^{2\lambda x}$$

— **Rindler Frame**



trajectories of observers with acc. $\lambda e^{-\lambda x}$

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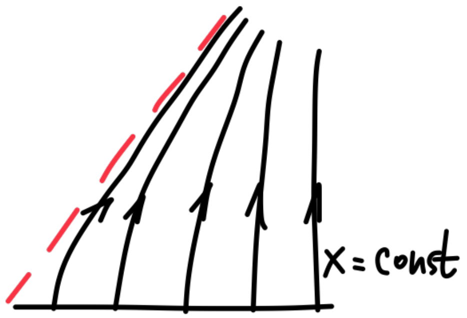
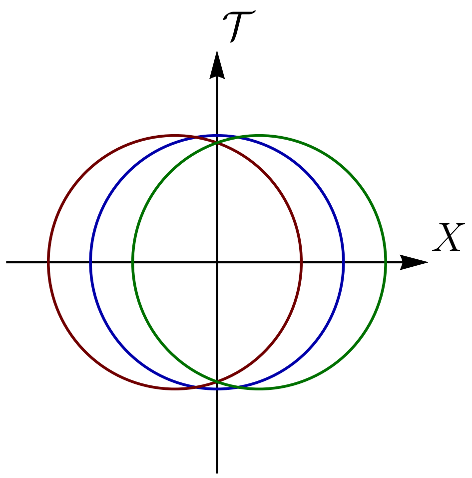


trajectories of observers with acc. $\lambda e^{-\lambda x}$

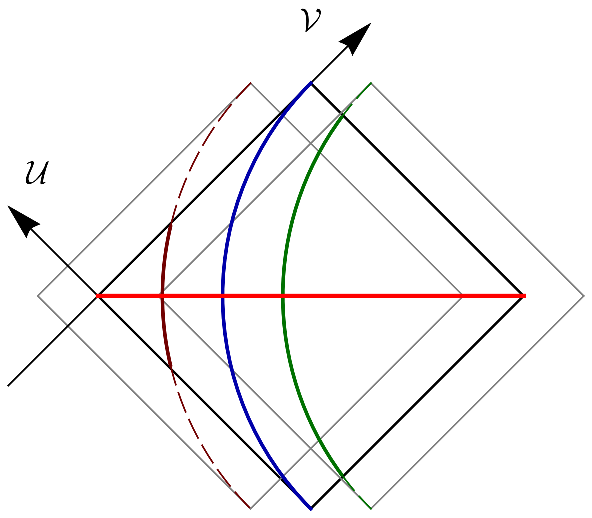
- Bounce in the Rindler Frame:

$$\varphi_B \left(\sqrt{X^2 + T^2} \right) \equiv \varphi^{(\lambda)}_{B,0}(x)$$

$$\varphi_B \left(\sqrt{(X - X_0)^2 + T^2} \right) \equiv \varphi^{(\lambda)}_{B,X_0}(x, \tau)$$



Bounce in the Minkowski space



Bounce in the Rindler wedge

- Since the decay rate $\Gamma = A_0 e^{-B_0}$, $B_0 = S_E[\varphi_B]$, is frame-independent, then

$S_E[\varphi^{(\lambda)}_{B,X_0}] = B_0$ — doesn't depend on λ and X_0 — **Flat direction**

Rindler Valley near BH horizon

- Generally, near the BH horizon, in the appropriate coordinates, $ds^2 = \Omega^2(x)\,(d\tau^2 + dx^2) + \dots$

$$\Omega^2(x) = e^{2\lambda x}$$

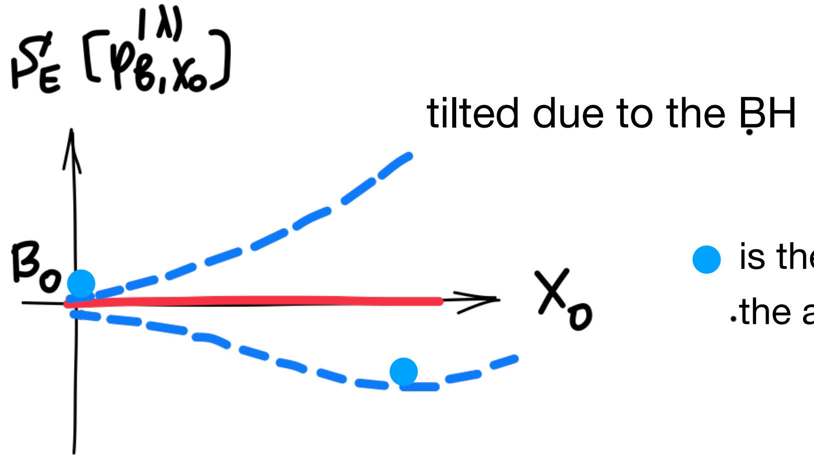
$$\lambda = 2\pi T_{BH}$$

becomes big at $x \approx r_h \propto \frac{1}{T_{BH}}$

- Rindler bounces with $R_0 \ll r_h$ are **almost** the true tunneling solution around the BH.

$$S_E[\varphi^{(\lambda)}_{B,X_0}] = B_0 + \alpha\, T_{BH}^2 X_0^2 + \dots$$

sign α is then crucial



● is the configuration closest to the actual tunneling solution

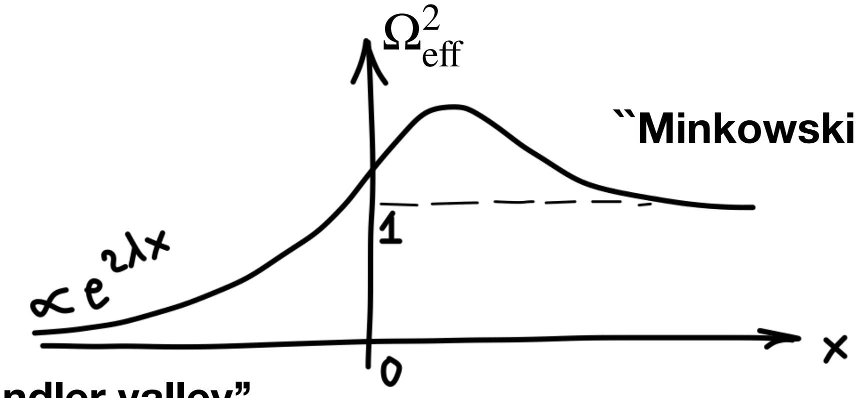
- The computation shows that $\alpha > 0$ for a large class of BHs (including dilaton BH in 2d and Schwarzschild BH in 4d)

Valleys and Mirrors

Can we do more?

emulates the centrifugal barrier for massive linear modes

Consider the 2d dilaton BH background: $\Omega_{\text{eff}}^2 = \frac{1}{e^{-2\lambda x}+1} + \frac{q e^{-2\lambda x}}{(e^{-2\lambda x}+1)^2}$



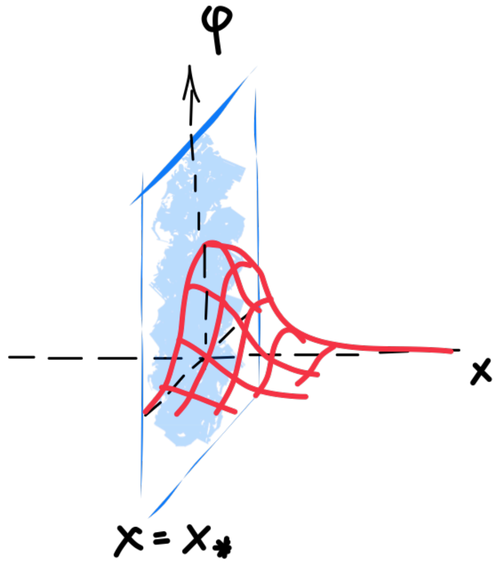
``Minkowski valley”

``Rindler valley”

- What if the bounce sits on the barrier?

(This is what actually happens for the Boulware bounce)

We can scan across this region with a one-parameter family of configurations.



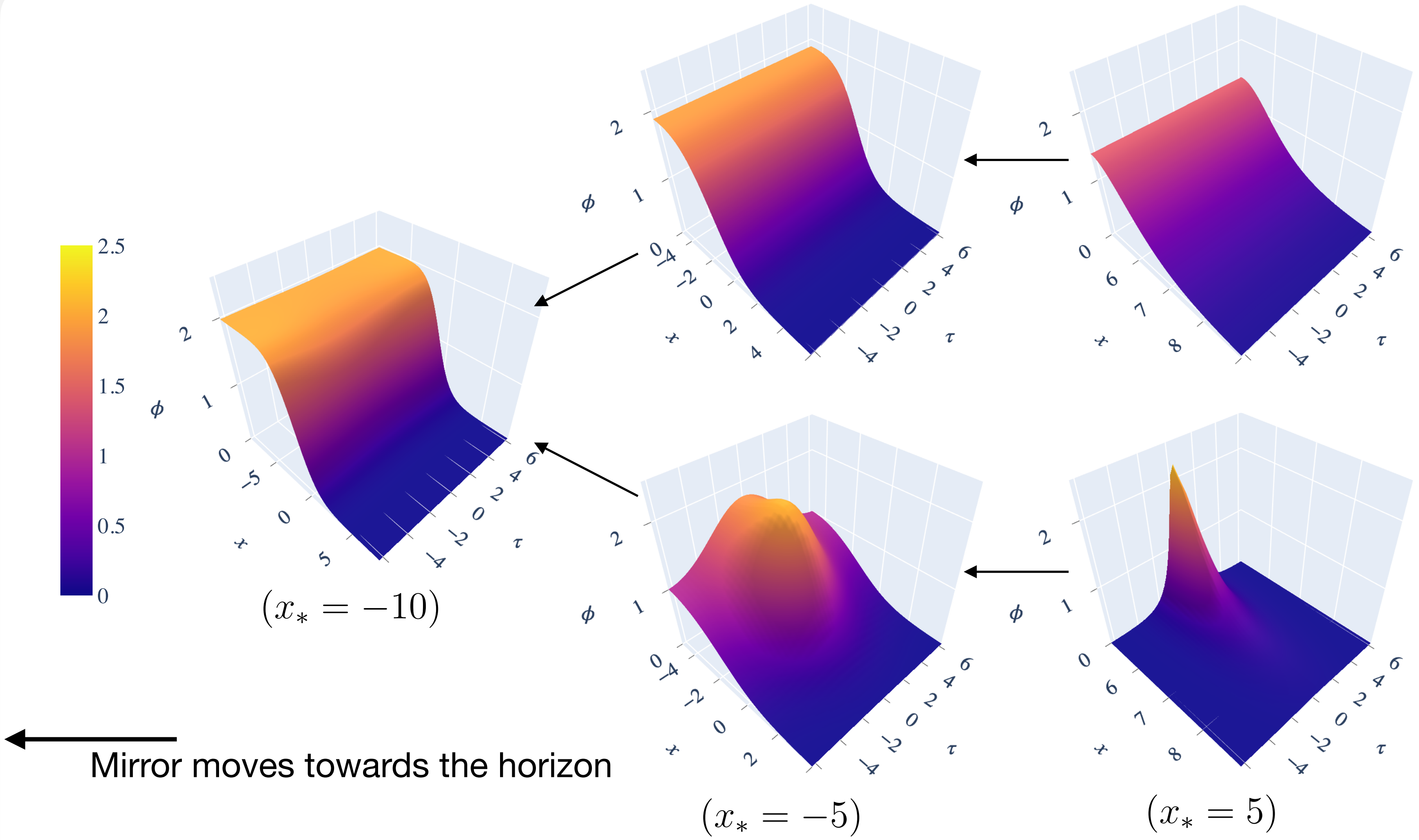
- Put the **Neumann mirror** at distance $X_* > 0$ from the horizon.
- Attach the periodic solution to the mirror. If $X_* \gg \lambda^{-1}$, this is the half-thermal bounce.
- Move the mirror towards the horizon, see what happens with the solution at $X_* \approx 0$

— whether it decouples from the mirror and retains time-dependence, or

— whether it collapses to the BH sphaleron

- Repeat the exercise with the half-flat space sphaleron as a seed.

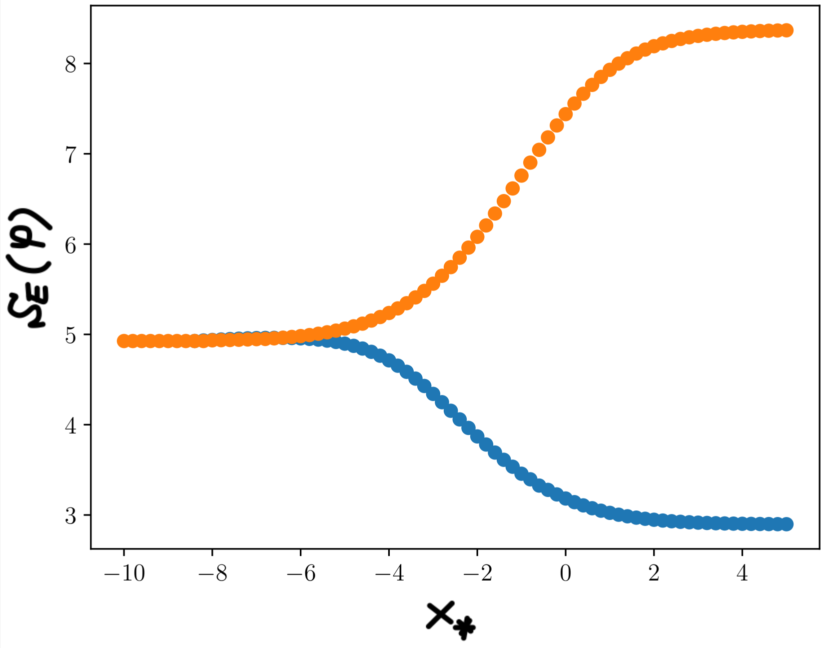
Fun with the mirrors



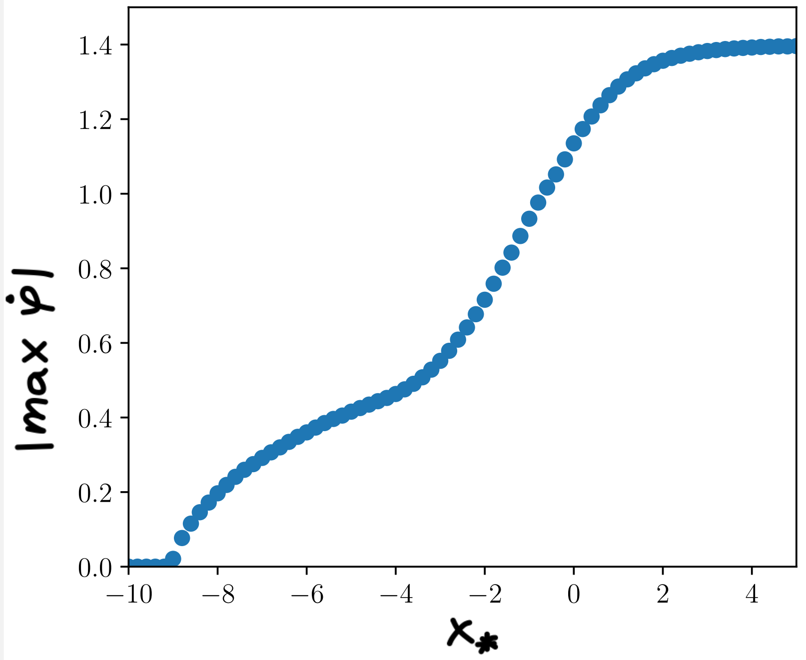
$(x_* = -10)$ $(x_* = -5)$ $(x_* = 5)$

Mirror moves towards the horizon

Evolution of the time-dependent (bottom) and time-independent (top) configurations attached to the moving mirror. Scalar field theory with quartic self-interaction in 2d dilaton BH background.



Action of the sphaleron (top line) and the periodic solution (bottom line), attached to the mirror, as a function of the position of the mirror.



Maximum value of the time derivative of the solution as a function of the position of the mirror.

Absence of dipole negative mode

Can we do more?

- The evidence comes from counting negative modes of the sphaleron.

  (Also see the talk by G. Lavrelashvili at this conference)

2210.08028

We can prove that, for a general **multi-scalar theory** and a wide class of **spherically-symmetric BHs** in *d* dimensions, the BH sphaleron has exactly one *O(d-1)*-symmetric (**monopole**) negative mode at any BH temperature.

(The proof is somewhat technical)

Let $ds^2 = f(r)\, d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma^2_{d-2}$

Then, the sufficient condition for the absence of negative modes in the **dipole** sector and above is

$$\mathcal{D}(r) \equiv \frac{f'(r_h)^2 - f'^2}{4} - \frac{(d-2)f^2}{r^2} + \frac{(d-2)ff'}{2r} + \frac{ff''}{2} > 0$$

- The condition holds for Schwarzschild (anti-)de Sitter BHs.

  Counter-example: the RN BH with the charge $Q^2 > Q_*^2 \approx 0.833\, M_{BH}^2$

- The condition says nothing about the negative modes violating the spherical symmetry. Such modes certainly exist for large BHs in *4d*.

Applications?

Maybe?…

- Holographic first-order phase transitions
- …

Creminelli, Nicolis, Rattazzi 2001

…

Mishra, Randall 2023

Thank you!

More on bounce at finite temperature

The bounce-sphaleron transition point was studied in QM and field theory

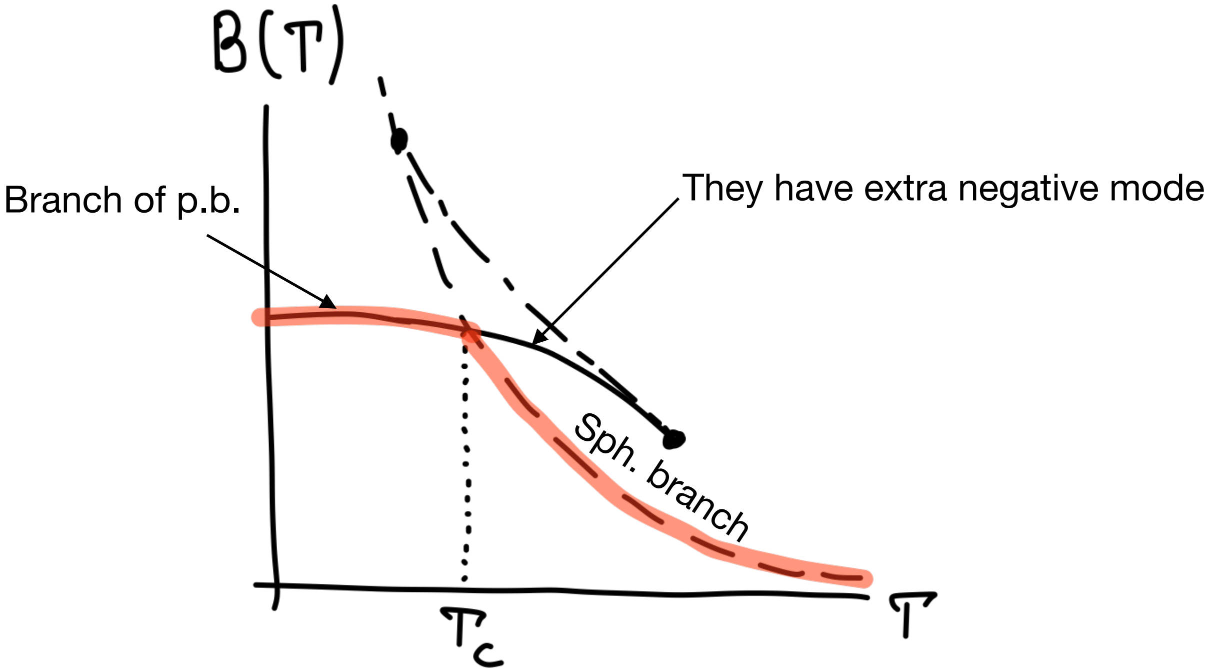
Chudnovsky 92

Garriga 94; Ferrera 95

In the thin-wall approximation, periodic bounces do not merge with the sphaleron — the transition is 1st order.

(The thin-wall sphaleron may not even exist)

This is not an artefact of the approximation.



Exponential suppression of vacuum decay in the thin-wall potential in 3+1 dimension.