False vacuum decay around a black hole

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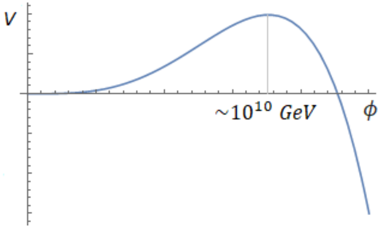
The Higgs vacuum

The effective Higgs potential has a false vacuum:

$$V_{eff}(\phi) = \frac{1}{4}\lambda_{eff}(\phi)\phi^4$$



The effective Higgs potential

D. Buttazzo et al (2013)

Coleman instantons

$$S = \int d^4x \left(\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_x \phi)^2 - V(\phi) \right)$$

The probability of false vacuum decay in flat space-time:

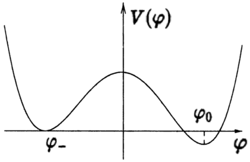
$$P \sim e^{-S_E}$$

Periodic instantons in a thermal bath (for the potential $V(\phi)$):

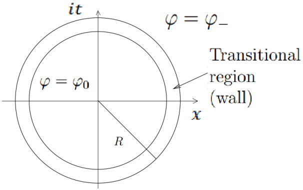
$$T_{Period} = \beta$$

For a very high temperature T:

$$P \sim e^{-E_{sph}/T}$$



The potential with false vacuum



The Euclidean solution

False vacuum decay in the presence of BH

Hawking temperature:

$$T_H = \frac{M_{Pl}^2}{8\pi M_{BH}}$$

Conjecture: small black holes have high temperatures $\implies$ significantly increase the decay probability.

P. Burda et al (2016)

A calculation from first principles is needed.

(see also A. Shkerin, S. Sibiryakov (2021))

Formulation from first principles

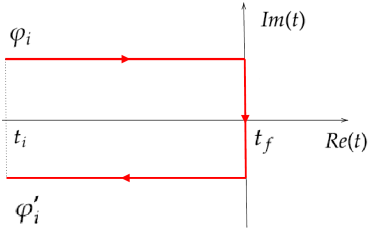
Decay probability from functional integral:

$$P = \int D\phi_f D\phi_i D\phi_i^{'} \langle\phi_f| \hat{S} |\phi_i\rangle \langle\phi_i| \hat{\rho} |\phi_i^{'}\rangle \langle\phi_i^{'}| \hat{S}^{\dagger} |\phi_f\rangle$$

Fields ϕ and $\phi^{'}$ can be written as a united field on the double-bent time contour.

Saddle-point approximation:

$$P \sim e^{iS[\phi_{cl}]+B[\phi_{cl}]}$$



The contour on the complex time plane

S.Yu. Khlebnikov et al (1991)

V.A. Rubakov, M.E. Shaposhnikov (1996)

Thermal equilibrium

Period for a thermal equilibrium case:

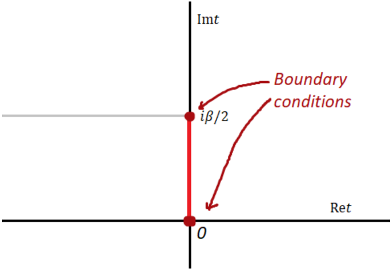
$$T_{period} = \beta_H = \beta_E$$

$\beta_H = 8\pi M_{BH}/M_{Pl}^2$ is Hawking inverse temperature.

Simplified boundary conditions:

$$\partial_\tau \phi(0, x) = \partial_\tau \phi(T_{period}/2, x) = 0$$

$$\partial_x \phi(t, -\infty) = \partial_x \phi(t, \infty) = 0$$



Euclidean part of the contour

Scalar field in Schwarzschild metric

$$S = \int \sqrt{-g} d^4 x \left(\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right)$$

$$ds^2 = f(r) dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega^2, \quad f(r) = 1 - \frac{2M}{r}, \quad M_{Pl} = 1$$

Substitution:

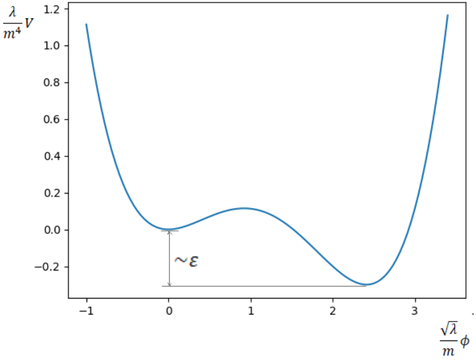
$$\phi = \frac{\varphi}{r}, \quad x = r + 2M \ln(r - 2M),$$

$$S = 4\pi \int dt dx \left(\frac{1}{2} (\partial_t \varphi)^2 - \frac{1}{2} (\partial_x \varphi)^2 - \frac{1}{2} U(x) \varphi^2 - r^2 f(r) V\left(\frac{\varphi}{r}\right) \right),$$

The toy potential:

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{m\sqrt{\lambda}}{2}\phi^3 + \frac{\lambda}{8}(1-\varepsilon)\phi^4$$

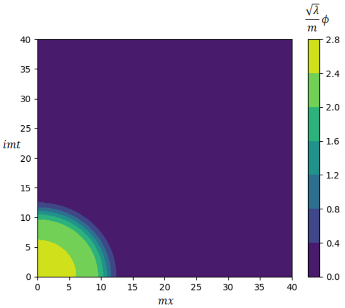
We set the parameter $\epsilon = 0.1$, then the thin wall approximation is valid.



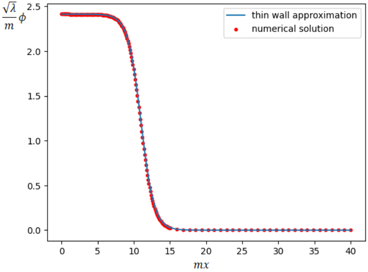
The potential V used in this work

Numerical results

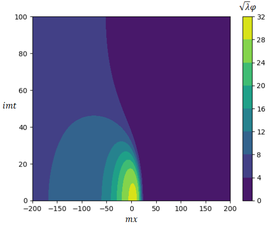
We use the Newton-Raphson method to solve the system of nonlinear equations.



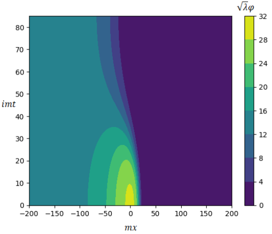
An instanton in flat space-time, $N_t * N_x = 150 * 150$



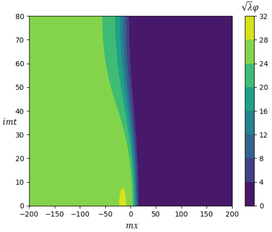
An instanton in flat space-time at $t = 0$ (dots), the thin wall approximation (line), $N_t * N_x = 150 * 150$



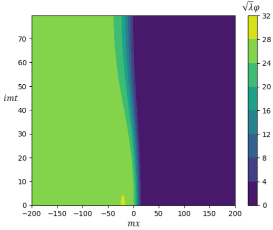
(a) $mT/2 = 100$



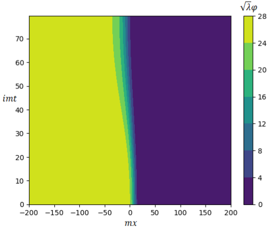
(b) $mT/2 = 85$



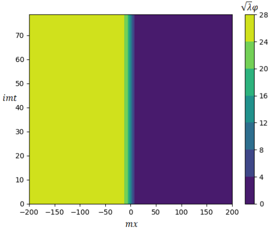
(c) $mT/2 = 80$



(d) $mT/2 = 79.7$

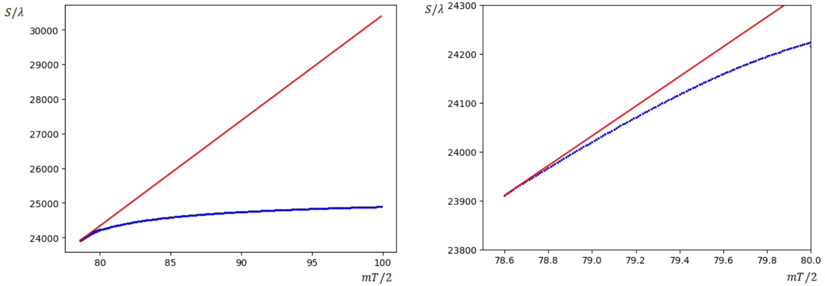


(e) $mT/2 = 79.6$



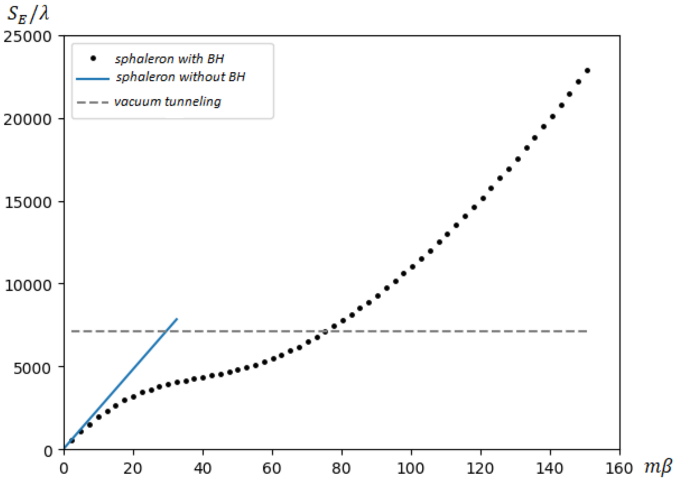
(f) $mT/2 = 78.6$

Periodic instantons with different periods in the presence of BH $mr_h = \frac{2Mm}{M_{Pl}^2} = 12$, $(N_t * N_x = 100 * 300)$. Period and mass of BH are independent parameters here.



(a) (b)

The dependence of the action of instantons S_E on the period T for non-trivial instantons (blue) and sphalerons (red), $mr_h = \frac{2Mm}{M_{Pl}^2} = 12$
$(mT_{phys} = m\beta_H = 8\pi Mm/M_{Pl}^2 = 75.4)$.



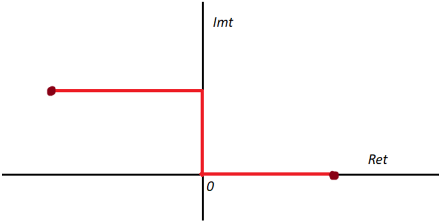
The dependence of the action of instantons S_E on the inverse temperature of the environment β in the presence of BH (dots) ($m\beta = m\beta_H = \frac{8\pi Mm}{M_{Pl}^2}$) and in the absence of BH (blue line).

Conclusions

- Physical solution at $\beta = \beta_H$ are always static sphalerons for the potential used in this work.
- $\beta \to 0$: sphalerons with BH approach flat-space sphalerons. It means that a small BH doesn't significantly change a sphaleron in a very hot environment.
- $\beta \to \infty$: a large massive BH changes the geometry of space $\implies$ sphalerons change too.

Further research

- Nonequilibrium case $\beta \neq \beta_H \implies$ a new contour and new boundary conditions



- More realistic potentials: $V(\phi) = -\lambda\phi^4/4$

Thank you for the attention

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