Non-perturbative probability distribution function for cosmological counts in cells

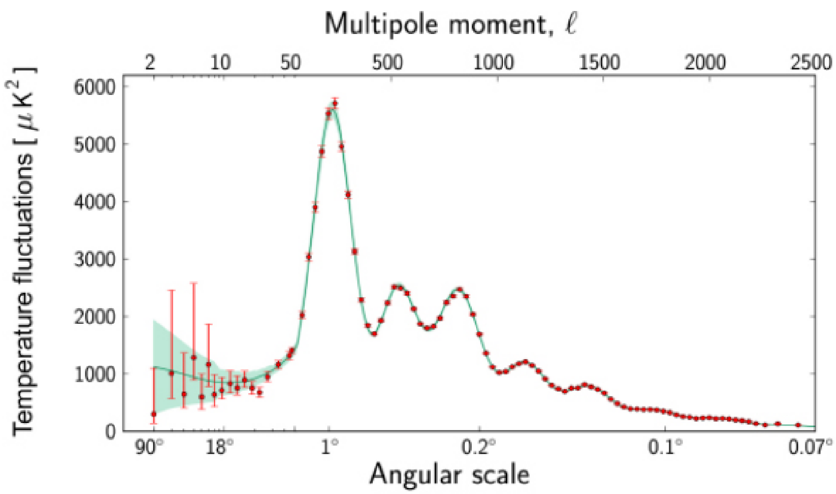
Anton Chudaykin

arXiv: 2212.09799

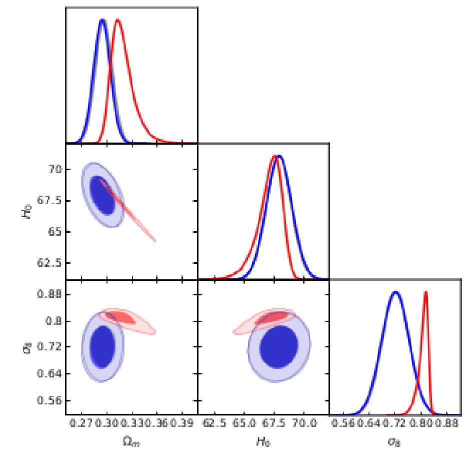
JCAP 08 (2023) 079

UNIVERSITÉ DE GENÈVE

CMB and LSS analysis



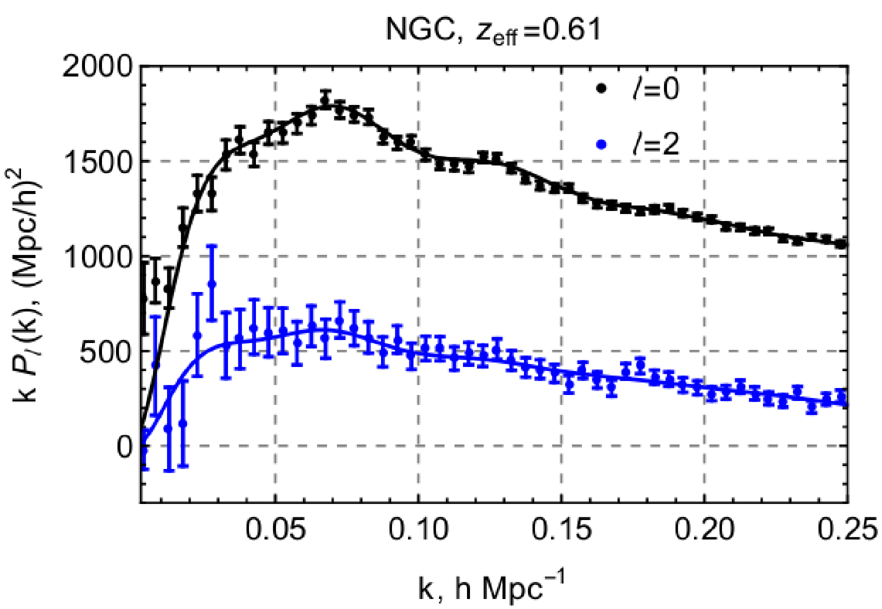
$$+C_\ell(H_0, \Omega_m, ...) =$$



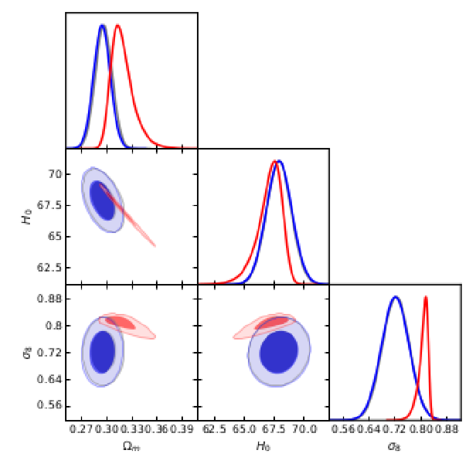
$$\Uparrow$$

CLASS,CAMB

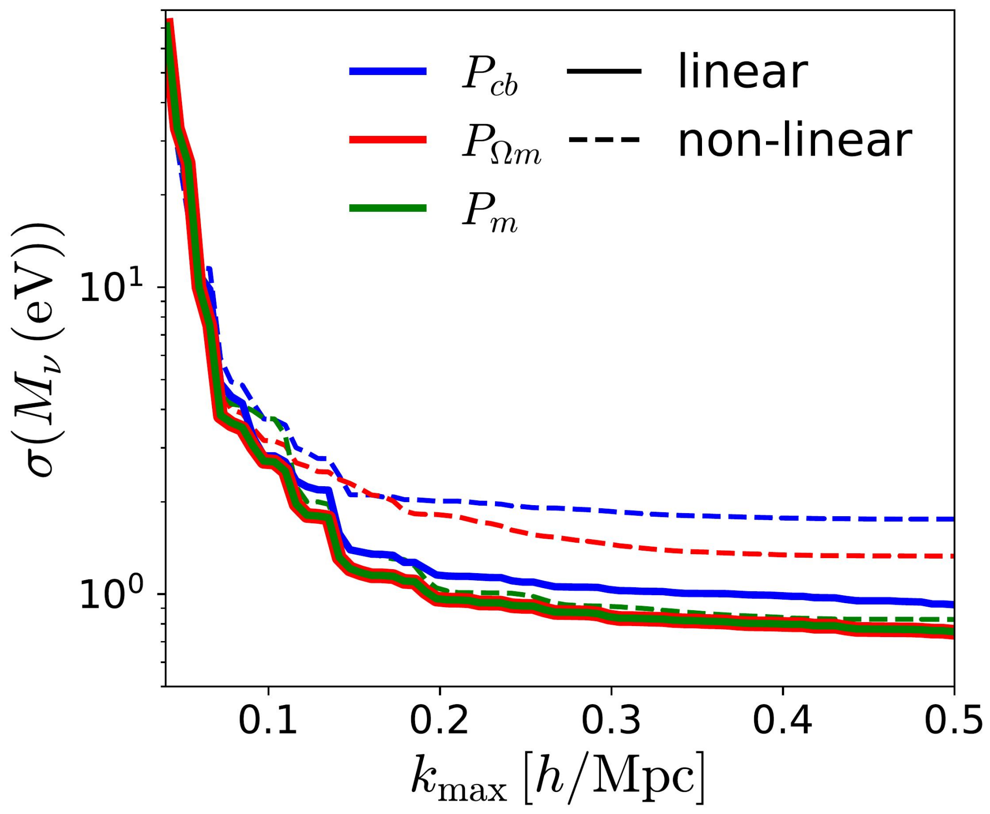
$$\Downarrow$$



$$+P_k(H_0, \Omega_m, ...) =$$

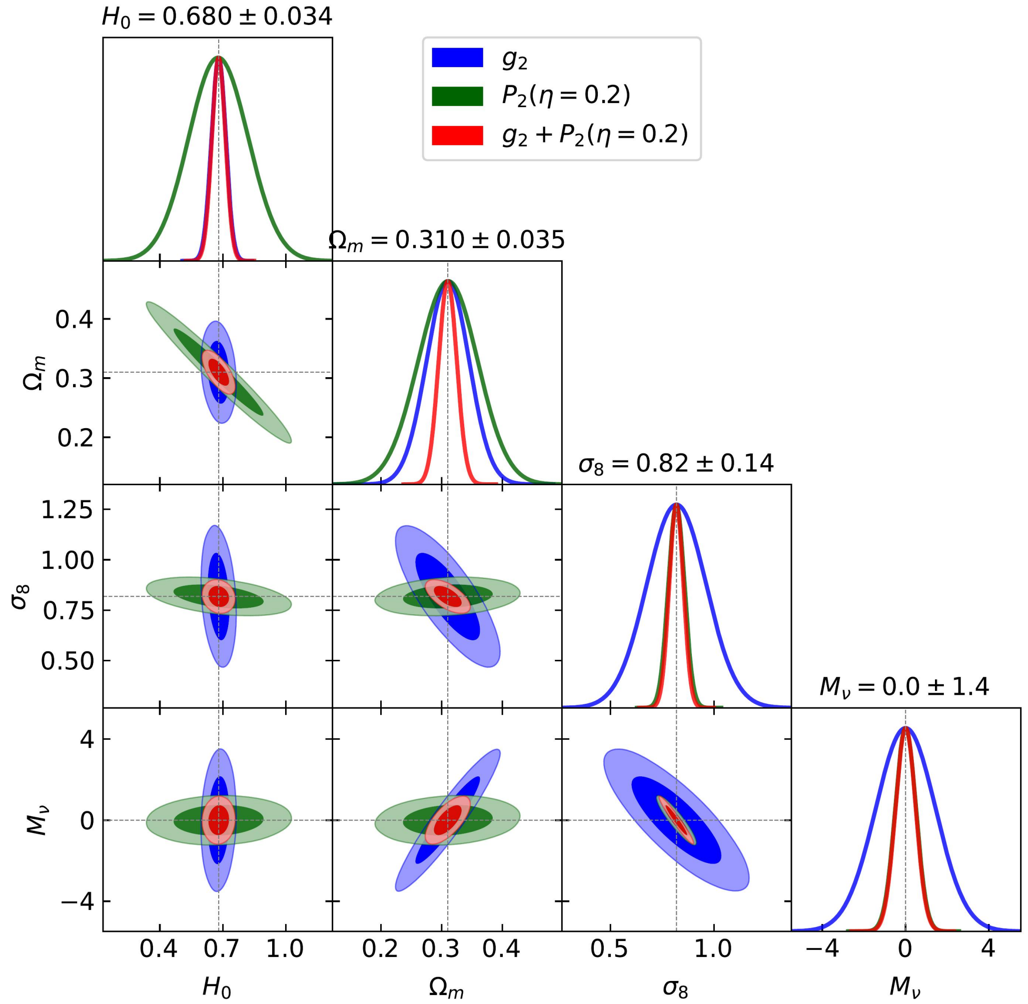


Information beyond linear physics



Bayer et al.,2022

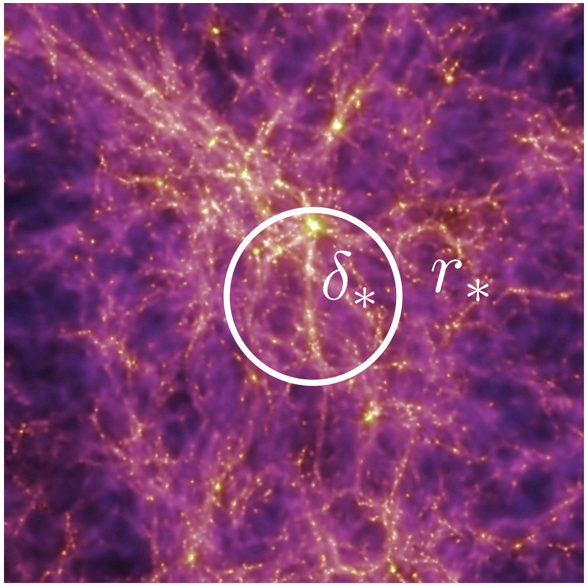
Fisher Forecast



Philcox, et al., 2023

Cosmological counts in cells

$\mathcal{P}(\delta_*)$ - probability that a cell of radius r_* has averaged density contrast δ_*



$$\alpha \sim g(z)^2 \sigma_{r_*}^2 \ll 1 \qquad \sigma_{r_*}^2 = \langle \delta^2 \rangle_{r_*}$$

$$\mathcal{P}(\delta_*) = \exp\left\{ -\frac{1}{\alpha}(a_0 + \alpha a_1 + \ldots) \right\}$$

Saddle point solution
('instanton')
defined by spherical collapse
(Valageas'02)

Prefactor
('determinant')
from perturbations around
the saddle point solution
(M.M. Ivanov'19)

Cosmological counts in cells

$$\mathcal{P}(\delta_*) = \mathcal{N}^{-1} \int \mathcal{D}\delta_L \exp\left\{ - \int_{\mathbf{k}} \frac{|\delta_L(\mathbf{k})|^2}{2g^2 P(k)} \right\} \delta_{\mathrm{D}}^{(1)}\left(\delta_* - \bar{\delta}_W[\delta_L]\right)$$

$$\bar{\delta}_W = \int \frac{d^3x}{r_*^3} \tilde{W}(r/r_*)\, \delta(\mathbf{x}) = \int_{\mathbf{k}} W(kr_*) \delta(\mathbf{k})$$

$$\mathcal{P}(\delta_*) \propto \exp\left\{ - \frac{F^2(\delta_*)}{2g^2\sigma^2_{R_*}} \right\}$$

$$\delta_L \equiv F \qquad\qquad R_* = r_*(1+\delta_*)^{1/3}$$

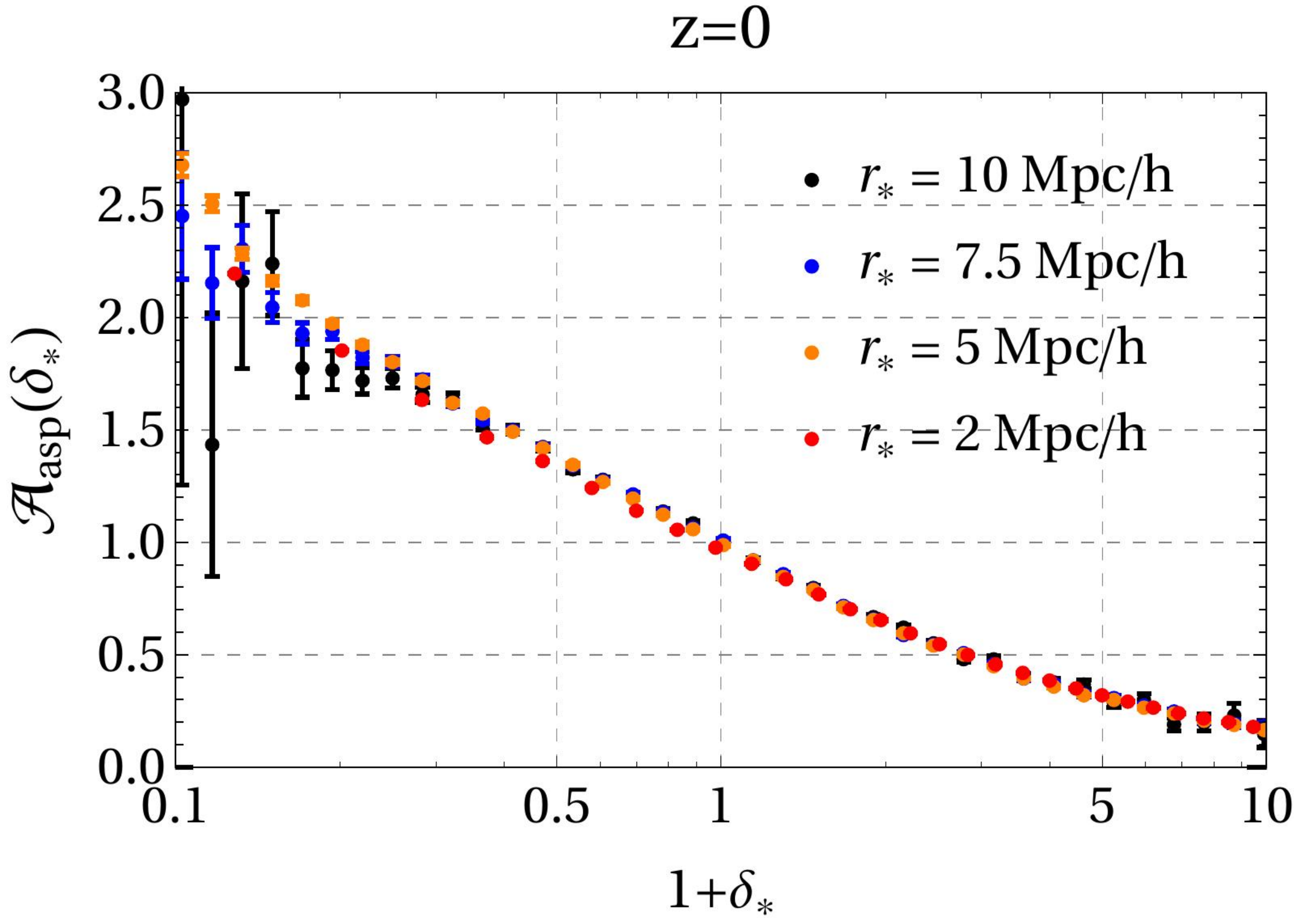
$$\mathcal{P}(\delta_*) = \mathcal{A}_0 \cdot \prod_{\ell>0} \mathcal{A}_\ell(\delta_*) \cdot \exp\left\{ - \frac{F^2(\delta_*)}{2g^2\sigma^2_{R_*}} \right\}$$

$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

Monopole prefactor
can be found precisely

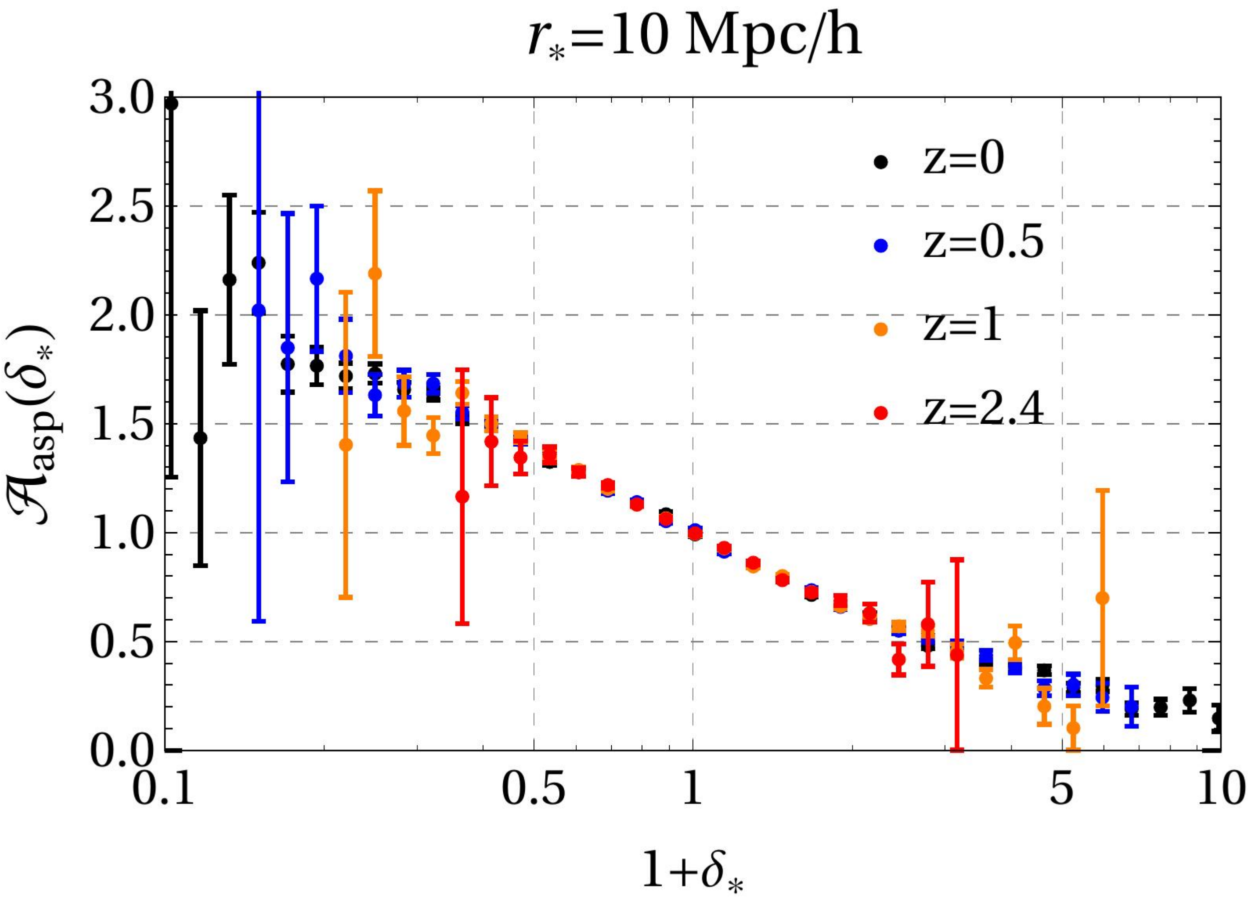
Aspherical prefactor
can be calculated on grid numerically

Semiclassical scaling



Semiclassical scaling persists down to very small r_*

Redshift dependence



Aspherical prefactor is almost redshift-independent

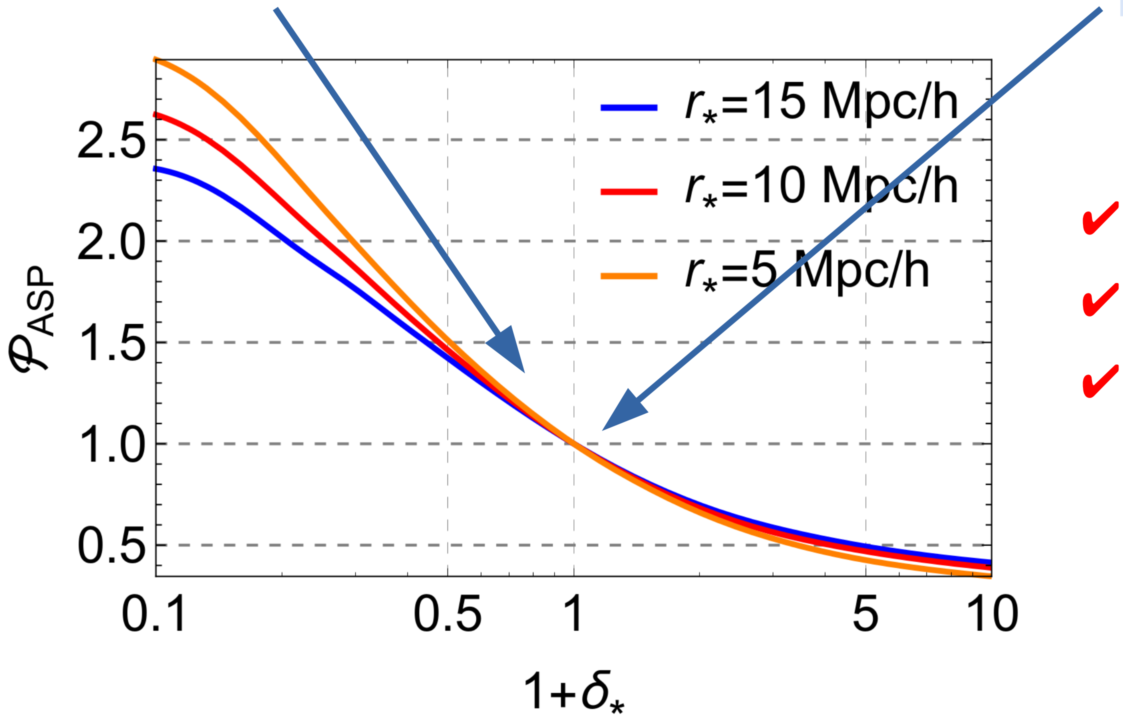
Fluctuation determinant

Value at the origin
controlled by unitarity

$$\int_{-1}^{\infty} d\delta_* \, \mathcal{P}(\delta_*) = 1$$

Slope at the origin
controlled by
translation invariance

$$\langle \delta_* \rangle \equiv \int_{-1}^{\infty} d\delta_* \, \mathcal{P}(\delta_*) \, \delta_* = 0$$



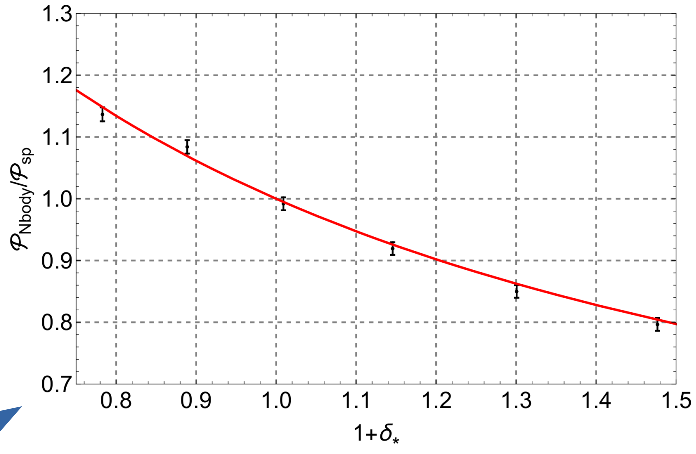
✔Unitarity
✔Translation invariance
✔Weakly depends on cosmology

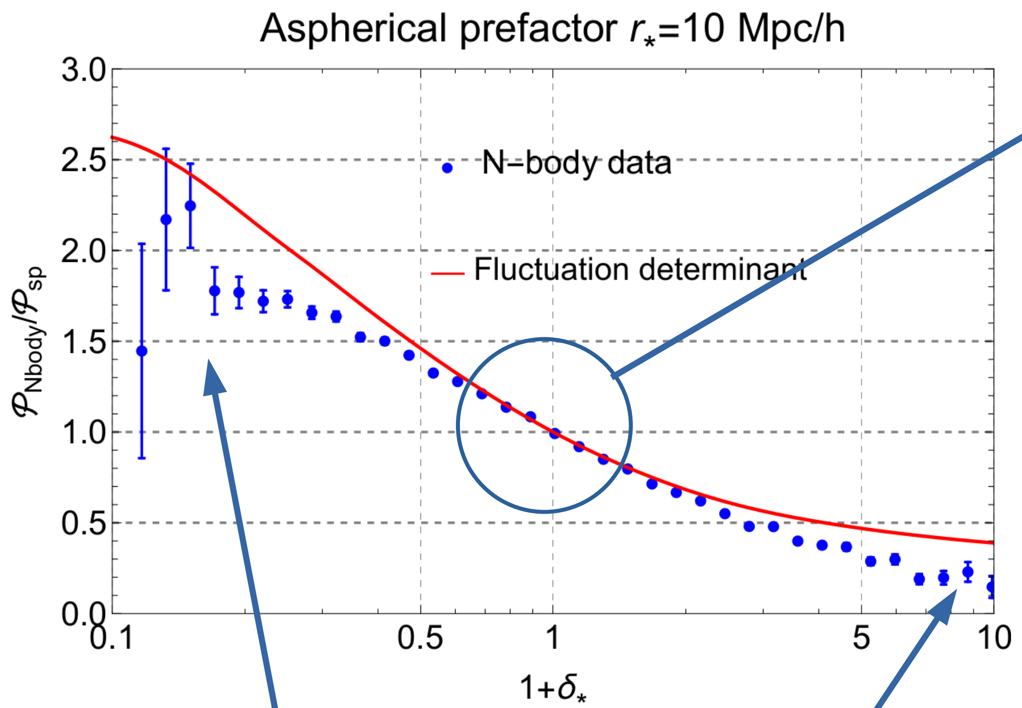
$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

Computations use https://github.com/Michalychforever/AsPy

Aspherical prefactor from data

$$\mathcal{A}_{\rm ASP}(\delta_*) = \frac{\mathcal{P}_{\rm data}(\delta_*)}{\mathcal{P}_{\rm SP}(\delta_*)}$$





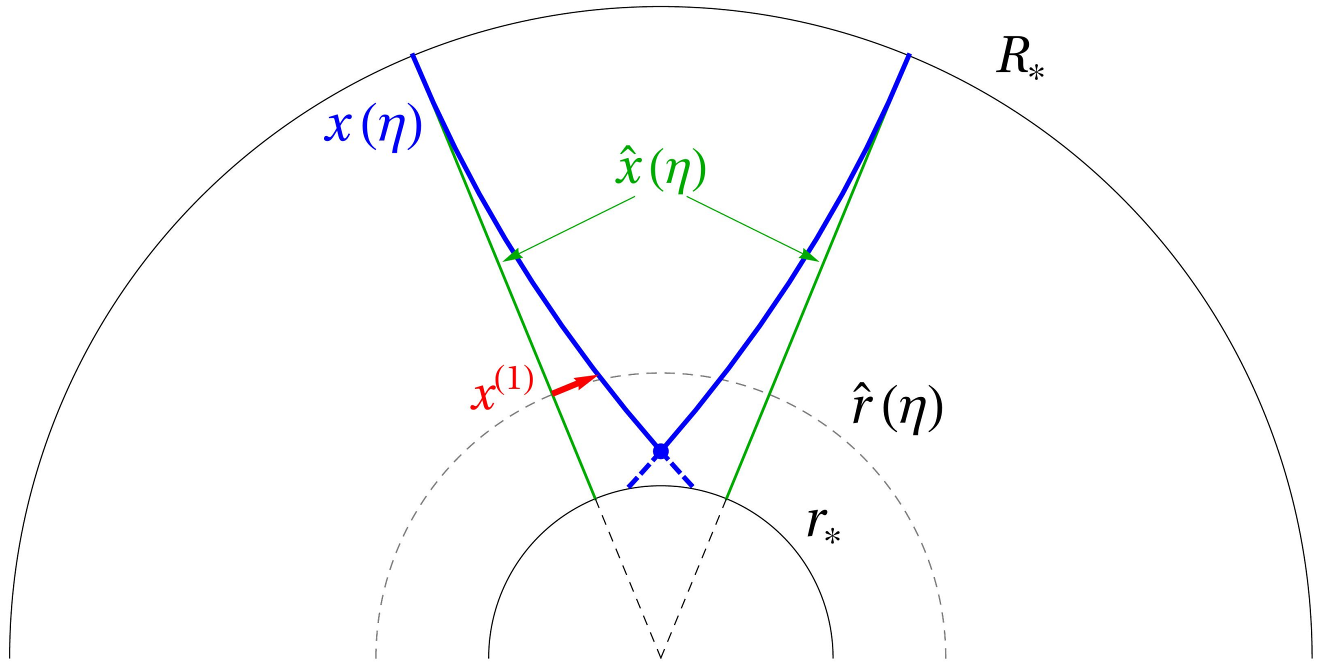
- ✔ Unitarity
- ✔ Translation invariance
- ✔ Weakly depend on cosmology
- ✘ Fluctuation determinant overpredicts the data

Renormalization of short-wavelength contributions is required!

Aspherical perturbations

$$u_i = -\mathcal{H}\partial_i \Psi$$

$$\Delta\Psi = \Theta$$



$$\frac{dx_i}{d\eta} = -\partial_i \hat{\Psi} - \partial_i \Psi^{(1)}$$

$$1 + \delta(\eta, \mathbf{x}) = \left(\det \frac{\partial x_i}{\partial X_j}\right)^{-1} = \left[\det\left(\frac{\partial \hat{x}_i}{\partial X_j} + \frac{\partial x_i^{(1)}}{\partial X_j}\right)\right]^{-1}$$
$$= \left(\det \frac{\partial \hat{x}_k}{\partial X_l}\right)^{-1} \left[\det\left(\delta_i^j + \frac{\partial x_i^{(1)}}{\partial \hat{x}_j}\right)\right]^{-1} .$$

$$\varpi_i^j \equiv \frac{\partial x_i^{(1)}}{\partial \hat{x}_j}$$

Consider its trace

$$\frac{d\varpi}{d\eta} = \Delta\Psi^{(1)} = \Theta^{(1)}$$

$$\varpi(\eta, \mathbf{X}) = \int_{-\infty}^{\eta} d\eta' \, \Theta^{(1)}\big(\eta', \hat{\mathbf{x}}(\eta', \mathbf{X})\big)$$

Growth factor in non-linear background

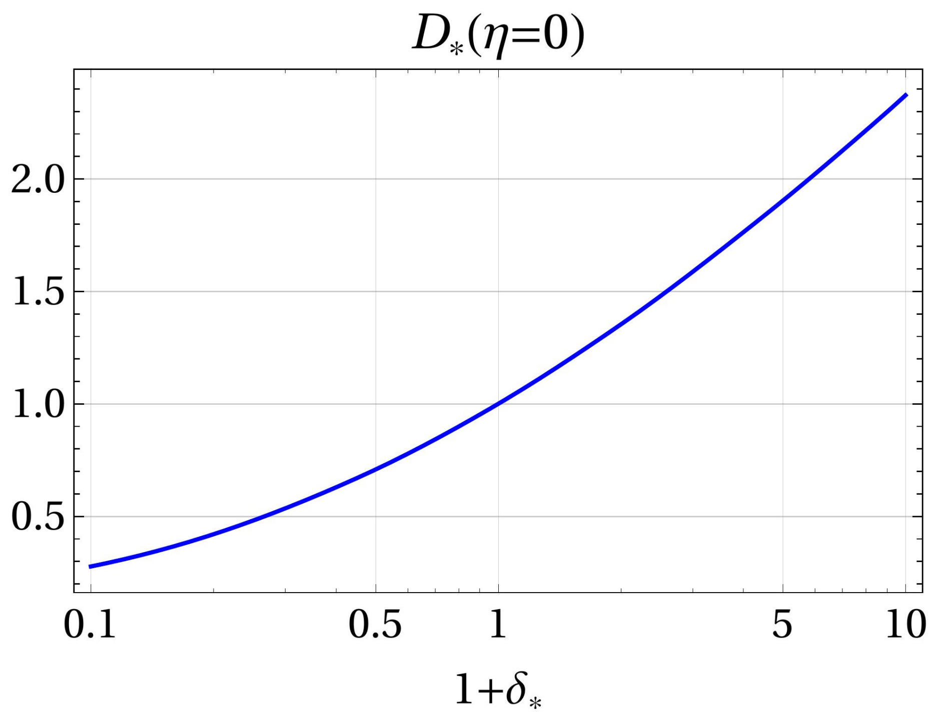
$$\langle \varpi^2 \rangle_{k_{\max}} \approx 4\pi \int^{k_{\max}} [dk] P(k) \left[D(\eta; R) \right]^2$$

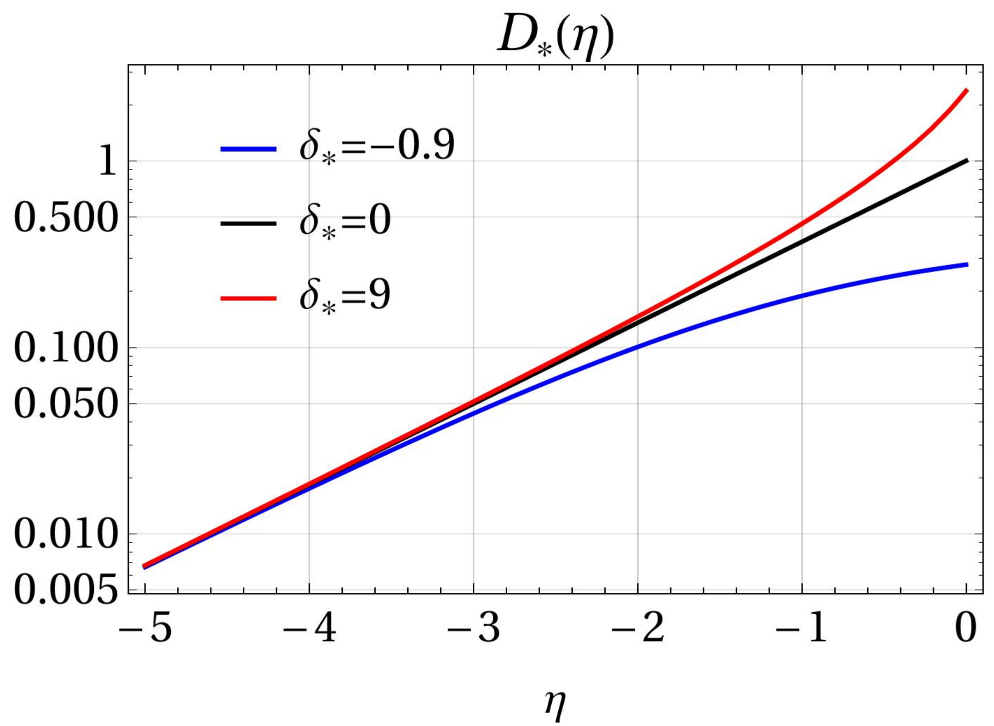
$$\varkappa \equiv \frac{k}{\ell + 1/2}$$

$$\tilde{\Theta}_\ell = \Theta_\ell (\ell + 1/2)$$

$$D(\eta; R) \approx \left(\int_{1/R_*}^{\infty} \frac{d\varkappa}{(2\pi)^2 \varkappa} \left| \int_{-\infty}^{\eta} d\eta' \tilde{\Theta}_{\ell 1}(\eta', R; \varkappa) \right|^2 \right)^{1/2}$$

Assuming power-law Universe $P(k) \propto k^n$ $D_*(\eta) \equiv D(\eta, R_*)$





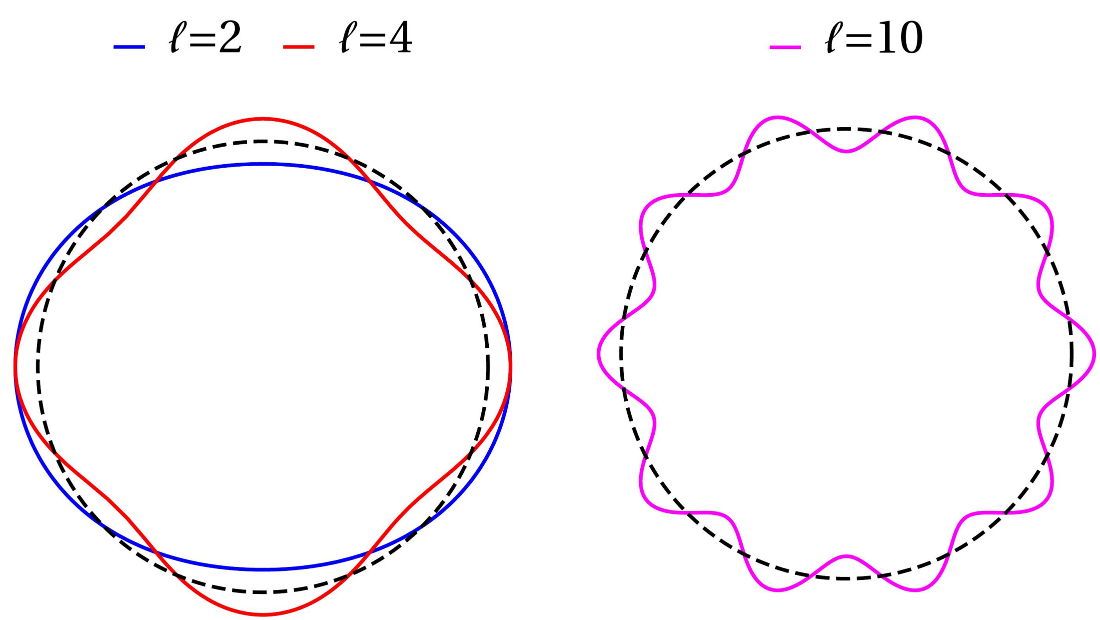
Shell crossing scale

$$\langle \varpi^2 \rangle_{k_{\rm sc}} \sim 1$$

$$k_{\rm sc}(\eta, R) \propto \left(D(\eta, R)\right)^{-m/2}$$

$$m = \frac{4}{3+n}$$

$$\gamma(z) = \gamma_0 g^m(z)$$



Effective stress tensor

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\delta_{\rm tot} = \delta + \delta^s$$
$$u_{{\rm tot},i} = u_i + u_i^s$$
$$\Phi_{\rm tot} = \Phi + \Phi^s$$

$$\delta(\mathbf{x}) \equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|)\delta_{\rm tot}(\mathbf{x}')$$

...

$$\frac{\partial \delta}{\partial t} + \partial_i\big((1+\delta)u_i\big) = 0$$
$$\frac{\partial u_i}{\partial t} + \mathcal{H}u_i + (u_j\partial_j)u_i + \partial_i\Phi = -\frac{1}{1+\delta}\partial_j\tau_{ij}$$

$$\tau_{ij} = (1+\delta)\sigma^l_{ij} + \frac{2}{3\mathcal{H}^2}\left([\partial_i\Phi^s\partial_j\Phi^s]^l - \frac{1}{2}\delta_{ij}[\partial_k\Phi^s\partial_k\Phi^s]^l\right)$$

D. Baumann et al., 2012

'kinetic' part

'potential' part
crucial for decoupling virial scales from long-wavelength dynamics

$$\sigma^l_{ij} = \frac{\int (v_i - u^l_i)(v_j - u^l_j) f^l \, d^3p}{\int f^l \, d^3p}, \qquad v_i \equiv \frac{p_i}{am}$$

Fluid description

$$1/R_* \ll k_1 < k < k_2 \ll k_{\rm sc}$$

Averaging over initial conditions

$$\sigma^l_{ij} = \langle u_i^{(1)} u_j^{(1)} \rangle = \mathcal{H}^2 \langle \partial_i \Psi^{(1)} \partial_j \Psi^{(1)} \rangle$$

Expanding up to the quadratic order
and averaging over the angles...

$$\delta_0 = \hat{\delta}_0 + \delta_0^{(2)}$$
$$\Theta_0 = \hat{\Theta}_0 + \Theta_0^{(2)}$$
$$\Phi_0 = \hat{\Phi}_0 + \Phi_0^{(2)}$$

$$\Upsilon^a(\eta) = \frac{1}{\mathcal{H}^2(1+\hat{\delta})} \partial_j \tau^a_{ij} \Big|_{i\to r}, \qquad a = \mathrm{kin}, \mathrm{pot}$$

$$\dot{\mu}^{(2)} + \dot{r}^{(2)}_\eta \hat{r}^2_\eta \big(1 + \hat{\delta}(\hat{r}_\eta)\big) + r^{(2)}_\eta \frac{d}{d\eta}\Big(\hat{r}^2_\eta \big(1+\hat{\delta}(\hat{r}_\eta)\big)\Big) = 0$$

$$\ddot{r}^{(2)}_\eta + \frac{\dot{r}^{(2)}_\eta}{2} + \left(1 + \frac{3}{2}\hat{\delta}(\hat{r}_\eta) - \frac{R_*^3}{\hat{r}^3_\eta}\right) r^{(2)}_\eta + \frac{3}{2\hat{r}^2_\eta}\mu^{(2)} = -\Upsilon^a(\hat{r}_\eta)$$

$$\hat{\mu} = \int_0^{r_\eta} dr\, r^2 \big(1 + \hat{\delta}_0(r)\big)$$

$$\mu^{(2)} = \int_0^{r_\eta} dr\, r^2 \delta_0^{(2)}(r)$$

$\Upsilon^{\rm pot}$, $\Upsilon^{\rm kin}$ receive sizable contributions from short modes and must be renormalized

Counterterm model

Counterterm should scale as $$\tau_\alpha^{a,\mathrm{ctr}} \sim 2\mathcal{H}^2 \int_{k_{\mathrm{sc}}}^{\infty} \frac{dk\,P(k)}{(2\pi)^3} \cdot \int \frac{d\varkappa}{\varkappa}\,\chi_\alpha^a$$

$$\tau_\alpha^{a,\mathrm{ctr}}(\eta,R) = \zeta^a \cdot 2\mathcal{H}^2\big(D(\eta,R)\big)^{m-2}\int_{R^{-1}}^{\infty} \frac{d\varkappa}{\varkappa}\,\chi_\alpha^a(\eta,R;\varkappa)\;, \qquad a = \mathrm{kin},\mathrm{pot}; \quad \alpha = \|,\perp$$

$$\Upsilon^a(\eta) = 2\zeta^a[D_*(\eta)]^{m-2}\left[\int_{R_*^{-1}}^{\infty}\frac{d\varkappa}{\varkappa}\,\upsilon^a(\eta;\varkappa) + \frac{(m-2)[\ln D_*(\eta)]'}{1+\hat{\delta}|_{\eta,\hat{r}(\eta)}}\int_{R_*^{-1}}^{\infty}\frac{d\varkappa}{\varkappa}\,\chi^a(\eta,R_*;\varkappa)\right]$$

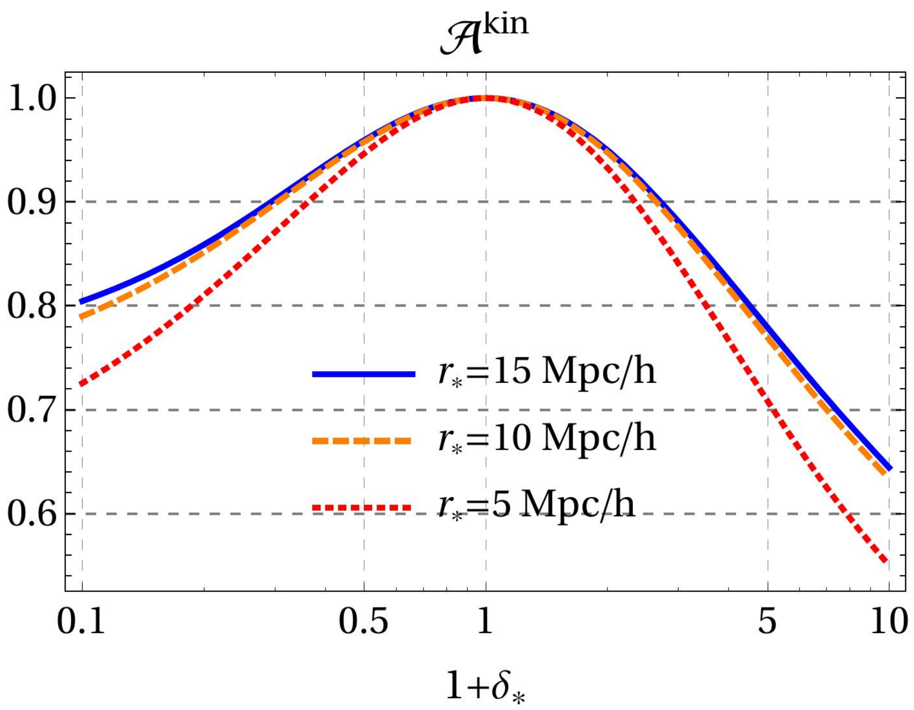
$$\bar{\delta}_W^a = \frac{3}{r_*^3}\mu^{(2),a}(\eta=0) \quad \longrightarrow \quad \mathcal{A}^a = \exp[-\hat{\lambda}\bar{\delta}_W^a]$$

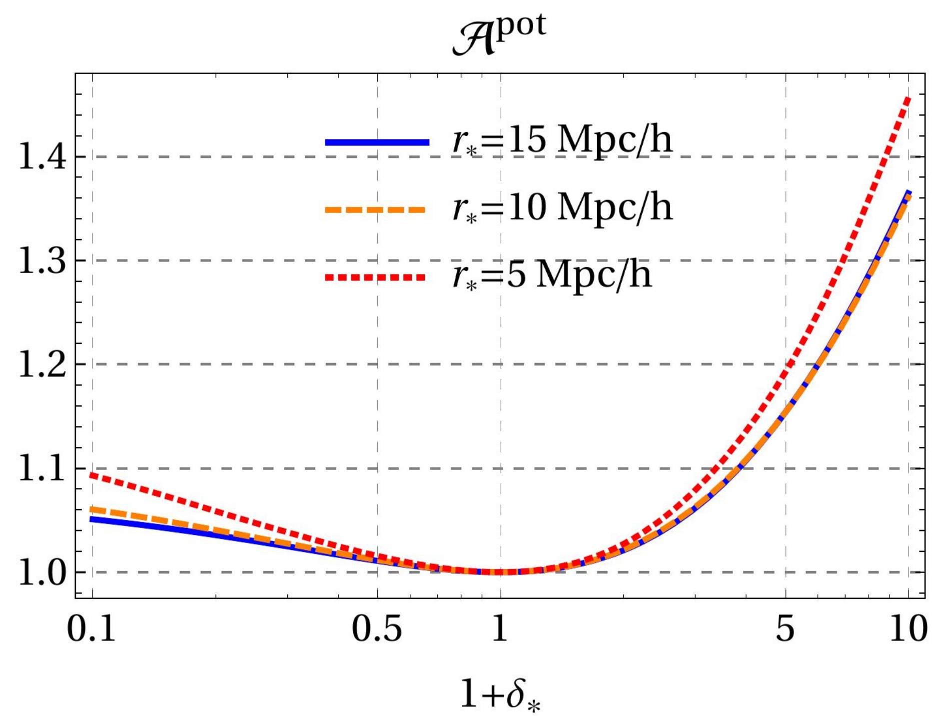
$$\mathcal{A}^{\mathrm{ctr}} = \mathcal{A}^{\mathrm{kin}} \cdot \mathcal{A}^{\mathrm{pot}}$$

Time dependence $\longrightarrow$ $\ln[\mathcal{A}^{\mathrm{ctr}}] \propto [g(z)]^{m-2}$

r_*-dependence of counterterm prefactor

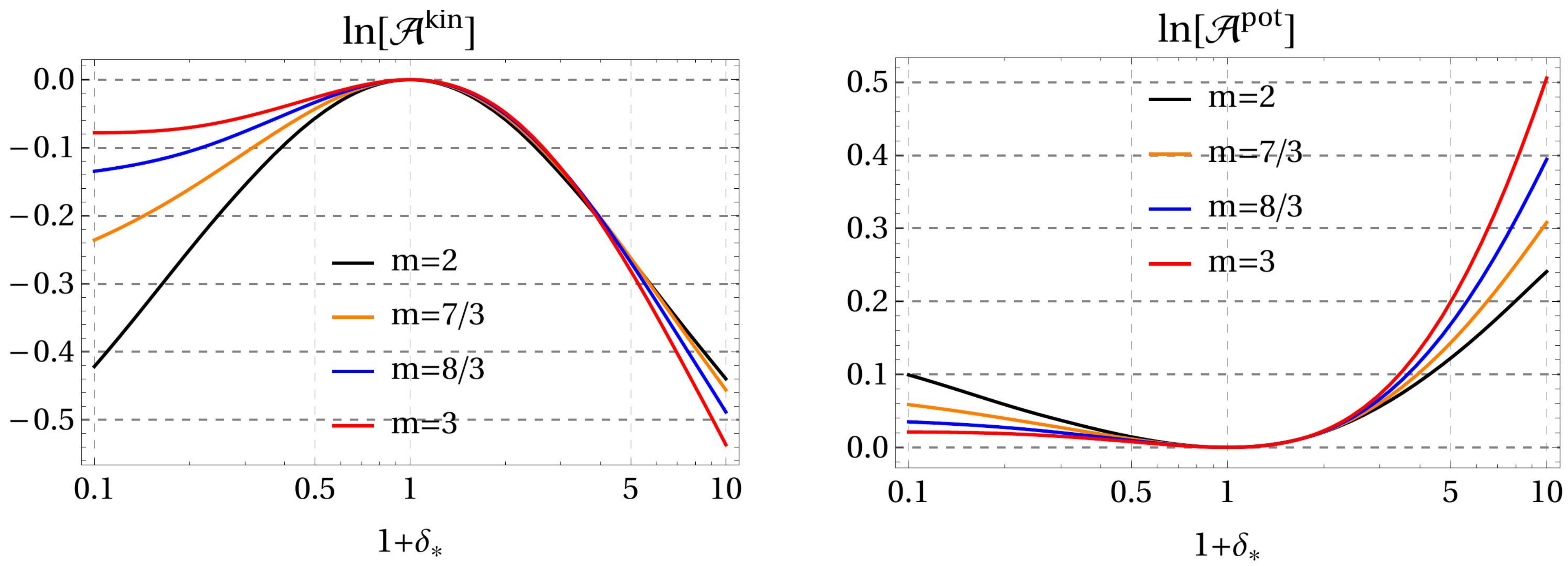
$$\zeta^{\rm kin} = \zeta^{\rm pot} = (1\,{\rm Mpc}/h)^2, \quad m = 2.33$$





m-dependence of counterterm prefactor

$$\zeta^{\rm kin} = \zeta^{\rm pot} = (1\,{\rm Mpc}/h)^2, \quad r_* = 10\,{\rm Mpc}/h$$



Theoretical model

$$\mathcal{P}(\delta_*) = \mathcal{P}_{\rm SP} \cdot \prod_{\ell>0} \mathcal{A}_\ell(\delta_*) \cdot \mathcal{A}_{\rm ctr}$$

Spherical PDF computed exactly

Fluctuation determinant calculated on grid numerically

Counterterm prefactor predicted given model parameters (α,β,m)

$$\mathcal{A}^{\rm theory}_{\rm ASP}(\delta_*) = \prod_{\ell>0} \mathcal{A}_\ell(\delta_*) \cdot \mathcal{A}^{\rm ctr}(\delta_*)$$

$$\mathcal{A}^{\rm ctr} = \mathcal{A}^{\rm kin} \cdot \mathcal{A}^{\rm pot}$$

$$\mathcal{A}^{\rm ctr}(z) = \exp\left(-\delta_*^2 \frac{\gamma(z)}{g^2(z)} \frac{\Sigma^2_{r_*}}{\sigma^4_{r_*}} + \mathcal{O}(\delta_*^2)\right)$$

$$\Sigma^2_{r_*} \equiv 4\pi \int [dk] k^2 \left|W_{\rm th}(kr_*)\right|^2 P(k)$$

Time-dependence: $\mathcal{A}_{\rm ctr}(z) = \mathcal{A}_{\rm ctr}(0)^{\frac{\gamma(z)}{\gamma_0}}$

$$m = \frac{4}{3+n}$$

$$\gamma(z) = \gamma_0 g^m(z)$$

$\langle \zeta^{\rm kin},\, \zeta^{\rm pot},\, m \rangle$ free parameters

Aspherical prefactor

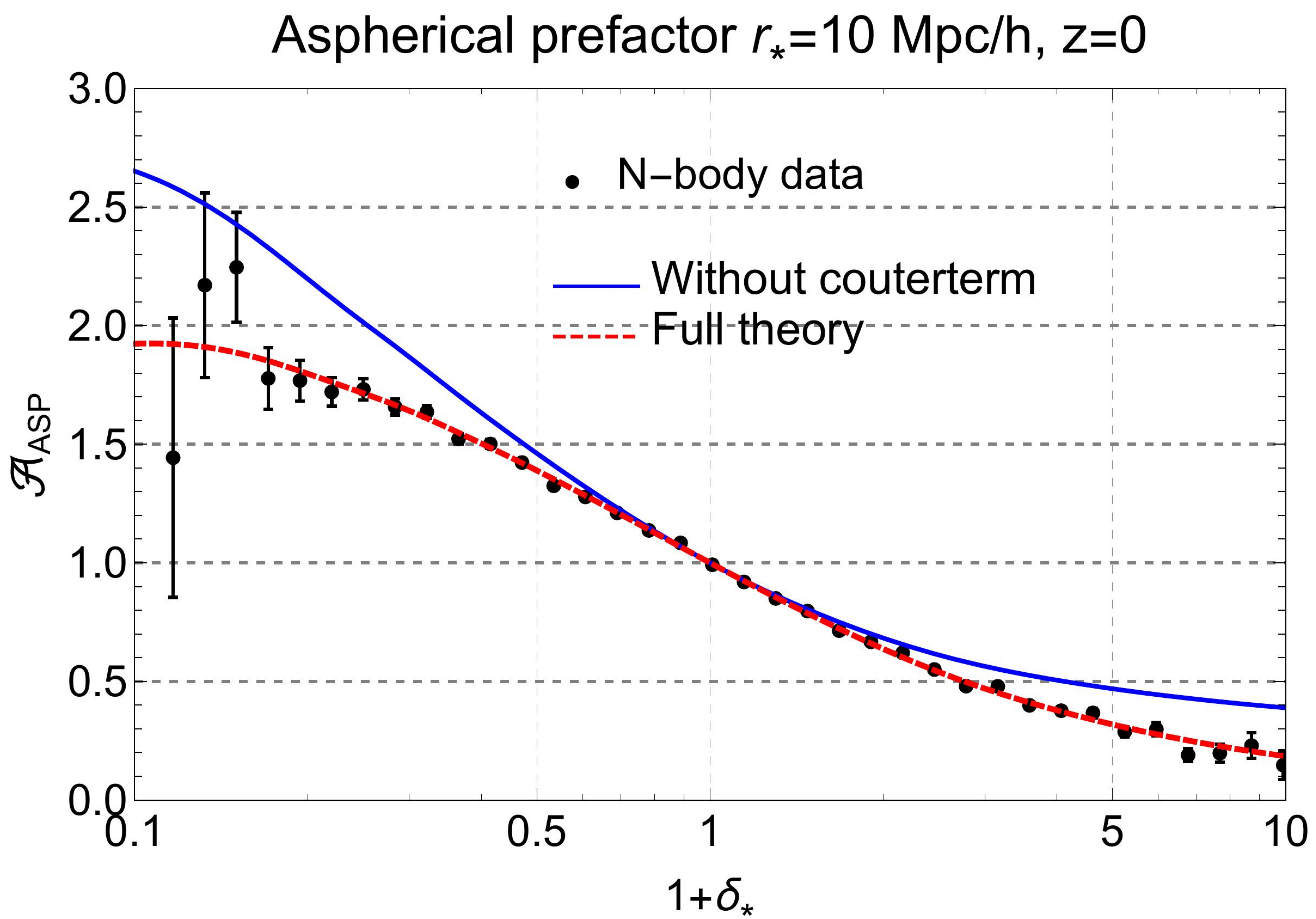
$$\mathcal{A}_{\rm ASP}^{\rm data}(\delta_i) = \frac{\mathcal{P}_{\rm data}(\delta_i)}{\langle \mathcal{P}_{\rm SP} \rangle_i}$$

$$-2 \ln \mathcal{L} = (\langle \mathcal{A}_{\rm ASP}^{\rm theory} \rangle_i - \mathcal{A}_{\rm ASP}^{\rm data}(\delta_i))(C_{ij}^{\rm stat})^{-1}(\langle \mathcal{A}_{\rm ASP}^{\rm theory} \rangle_j - \mathcal{A}_{\rm ASP}^{\rm data}(\delta_j)) + (\gamma_0, m)^{\rm T} C(\gamma_0, m)$$

Aspherical prefactor

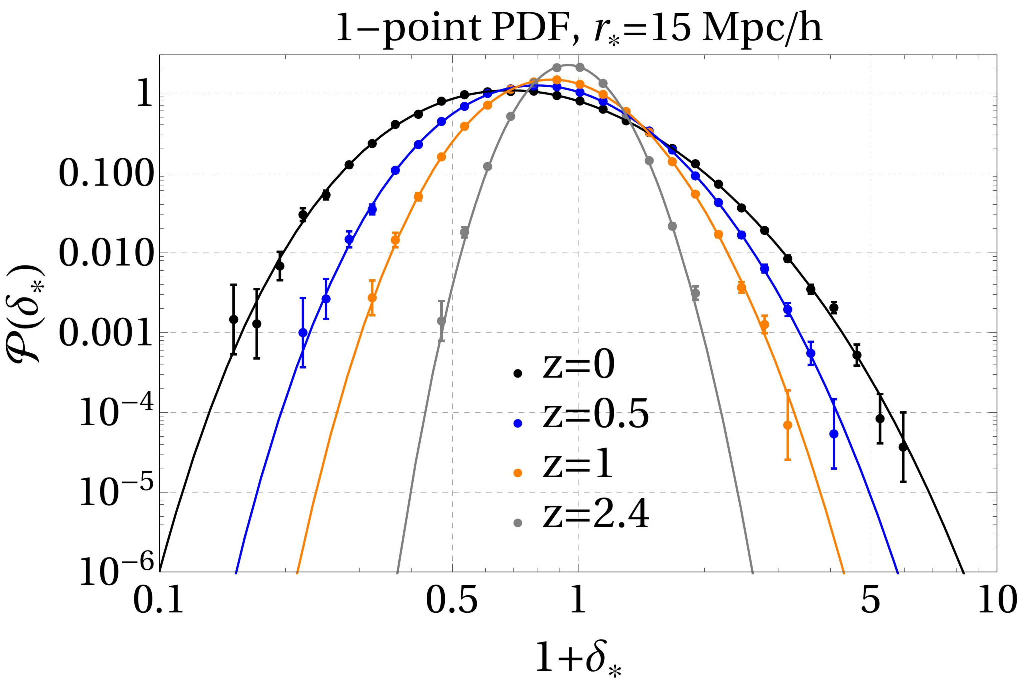
$$\mathcal{A}^{\rm data}_{\rm ASP}(\delta_i) = \frac{\mathcal{P}_{\rm data}(\delta_i)}{\langle \mathcal{P}_{\rm SP}\rangle_i}$$

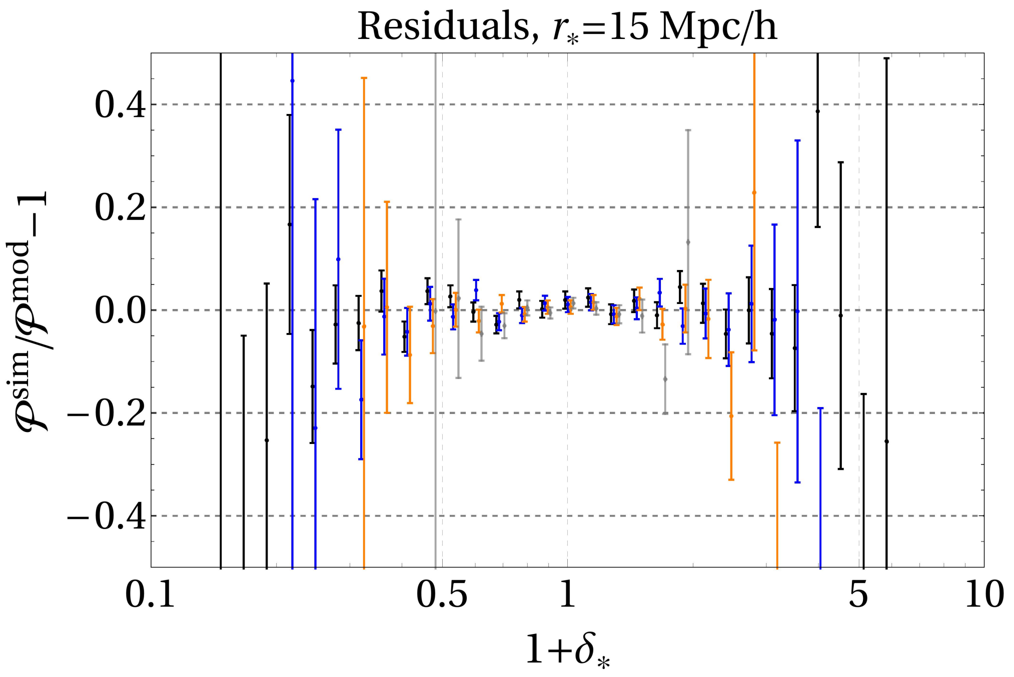
$$-2\ln\mathcal{L} = (\langle \mathcal{A}^{\rm theory}_{\rm ASP}\rangle_i - \mathcal{A}^{\rm data}_{\rm ASP}(\delta_i))(C^{\rm stat}_{ij})^{-1}(\langle \mathcal{A}^{\rm theory}_{\rm ASP}\rangle_j - \mathcal{A}^{\rm data}_{\rm ASP}(\delta_j)) + (\gamma_0, m)^{\rm T} C(\gamma_0, m)$$



Results for R=15 Mpc/h

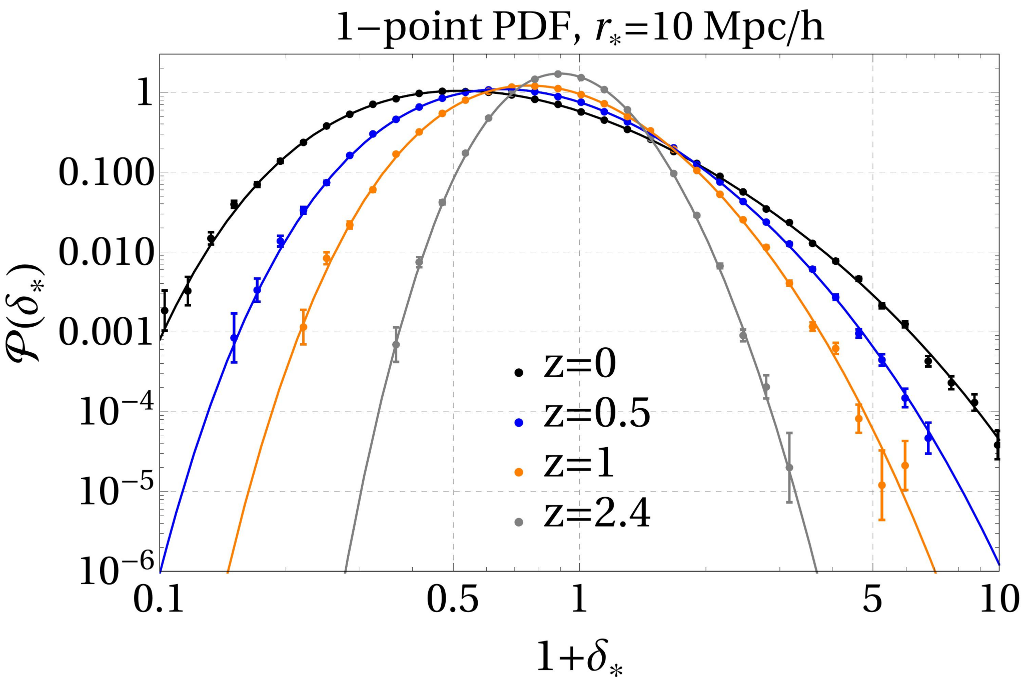
$\chi^2_{\rm best-fit}/N_{\rm dof} = 0.88\ (0.3\sigma)$

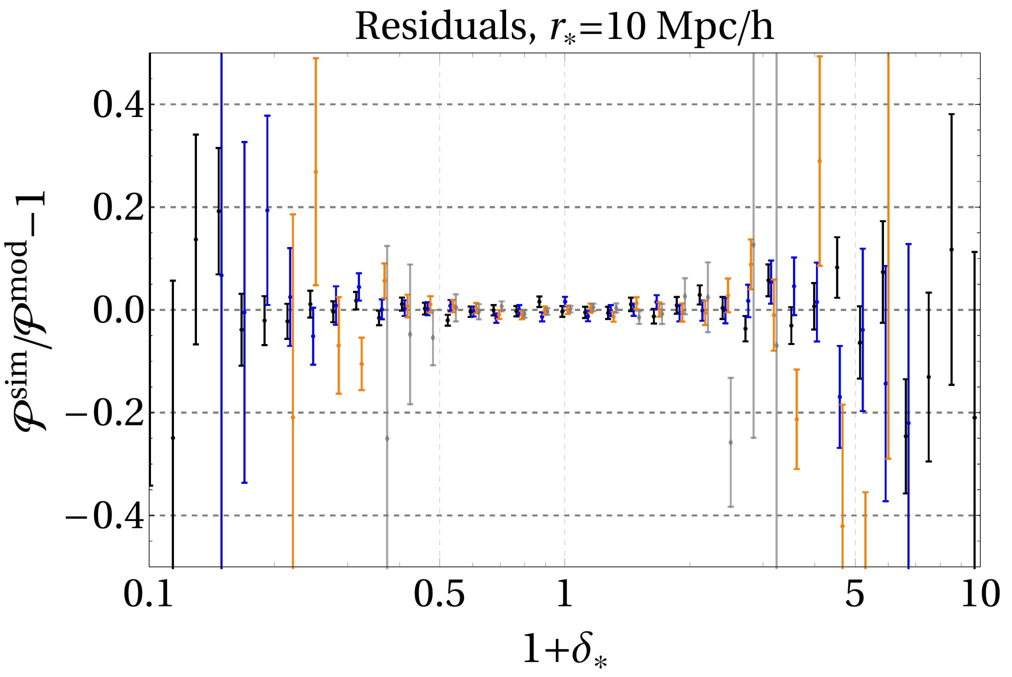




Results for R=10 Mpc/h

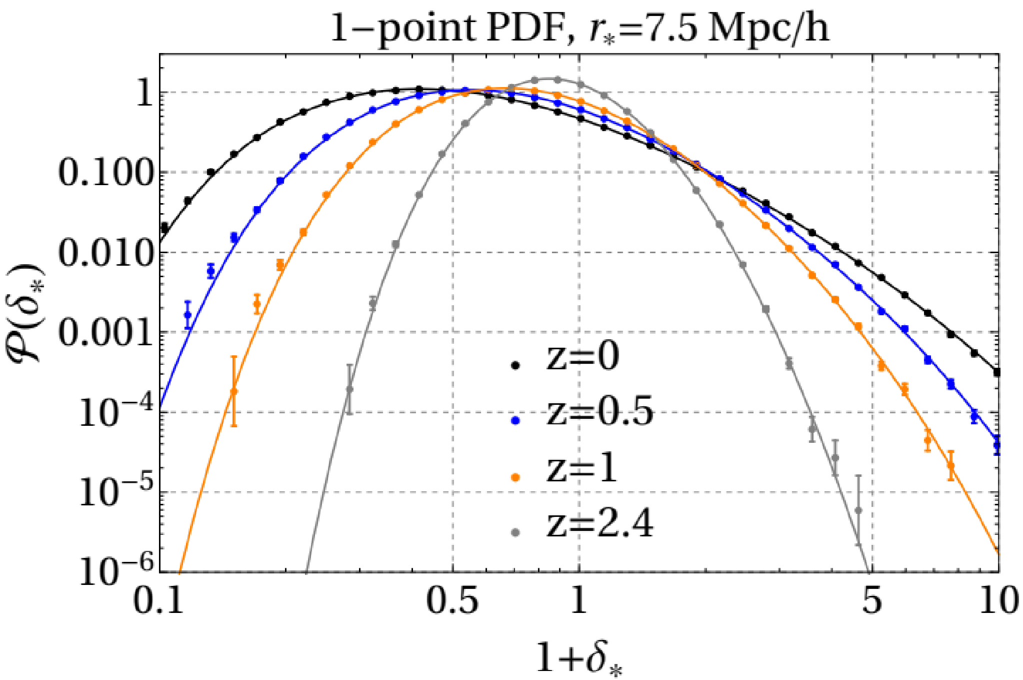
$\chi^2_{\rm best-fit}/N_{\rm dof} = 0.99$ (0.6σ)

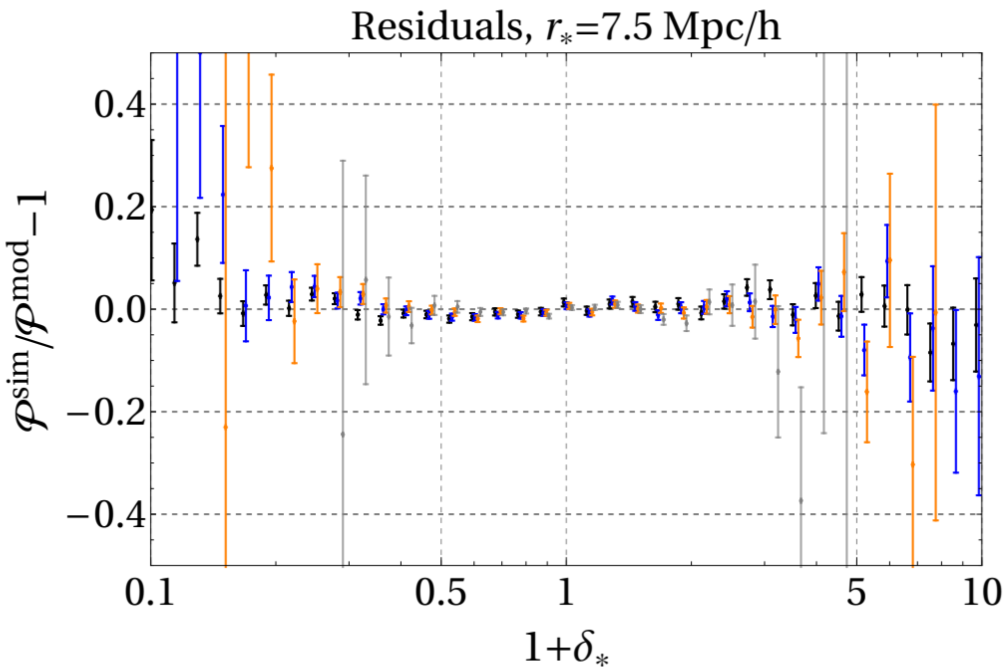




Results for R=7.5 Mpc/h

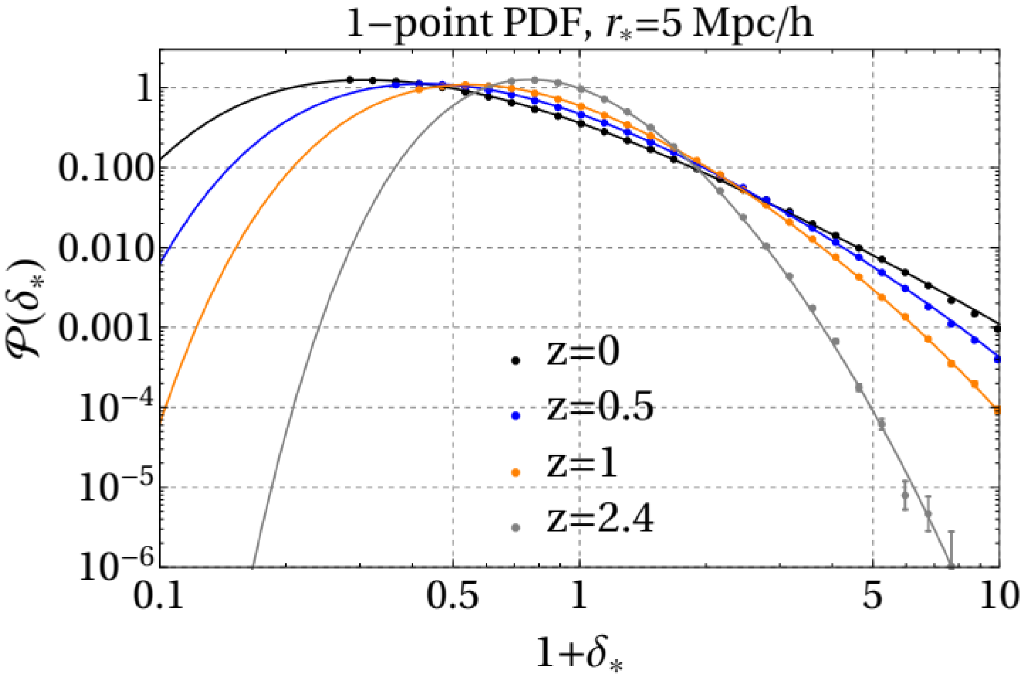
$\chi^2_{\text{best-fit}}/N_{\text{dof}} = 1.80\ (5.3\sigma)$

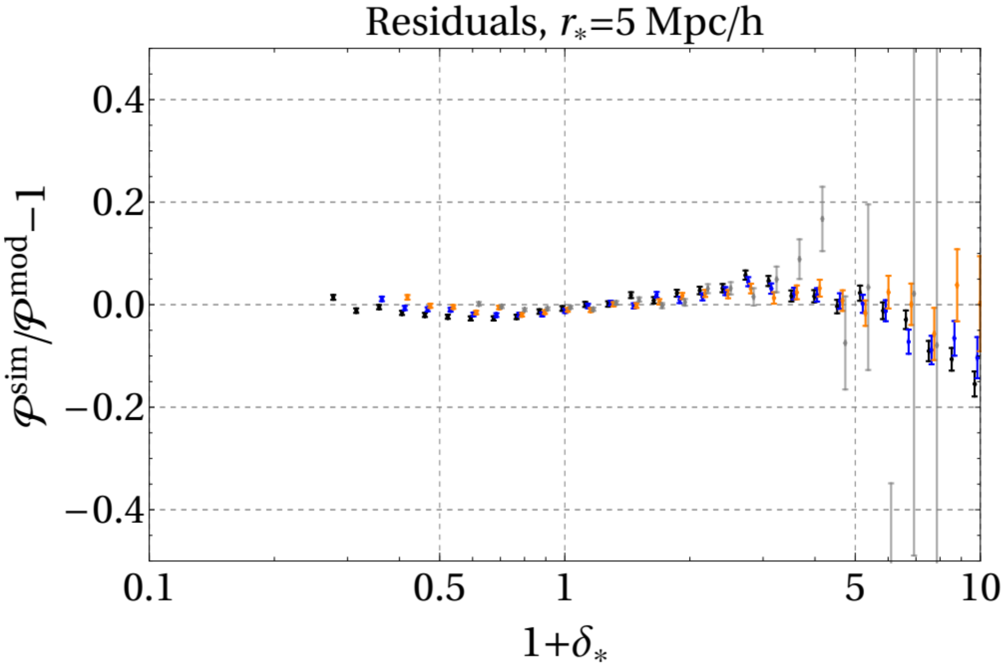




Results for R=5 Mpc/h

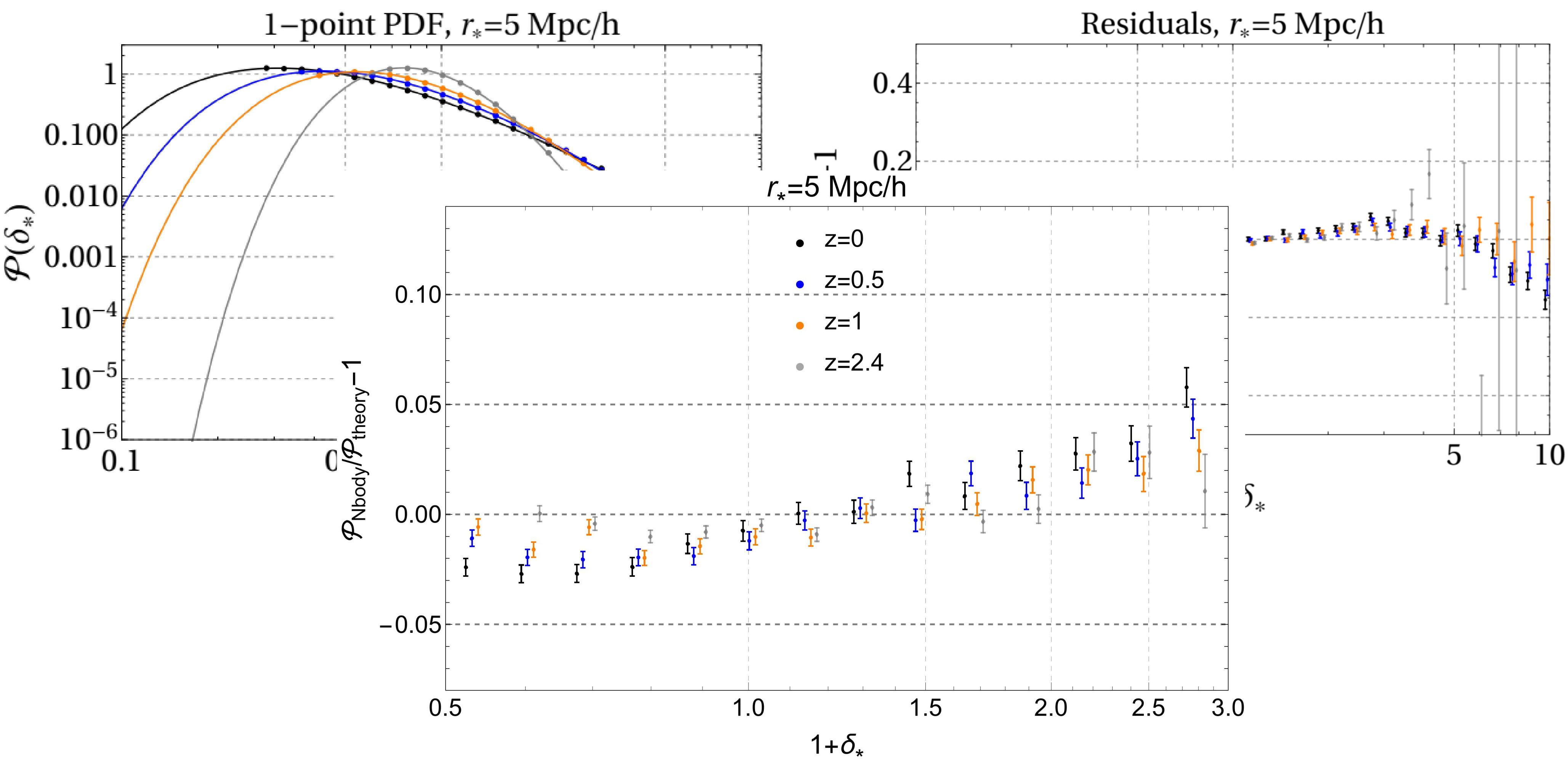
$$\chi^2_{\mathrm{best-fit}}/N_{\mathrm{dof}} = 9.74\ (25\sigma)$$



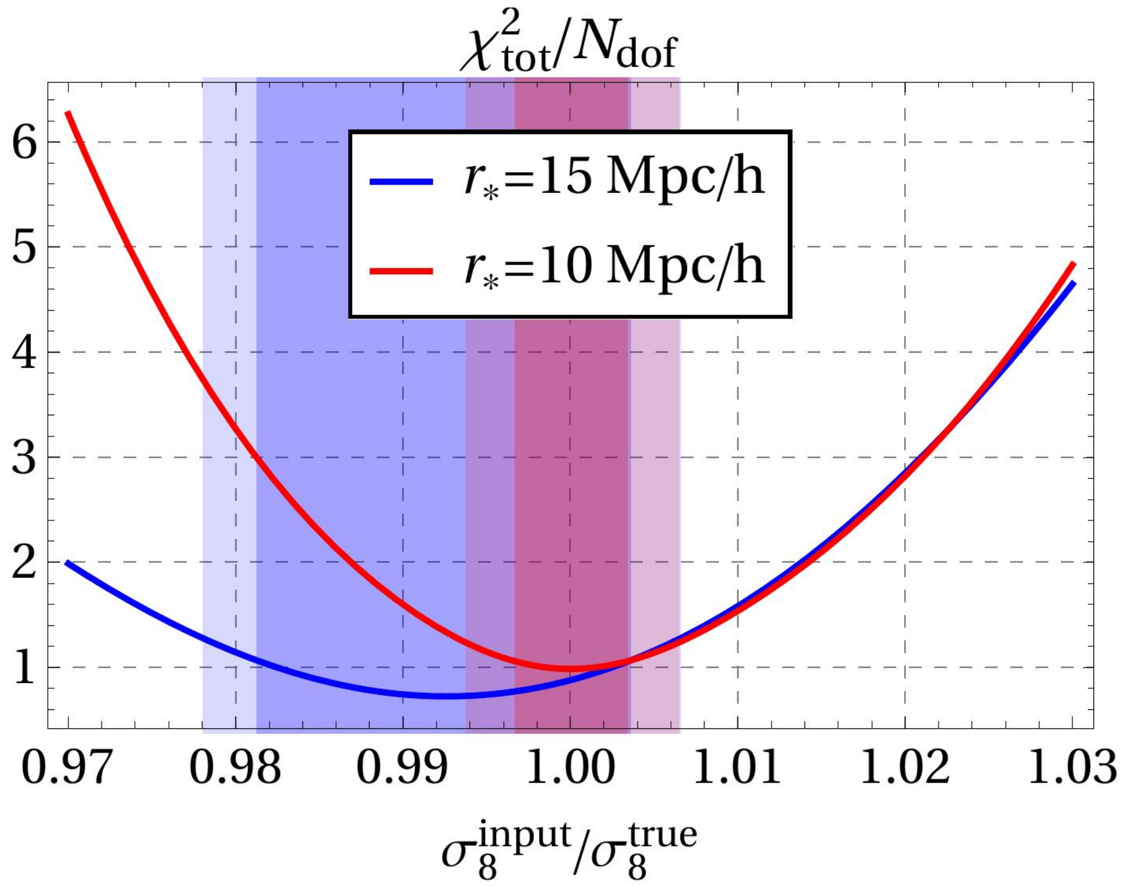


Results for R=5 Mpc/h

$\chi^2_{\text{best-fit}}/N_{\text{dof}} = 9.74\ (25\sigma)$



Sensitivity to σ_8



PDF is sensitive to the value of σ_8 at sub-per cent level

Filtered n-point correlators

		norm − 1	$\langle\delta_*\rangle$	$\langle\delta_*^2\rangle$	$\sigma^2_{\rm EFT}$	$\langle\delta_*^3\rangle/\langle\delta_*^2\rangle^2$
$r_* = 15\,{\rm Mpc}/h$	$z=0$	$-6.2\cdot10^{-3}$	$-7.1\cdot10^{-3}$	0.260	0.262	3.35
	$z=0.5$	$-2.9\cdot10^{-3}$	$-3.1\cdot10^{-3}$	0.153	0.154	3.30
	$z=1$	$-1.3\cdot10^{-3}$	$-1.3\cdot10^{-3}$	0.095	0.095	3.27
	$z=2.4$	$-3.3\cdot10^{-5}$	$-5.6\cdot10^{-5}$	0.035	0.035	3.23
$r_* = 10\,{\rm Mpc}/h$	$z=0$	$-3.7\cdot10^{-4}$	$-2.1\cdot10^{-3}$	0.533	0.532	3.63
	$z=0.5$	$1.2\cdot10^{-4}$	$-6.7\cdot10^{-4}$	0.306	0.304	3.65
	$z=1$	$2.8\cdot10^{-4}$	$-2.9\cdot10^{-4}$	0.185	0.185	3.56
	$z=2.4$	$3.3\cdot10^{-4}$	$6.4\cdot10^{-5}$	0.067	0.067	3.45

PDF reproduces the EFT filtered density variance with
sub-per cent accuracy

Conclusions

- ☑ Three-parametric model for couterterm prefactor is in excellent agreement with N-body data for $r_* \geq 10$ Mpc/h

- ☑ For $r_*<10$ Mpc/h the 2-loop order correction at the the origin comes into play – theoretical error is needed!

- ☑ The renormolized theory describe the N-body data for $r_*=5$ Mpc/h with 10% accuracy

Priors:

$$\gamma_0 = (1.95 \pm 0.26)(\,\mathrm{Mpc}/h)^2\ , \qquad m = 2.26 \pm 0.21\ , \qquad \mathrm{corr}(\gamma_0, m) = 0.85$$