AdS/CFT, Wilson loops and M2-branes

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arXiv:2303.15207, 2307.14112, 2309.10786


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Modelling macroscopic and baby universes by fundamental strings

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May 1995

Abstract

We develop a model of $(1+1)$-dimensional parent and baby universes as macroscopic and microscopic fundamental closed strings. We argue, on the basis of understanding of strings from the point of view of target D-dimensional space-time, that processes involving baby universes/wormholes not only induce c-number "α-parameters" in $(1+1)d$ action, but also lead to loss of quantum coherence for a $(1+1)d$ observer in the parent universe.

hep-th/9505159

hep-th/9510107
INR 15/10/95
October, 1995

STRINGS AS A MODEL FOR PARENT AND BABY UNIVERSES: TOTAL SPLITTING RATES

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Abstract

Emission of hard microscopic string (graviton) by an excited macroscopic string may be viewed as a model of branching of a (1+1)-dimensional baby universe off large parent one. We show that, apart from a trivial factor, the total emission rate is not suppressed by the size of the macroscopic string. This implies unsuppressed loss of quantum coherence in (1+1)-dimensional parent universe.

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QFT: major problem – beyond perturbation theory
how to compute path integral exactly?
e.g. $SU(N)$ SYM : $\lambda = g_{YM}^2 N$, large N

$$F(\lambda, N) = N^2 F_0(\lambda) + F_1(\lambda) + \frac{1}{N^2} F_2(\lambda) + ..., \qquad F_n(\lambda) = ?$$

remarkable progress in superconformal theories
using combination of different methods

1. Integrability: anomalous dims in 4d and 3d conformal theories as exact functions of $\lambda = g_{YM}^2 N$ at large N
2. Localization: some supersymmetric observables (free energy on S^d, Wilson loop expectation value, few correlators) computed exactly in $g_{\rm YM}$ and N
3. Bootstrap: constraints from symmetries and general principles
4. AdS/CFT as a guide

(un)related questions:

non-perturbative string theory?

what is 11d M-theory?
theory of quantum supermembranes?

recent developments provide novel clues ...

AdS/CFT: 26 years

basic examples:

$\mathcal{N} = 4,\ d = 4$ SYM $\leftrightarrow$ $\mathrm{AdS}_5 \times S^5$ string

$\mathcal{N} = 6,\ d = 3$ Chern-Simmons + matter (ABJM)

$\leftrightarrow$ $\mathrm{AdS}_4 \times \mathrm{CP}^3$ string or M-theory on $\mathrm{AdS}_4 \times S^7$

- quantitative understanding of duality in planar limit based on integrability

- beyond planar limit: recent progress using localization

$\mathbf{N = 4}$ **SYM**	**String theory in** $\mathbf{AdS^5 \times S^5}$
Yang-Mills coupling: g_{YM} Number of colors: N	String coupling: g_s String tension: T
Level 1: Exact equivalence	
$g_s = g_{YM}^2/4\pi, \quad T = \sqrt{g_{YM}^2 N}/2\pi$	
Level 2: Equivalence in the 't Hooft limit	
$N \to \infty, \quad \lambda = g_{YM}^2 N$-fixed (planar limit)	$g_s \to 0, \quad T$-fixed (non-interacting strings)
Level 3: Equivalence at strong coupling	
$N \to \infty, \quad \lambda \gg 1$	$g_s \to 0, \quad T \gg 1$

Integrability:

- spectrum of classical (genus 0) $\mathrm{AdS}_5 \times S^5$ string theory

- anomalous dimensions of $N = \infty$ gauge theory:

$$\mathcal{O} = \mathrm{Tr}\,(\Phi^* D^S \Phi) : \qquad \Delta(S,\lambda) = S + 2 + f(\lambda) \log S + ...$$

$$f(\lambda) = c_1 \sqrt{\lambda} + c_2 + \frac{c_3}{\sqrt{\lambda}} + ... , \qquad T = \frac{L^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$$

- beyond planar limit – finite N or string loop corrections?

Localization:

• reduction of SYM path integral to $N \times N$ matrix integral

$$\int [da]\, M[a]\ \exp[-\frac{N}{g^2_{\rm YM}} {\rm Tr}\, a^2]$$ [Pestun 2007]

computes special susy observables for any N and $\lambda = g^2_{\rm YM} N$

• examples: free energy on S^4 and susy Wilson loop expectation value

• $\mathcal{N} = 2$ superconformal 4d models: SYM + hypermultiplets:

$F(N, \lambda)$ from matrix model in $1/N$ expansion

e.g. for $SU(N) \times SU(N)$ quiver model ($\mathbb{Z}_2$ quotient of $\mathcal{N} = 4$ SYM)

$$F = N^2 F_0 + F_1 + \frac{1}{N^2} F_2 + ..., \qquad F_0 \sim \log\lambda = F_{\mathcal{N}=4\ SYM}$$

$$F_1 = \frac{1}{4}\lambda^{1/2} - \log\lambda^{1/2} - \frac{3}{32}\frac{\zeta(3)}{\lambda^{3/2}} - \frac{135}{256}\frac{\zeta(5)}{\lambda^{5/2}} + ...$$

match string theory on $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$ [Beccaria, Korchemsky, AT 2022]

AdS$_4$/CFT$_3$: 3d $U_k(N) \times U_{-k}(N)$ CS + matter (ABJM)
dual to superstring theory in AdS$_4 \times$ CP^3
in planar limit ($N \to \infty,\ k \to \infty,\ \lambda = \frac{N}{k}$=fixed)
beyond planar limit – finite k – dual to 11d M-theory in AdS$_4 \times S^7/\mathbb{Z}_k$

Aims:

- use localization to check AdS$_4$/CFT$_3$ for finite k:
match quantum M2-brane correction
and ABJM theory localization results
(1-loop M2: sum of ∞ set of string loop corrections)
- existence of quantum supermembrane theory?
formally non-renormalizable
but semiclassical 1-loop computations are well-defined
- novel evidence that semiclassical quantization of
M2 brane is under control: matching localization results
highly non-trivial check of AdS$_4$/CFT$_3$

bosonic membrane action [Dirac 1962]

$$S = -T_2 \int d^3\sigma \sqrt{-\det g}\,, \qquad g_{ab} = \eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

can gauge-fix only 3 out of 6 components of 3d metric
$\rightarrow$ non-linear action in any gauge (cf. string)

formally non-renormalizable; instabilities
but susy version may have improved quantum properties

UV finite despite formal power-counting nonrenormalizability?
may be true for supermembrane in supersymmetric $\mathrm{AdS}_4 \times S^7$
dual to superconformal 3d gauge theory

11d supergravity and M2 brane

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left(R - \tfrac{1}{2\cdot 4!} F_{mnk\ell} F^{mnk\ell} + \cdots \right), \qquad F = dC_3$$

- M2 brane solution [Duff, Stelle 90]

$$ds^2 = H^{-2/3}(y) dx^m dx_m + H^{1/3}(y) dy^r dy_r, \qquad C_{mnk} = H^{-1} \epsilon_{mnk}$$

$$H = 1 + \frac{Q}{y^6}, \qquad Q \sim N$$

"near-horizon" limit is $\mathrm{AdS}_4 \times S^7$: $ds^2_{11} \sim (y^4 dx^m dx_m + \frac{dy^2}{y^2}) + dS^7$

$$ds^2_{11} = \mathrm{L}^2 \left(\tfrac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{S^7} \right), \quad F_4 = dC_3 \sim N\,\epsilon_4, \quad \left(\frac{\mathrm{L}}{\ell_P}\right)^6 = 32\pi^2 N$$

- collective coordinates $\to$ M2 action: [Bergshoeff, Sezgin,Townsend 87]

$$S_{\mathrm{M2}} = T_2 \int d^3\sigma \left[\sqrt{-\det g_{mn}} + \hat{C}_3 \right]$$

$$g_{mn} = G_{MN}(x)\,\Pi_m^M \Pi_n^N + \dots\,, \qquad \hat{C}_3 = \tfrac{1}{6}\epsilon^{mnk}\,C_{MNK}(x)\,\Pi_m^M \Pi_n^N \Pi_k^K$$

$$\Pi_m^M = \partial_m x^M - i\bar{\theta}\Gamma^M \partial_m \theta\,, \qquad x^M = x^M(\sigma)$$

• parameters

$$2\kappa_{11}^2 = (2\pi)^8\,\ell_P^9\,, \qquad T_2 = \Big(\frac{2\pi^2}{\kappa_{11}^2}\Big)^{1/3} = \frac{1}{(2\pi)^2 \ell_P^3}$$

• relation to 10d string: $S = \frac{1}{2\mathrm{r}_{10}^2}\int d^{10}x\,\sqrt{G}\,e^{-2\phi}(R + \dots)$

$$ds_{11}^2 = e^{-\frac{2}{3}\phi}\,ds_{10}^2 + e^{\frac{4}{3}\phi}(dx^{11} + e^{-\phi}A)^2, \qquad x_{11} \sim x_{11} + 2\pi R_{11}$$

$$g_{\mathrm{s}} = e^{\phi}; \qquad 2\kappa_{10}^2 = (2\pi)^7 g_{\mathrm{s}}^2 \alpha'^4$$

• "double dimensional reduction": [Duff, Howe, Inami, Stelle 87]

M2 action in 11d background → superstring action in 10d background

• string theory – theory of quantum strings
M-theory – theory of quantum M2 branes?
• analogy: theory on N D3 branes $\to SU(N)\ \mathcal{N}=4$ SYM
one M2-brane: $\mathcal{N}=8$ 3d scalar multiplet (x^i, θ^i)
theory on N coincident M2 branes?
3d superconf theory dual to M-theory in $\mathrm{AdS}_4 \times S^7$ [Maldacena 97]
• can be defined as $k=1$ case of more general 3d theory
dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$
to have perturbative theory requires extra parameter k

ABJM theory: [Aharony, Bergman, Jafferis, Maldacena 08]
N M2 branes on $M^{11} = R^{1,2} \times \mathbb{R}^8/\mathbb{Z}_k$
• described by $U_k(N) \times U_{-k}(N)$
3d Chern-Simons + matter $\mathcal{N}=6$ superconformal theory
$\mathcal{N}=8$ susy restored for $k=1$
• fields: $A_m,\ \tilde{A}_m$; bi-fundamental 4 scalars ϕ^A and 4 fermions ψ_A

$$S = k \int d^3x \left[L_{CS}(A) - L_{CS}(\tilde{A}) + |D\phi|^2 + V(\phi) + \bar{\psi} D\psi + \bar{\psi}\psi\phi^\dagger\phi \right]$$

$$L_{CS} = \epsilon^{mnk} \mathrm{Tr}\,(A_m \partial_n A_k + \tfrac{2}{3} A_m A_n A_k)\ , \qquad V = \mathrm{Tr}\,(\phi\phi^\dagger\phi\phi^\dagger\phi\phi^\dagger) + ...$$

integer parameters N and k (analog of $\frac{1}{g^2_{\rm YM}}$ in YM case)

• 3d superconformal gauge theory is dual to:

(i) in "string" regime = large N, large k, $\lambda \equiv \frac{N}{k}$ = fixed:

10d superstring on $\mathrm{AdS}_4 \times \mathrm{CP}^3$

(ii) in "M-theory" regime = large N, fixed k:

M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

$$ds^2_{11} = \mathrm{L}^2\Big(\tfrac{1}{4}ds^2_{AdS_4} + ds^2_{S^7/\mathbb{Z}_k}\Big), \qquad \mathrm{L} = (2^5\pi^2 Nk)^{1/6}\ell_P$$

S^7 as S^1 fibration over CP^3 and $\mathbb{Z}_k$ quotient

$$ds^2_{S^7/\mathbb{Z}_k} = ds^2_{\mathrm{CP}^3} + (d\varphi + \mathrm{A})^2\,, \qquad \varphi \equiv \varphi + 2\pi k$$

$$ds^2_{\mathrm{CP}^3} = \tfrac{dw^s d\bar{w}^s}{1+|w|^2} - \tfrac{w_r\bar{w}_s}{(1+|w|^2)^2} dw^s d\bar{w}^r, \qquad d\mathrm{A} = i\big[\tfrac{\delta_{sr}}{1+|w|^2} - \tfrac{w_s\bar{w}_r}{(1+|w|^2)^2}\big] dw^r \wedge d\bar{w}^s$$

$$ds^2_{10} = L^2\Big(\tfrac{1}{4}ds^2_{AdS_4} + ds^2_{\mathrm{CP}^3}\Big), \qquad L = g_{\mathrm{s}}^{1/3}\,\mathrm{L}$$

$$g_{\mathrm{s}} = \big(\tfrac{\mathrm{L}}{k\,\ell_P}\big)^{3/2} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \quad \lambda = \frac{N}{k}\,, \qquad T = \frac{L^2_{\mathrm{ads}}}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{\sqrt{2}}$$

$$\frac{g_{\mathrm{s}}^2}{8\pi\, T} = \frac{\lambda^2}{N^2} = \frac{1}{k^2}$$

- M-theory expansion: $\frac{\mathrm{L}}{\ell_P} \gg 1$ or large N for fixed $k = 1, 2, ...$

$$\mathrm{T}_2 \equiv \mathrm{L}^3 T_2 = \frac{1}{\pi}\sqrt{Nk} \;\; \gg 1$$

special "observable": $\frac{1}{2}$ supersymmetric Wilson loop

- find dual minimal 3-surface : probe M2 brane

intersecting AdS_4 boundary (multiple M2's) over line or circle

- compute M2 partition function for $T_2 \gg 1$

compare to large N, fixed k expansion of WL from localization

Plan:

- localization results for WL in SYM and ABJM
- matching leading order string theory results
- higher genus strong coupling terms $\sum_n c_n (\frac{g_s^2}{T})^n$:

$\exp(c_1 \frac{g_s^2}{T})$ in SYM and $(\sin \frac{2\pi}{k})^{-1}$=$(\sin \frac{g_s}{\sqrt{T}})^{-1}$ in ABJM

- $(\sin \frac{2\pi}{k})^{-1}$ as 1-loop M2 brane contribution
- generalizations

$\frac{1}{2}$ BPS circular WL in SYM and ABJM

- $\mathcal{N}=4\ \ SU(N)$ SYM: $\qquad \mathcal{W} = \mathrm{Tr}\, Pe^{\int (iA+\Phi)}$

Localization $\to$ Gaussian matrix model: any $N,\ g_{\rm YM}^2$

[Erickson, Semenoff, Zarembo 00; Drukker, Gross 01; Pestun 07]

$$\langle \mathcal{W} \rangle = \ e^{\frac{N-1}{8N} g_{\rm YM}^2}\, L_{N-1}^1(-\tfrac{1}{4} g_{\rm YM}^2)$$

$$L_n^1(x) \equiv \tfrac{1}{n!} \tfrac{d^n}{dx^n} (x^n e^{-x})$$

Large N, fixed $\lambda = N g_{\rm YM}^2$:

$$\langle \mathcal{W} \rangle = N \Big[\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48 N^2} I_2(\sqrt{\lambda}) + ... \Big]$$

$$\lambda \gg 1: \qquad \langle \mathcal{W} \rangle = \frac{N}{\lambda^{3/4}} \sqrt{\tfrac{2}{\pi}}\, e^{\sqrt{\lambda}} + ...$$

ABJM: analogous $\frac{1}{2}$ BPS operator $\mathcal{W} = \mathrm{Tr}\, P e^{\int (iA + \phi^* \phi + \ldots)}$

Localization matrix model (two bi-fundamental scalars)

$Z(N,k) = \int d^N x_i \, d^N y_i \; M(x_i, y_j) \; \exp\left[i \frac{k}{4\pi} \sum_{i=1}^{N} (x_i^2 - y_i^2) \right]$

$M(x_i, y_j) = \prod_{i,j=1}^{N} \left[\sinh \frac{x_i - x_j}{2} \; \sinh \frac{y_i - y_j}{2} \; (\cosh \frac{x_i - y_j}{2})^{-2} \right]$

for any N, $k > 2$: $\langle \mathcal{W} \rangle = \langle \exp \sum_i x_i \rangle$

[Drukker, Marino, Putrov 10; Klemm, Marino, et al 12]

$$\langle \mathcal{W} \rangle = \frac{1}{2\, \sin \frac{2\pi}{k}} \, \frac{\mathrm{Ai}\left[(\frac{\pi^2}{2} k)^{1/3} \left(N - \frac{k}{24} - \frac{7}{3k} \right) \right]}{\mathrm{Ai}\left[(\frac{\pi^2}{2} k)^{1/3} \left(N - \frac{k}{24} - \frac{1}{3k} \right) \right]}$$

- "M-theory" regime: large N at fixed k:

$$\mathrm{Ai}(x) \Big|_{x \gg 1} \simeq \frac{e^{-\frac{2}{3} x^{3/2}}}{2\sqrt{\pi}\, x^{1/4}} \sum_{n=0}^{\infty} \frac{(-\frac{3}{4})^n \, \Gamma(n+\frac{5}{6}) \Gamma(n+\frac{1}{6})}{2\pi \, n! \, x^{3n/2}}$$

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi}{k}} \, e^{\pi \sqrt{\frac{2N}{k}}} \left[1 - \frac{\pi \left(k^2 + 32 \right)}{24\sqrt{2}\, k^{3/2}} \frac{1}{\sqrt{N}} + \mathcal{O}(\frac{1}{N}) \right]$$

- "string" regime: $N, k \gg 1, \quad \lambda = \frac{N}{k}$ = fixed

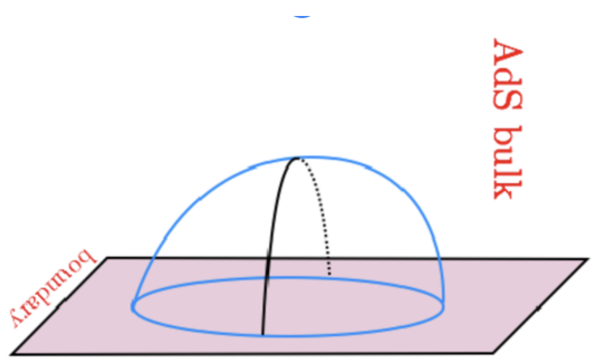
$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi\lambda}{N}} e^{\pi\sqrt{2\lambda}} \Big[1 - \frac{\pi}{24\sqrt{2}} \frac{1}{\sqrt{\lambda}} + \mathcal{O}(\frac{1}{N})\Big] = \frac{N}{4\pi\lambda} e^{\pi\sqrt{2\lambda}} [1 + ...]$$

- dual string in $AdS_5 \times S^5$ and $AdS_4 \times CP^3$

$$\text{SYM}: \quad g_s = \frac{g_{YM}^2}{4\pi} = \frac{\lambda}{4\pi N}, \qquad T = \frac{\sqrt{\lambda}}{2\pi}, \qquad \lambda = g_{YM}^2 N$$

$$\text{ABJM}: \quad g_s = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}, \qquad T = \frac{\sqrt{\lambda}}{\sqrt{2}}, \qquad \lambda = \frac{N}{k}$$

$\langle \mathcal{W} \rangle$ = disk partition function near AdS_2 minimal surface



$$\langle \mathcal{W} \rangle = Z_{\rm str} = \frac{1}{g_{\rm s}} Z_1 + \mathcal{O}(g_{\rm s})\,, \qquad\qquad Z_1 = \int [dx] ...\, e^{-T\int d^2\sigma\, L}$$

$$\text{SYM}: \quad \langle \mathcal{W} \rangle = \sqrt{\frac{2}{\pi}} \frac{N}{\lambda^{3/4}}\, e^{\sqrt{\lambda}} + ... \; = \frac{1}{2\pi} \, \frac{\sqrt{T}}{g_{\rm s}} \;\; e^{2\pi T} + ...$$

$$\text{ABJM}: \quad \langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}} + ... \; = \frac{1}{\sqrt{2\pi}} \, \frac{\sqrt{T}}{g_{\rm s}} \;\; e^{2\pi T} + ...$$

universal form at strong coupling [Giombi, AT 2020]

$$\langle \mathcal{W} \rangle = c_0 \, \tfrac{\sqrt{T}}{g_{\rm s}} \;\; e^{2\pi T} \Big[1 + \mathcal{O}(T^{-1}) \Big] + \mathcal{O}(g_{\rm s}), \qquad\quad c_0 = \tfrac{1}{(\sqrt{2\pi})^{d-3}}$$

dual string theories in $\text{AdS}_d \times M^{10-d}$ ($d = 4, 5$) have similar structure

$c_0\sqrt{T}$ from 1-loop superstring partition function in $\text{AdS}_d \times M^{10-d}$

det's of fluctuation operators near AdS_2 minimal surface

$$\log Z_1 = -\frac{1}{2} \log \frac{[\det(-\nabla^2 + 2)]^{d-2} \; [\det(-\nabla^2)]^{10-d}}{[\det(-\nabla^2 + \frac{1}{2})]^{2d-2} \; [\det(-\nabla^2 - \frac{1}{2})]^{10-2d}}$$

$$Z_1 \sim (\sqrt{T})^{\chi}\,, \qquad (Z_1)_{\text{disk}} \sim \sqrt{T}$$

disk with h handles $\chi = 1 - 2h$: $g_s^{-1} \to g_s^{\chi}$, $\sqrt{T} \to (\sqrt{T})^{\chi}$

• thus prediction on string side:

$$\langle \mathcal{W} \rangle = e^{2\pi T} \sum_{h=0}^{\infty} c_h \left(\frac{g_s}{\sqrt{T}} \right)^{2h-1} \left[1 + \mathcal{O}(T^{-1}) \right]$$

remarkably, consistent with form of $\frac{1}{N}$ terms on gauge theory side

• SYM: $N \gg 1$, then $\lambda \gg 1$

$$\langle \mathcal{W} \rangle = e^{\frac{(N-1)\lambda}{8N^2}} L^1_{N-1}(-\tfrac{\lambda}{4N}) = e^{\sqrt{\lambda}} \sum_{h=0}^{\infty} \frac{\sqrt{2}}{96^h \sqrt{\pi}\, h!} \frac{\lambda^{\frac{3}{4}(2h-1)}}{N^{2h-1}} \left[1 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}}) \right]$$

• $\frac{g_s}{\sqrt{T}} \sim \frac{\lambda^{3/4}}{N}$ appears as expansion parameter

• from localization result: $c_h = \frac{1}{2\pi\, h!} \left(\frac{\pi}{12} \right)^h$

• large $T = \frac{\sqrt{\lambda}}{2\pi}$ terms at each order in $g_s = \frac{\lambda}{N}$ exponentiate:

$$\langle \mathcal{W} \rangle = W_1 \, e^H \left[1 + \mathcal{O}(T^{-1}) \right], \qquad W_1 = \frac{1}{2\pi} \frac{\sqrt{T}}{g_s} \, e^{2\pi T}$$

$$H \equiv \frac{\pi}{12} \frac{g_s^2}{T} = \frac{1}{96\pi} \frac{\lambda^{3/2}}{N^2}$$

conjectured interpretation: H= "handle operator"

• computing even 1-loop string term is challenge

but will derive analog of $e^{\frac{\pi}{12} \frac{g_s^2}{T}}$ in ABJM case

from 1-loop M2 brane partition function

$1/N$ expansion of $\frac{1}{2}$ BPS circular WL in ABJM

- string side: universal form of expansion in small g_s, large T

$$\langle \mathcal{W} \rangle = e^{2\pi T} \frac{\sqrt{T}}{g_s} \Big(c_0 + ... + \frac{g_s^2}{T} [c_1 + ...] + \big(\frac{g_s^2}{T}\big)^2 [c_2 + ...] + ... \Big)$$

ABJM: $\frac{g_s^2}{T} \sim \frac{\lambda^2}{N^2} = \frac{1}{k^2}$, corrections $T^{-1} \sim \frac{\sqrt{k}}{\sqrt{N}}$

- gauge side (localization): exponentiation of leading terms?

no, summed by $\frac{1}{\sin \frac{2\pi}{k}}$: $\frac{2\pi}{k} = 2\pi \frac{\lambda}{N} = \sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}}$ [Beccaria, AT 20]

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi}{k}} e^{\pi \sqrt{\frac{2N}{k}}} \Big[1 + \mathcal{O}\big(\tfrac{1}{\sqrt{N}}\big) \Big] = \frac{1}{2 \sin \Big(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \Big)} e^{2\pi T} \Big[1 + O(T^{-1}) \Big]$$

$$\frac{1}{2 \sin \Big(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \Big)} = \frac{\sqrt{T}}{\sqrt{2\pi} g_s} \Big[1 + \frac{\pi}{12} \frac{g_s^2}{T} + \frac{7\pi^2}{1440} \big(\frac{g_s^2}{T} \big)^2 + ... \Big]$$

Main result: $\frac{1}{\sin\frac{2\pi}{k}}$ is 1-loop M2 brane contribution [Giombi, AT 2023]

- large N, fixed k: WL described by M2-brane on $\mathrm{AdS}_2 \times S^1$
  $e^{-S_{\mathrm{M2}}} = e^{\pi\sqrt{\frac{2N}{k}}}$ from classical M2 action

- 1-loop M2 correction $\to Z_1 = \frac{1}{\sin\frac{2\pi}{k}}$

- leading quantum M2 correction in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$
sums large T terms at all orders in g_{s} in string theory on $\mathrm{AdS}_4 \times \mathrm{CP}^3$

- highly non-trivial check of $\mathrm{AdS}_4/\mathrm{CFT}_3$ duality at all orders in $1/N$

1-loop M2 brane partition function

• $\mathrm{AdS}_2 \times S^1$ M2 solution dual to Wilson loop:
wrapping AdS_2 of AdS_4 and S^1_φ of $S^7/\mathbb{Z}_k$

$S_{\mathrm{M2}} = \frac{1}{4}\mathrm{T}_2 \mathrm{vol}(\mathrm{AdS}_2)\, \frac{2\pi}{k} = -\pi\sqrt{\frac{2N}{k}}$

$e^{-S_{\mathrm{M2}}}$ matches leading factor in $\langle \mathcal{W} \rangle$

• expand M2 brane action near $\mathrm{AdS}_2 \times S^1$ solution
static gauge: M2 coordinates $\sigma_1, \sigma_2 = \mathrm{AdS}_2$; $\sigma_3 = 11\mathrm{d}$ circle φ
κ-symmetry gauge: 8+8 3d fluctuations [Sakaguchi, Shin, Yoshida 2010]

• Fourier expansion of 3d fields in $\sigma_3 = (0, 2\pi)$:
tower $(n = 0, \pm 1, ...)$ of bosonic + fermionic 2d fields on AdS_2

• fluctuations in 2 $\perp$ AdS_4 directions: $m^2 = \frac{1}{4}(kn-2)(kn-4)$

• fluctuations of CP^3 directions: $m^2 = \frac{1}{4}kn(kn+2)$

• fermions: 6+2 towers of 2d spinors: $m = \frac{1}{2}kn \pm 1, \quad m = \frac{1}{2}kn$

• string theory limit $k \to \infty$: $n \neq 0$ modes decouple
$n = 0$: same as 2d fluctuationsin superstring on $AdS_4 \times CP^3$:
B: 2 of $m^2 = 2$; 6 of $m^2 = 0$; F: 3+3 of $m = \pm 1$ and 2 of $m = 0$

• 1-loop M2 partition function on $AdS_2 \times S^1$

$$Z_{M2} = Z_1 \, e^{-S_{M2}} \left[1 + \mathcal{O}\left(T_2^{-1}\right)\right], \qquad S_{M2} = -\frac{\pi}{k} T_2$$

$$Z_1 = \prod_{n=-\infty}^{\infty} \mathcal{Z}_n \,, \qquad \mathcal{Z}_0 = AdS_4 \times CP^3 \text{ string on } AdS_2$$

$$\mathcal{Z}_n = \frac{\left[\det\left(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}+1)^2\right)\right]^{\frac{3}{2}} \left[\det\left(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}-1)^2\right)\right]^{\frac{3}{2}} \det\left(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2})^2\right)}{\det\left(-\nabla^2 + \frac{1}{4}(kn-2)(kn-4)\right) \left[\det\left(-\nabla^2 + \frac{1}{4}kn(kn+2)\right)\right]^3}$$

• dets by spectral ζ-function in AdS_2 [Drukker, Gross, AT 00]
$$\Gamma_1 = \tfrac{1}{2} \log \det(-\nabla^2 + m^2) = -\tfrac{1}{2}\zeta(0; m^2) \log \Lambda^2 - \tfrac{1}{2}\zeta'(0; m^2)$$
$$\zeta'_B(0; m^2) = -\tfrac{1}{12}(1 + \log 2) - \int_0^{m^2+\frac{1}{4}} dx \, \psi(\sqrt{x} + \tfrac{1}{2})$$

• cancellation of log UV ∞ in $\Gamma_1 = -\log Z_1$:

$$\zeta_{\rm tot}(0) = \tfrac{1}{2} \sum_{n=-\infty}^{\infty} \big(-2 + 4 \big) = \sum_{n=-\infty}^{\infty} 1 = 1 + 2\zeta_R(0) = 0$$

$n \neq 0$ massive KK modes cancel UV div of $AdS_4 \times CP^3$ string ($n = 0$)
(as expected: no 1-loop log UV div in 3d theory)

$$\Gamma_1 = -\log Z_1 = -\tfrac{1}{2}\zeta'_{\rm tot}(0)\,, \qquad \zeta'_{\rm tot}(0) = \sum_{n=-\infty}^{\infty} \zeta'_{\rm tot}(0; n)$$

• combining B and F contributions: remarkable simplifications

$$\zeta'_{\rm tot}(0; n) + \zeta'_{\rm tot}(0; -n) = -2\log\big(\tfrac{k^2n^2}{4} - 1\big)\,, \;\; kn > 2$$

$$\Gamma_1 = \sum_{n=1}^{\infty} \log\big(\tfrac{k^2n^2}{4} - 1\big) = 2\sum_{n=1}^{\infty} \log\tfrac{kn}{2} + \sum_{n=1}^{\infty} \log\big(1 - \tfrac{4}{k^2n^2}\big)$$

$\zeta_R(0) = -\frac{1}{2}$, $\zeta'_R(0) = -\frac{\log(2\pi)}{2}$: $\qquad 2\sum_{n=1}^{\infty} \log\frac{kn}{2} = -\log\frac{k}{4\pi}$

• use Euler's relation: $\sin \pi x = \pi x \prod_{n=1}^{\infty}(1 - \frac{x^2}{n^2})$

$$\sum_{n=1}^{\infty} \log\left(1 - \tfrac{4}{k^2 n^2}\right) = \log\left[\prod_{n=1}^{\infty}\left(1 - \tfrac{4}{k^2 n^2}\right)\right] = \log\left(\frac{k}{2\pi}\sin\frac{2\pi}{k}\right)$$

• final result for $k > 2$

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin\frac{2\pi}{k}}$$

precise agreement with localization

• cases of $k = 1, 2$ require a separate treatment: $Z_1^{k=1} = \frac{1}{4}$, $Z_1^{k=2} = 1$
but localization result for $\langle \mathcal{W} \rangle$ is singular for $k = 1, 2$
need further consideration – may be subtle as susy $\mathcal{N} = 6 \to 8$

Generalizations:

• $\frac{1}{\sqrt{N}}$ corrections: from higher M2 loops?

expansion parameter: effective M2 brane tension $\frac{1}{\mathrm{T}_2} = \frac{\pi}{\sqrt{2}\,k}\frac{1}{\sqrt{N}}$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\Big[1 - \frac{\pi(k^2+32)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + \mathcal{O}\big(\frac{1}{N}\big)\Big]$$

$$= \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\frac{\pi^2}{k}\mathrm{T}_2}\Big[1 - \frac{k^2+32}{24k}\frac{1}{\mathrm{T}_2} + \mathcal{O}(\mathrm{T}_2^{-2})\Big]$$

• $\mathrm{T}_2^{-1} \sim \frac{1}{\sqrt{N}}$ 2-loop M2 contribution UV finite?

• conjecture: div's cancel also at higher M2 loops
as in GS action in $AdS_5 \times S^5$ or $AdS_4 \times CP^3$

hidden symmetry in M2 brane theory?

• Lesson: take quantum M2 brane seriously
use it to derive strong coupling corrections to
non-BPS observables not known from localization or integrability

• Example: cusp anom dim in ABJM
at strong coupling beyond planar limit [Giombi, AT]

$$f(\lambda, N) = \sqrt{2\lambda} - \frac{5}{2\pi}\log 2 + q_1(k) + \mathcal{O}(\frac{1}{\sqrt{\lambda}})$$

$$q_1 = \frac{2\pi}{3k^2} + \frac{2\pi^3}{45k^4} + ... = \frac{2\pi\lambda^2}{3N^2} + \frac{2\pi^3\lambda^4}{45N^4} + ...$$

M2 brane instanton contrubution to ABJM free energy

- semiclassical quantization of M2 brane in $AdS_4 \times S^7/\mathbb{Z}_k$ reproduces also localization result for instanton contribution to free energy F in ABJM theory on S^3 [Beccaria, Giombi, AT 23]
- non-perturbative part of F [Drukker et al, Hatsuda 12]

$$F^{\rm np}(N,k) = F_1^{\rm inst}(N,k)\left[1 + \frac{\pi}{\sqrt{2k}}\frac{k^2-40}{12k}\frac{1}{\sqrt{N}} + ...\right] + ...$$

$$F_1^{\rm inst}(N,k) = -\frac{1}{\sin^2\frac{2\pi}{k}}\, e^{-2\pi\sqrt{\frac{2N}{k}}}$$

- in string theory regime (large N and k with $\lambda = \frac{N}{k}$=fixed) interpreted as string world-sheet instanton (wrapping $CP^1 \subset CP^3$)
- in M-theory regime (large N with fixed k) corresponds to M2 instanton on 11d circle and $CP^1 \subset CP^3$, i.e. $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$
- classical action of $S^3/\mathbb{Z}_k$ M2 brane solution matches exponential 1-loop fluctuation determinants reproduce $\frac{1}{\sin^2\frac{2\pi}{k}}$ prefactor

Conclusions

• novel precision tests of AdS_4/CFT_3
in M-theory regime (non-planar corrections on gauge side)

• remarkable duality: relation between very different QFT's:
3d superconformal non-abelian CS+matter on S^3
and non-linear M2 theory for abelian 3d scalar multiplet
in curved $AdS_4 \times S^7/\mathbb{Z}_k$ space
• evidence that quantum M2 brane theory is well defined
at least in semiclassical expansion
sheds light on M-theory as theory of quantum supermembranes

• similar results for AdS_7/CFT_6 generalization:
6d (2,0) superconformal theory on multiple M5 branes
dual to M-theory on $AdS_7 \times S^4$
quantum effects in free energy on $S^5 \times S^1$ captured by
semiclassical M2 branes in $AdS_7 \times S^4$ [Beccaria, Giombi, AT 23]