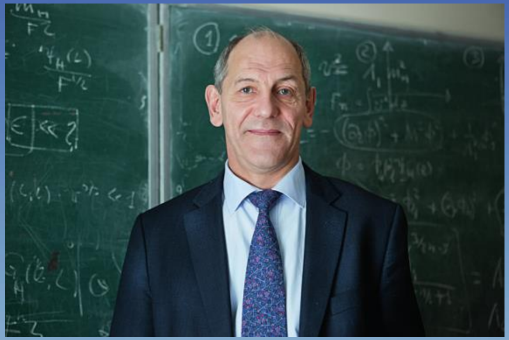

International Conference
on
Particle Physics and Cosmology

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Yerevan, Armenia

Valery Rubakov and quantum cosmology: origin of the Universe

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Plan

Schroedinger equation vs Wheeler-DeWitt equation(s)

No-boundary (Hartle-Hawking) vs tunneling wavefunction

Cosmological initial conditions: microcanonical density matrix of the Universe

CFT driven cosmology: quasi-thermal cosmological instantons and UV bounded range of Λ

New type of hill-top inflation, ¤$\rightarrow$ $V(\phi)$ – selection of inflaton potential $V(\phi)$ maxima

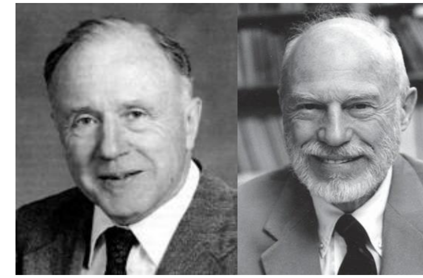
Mechanism of hill-top potential: origin of non-minimal Higgs inflation and R^2 gravity

Conformal higher spin fields (CHS): solution of hierarchy problem; justification of semiclassical expansion

Thermally corrected CMB spectrum: temperature of the CMB temperature – cool Universe

Schroedinger equation vs Wheeler-DeWitt equation(s)

$$i\frac{\partial}{\partial t}|\,\Psi(t)\,\rangle = \hat{H}|\,\Psi(t)\,\rangle \quad \Longleftrightarrow \quad \hat{H}_\mu|\,\Psi\,\rangle = 0$$



$$\underbrace{\hat{H}_\mu = \hat{H}_\perp(\mathbf{x}),\ \hat{H}_i(\mathbf{x})}$$

Hamiltonian and momentum quantum Dirac constraints constraints, ***NO TIME !***

Semiclassical gravity factor

$$\hat{H}_\mu = \hat{H}_\mu^{grav} + \hat{H}_\mu^{matter}, \qquad |\,\Psi\,\rangle = \Psi[\,g_{ij},\phi\,] = e^{iS[\,g_{ij}\,]}\,\Psi_{matter}[\,g_{ij},\phi\,]$$

Quantum matter wave function in external gravitational field

$$||\,\Psi(t)\,\rangle\rangle = \Psi_{matter}[\,g_{ij}(t),\phi\,]$$

Solution of classical vacuum Einstein eqs. with ADM lapse and shift $N^\perp(t),\ N^i(t)$

$$\hat{H}_\mu\,|\Psi\rangle = 0 \quad \Rightarrow \quad i\frac{\partial}{\partial t}||\,\Psi(t)\,\rangle\rangle = \hat{H}_{matter}||\,\Psi(t)\,\rangle\rangle + \text{graviton loops}$$

$$\hat{H}_{matter} = \int d^3x\,\left(N^\perp \hat{H}_\perp^{matter} + N^i \hat{H}_i^{matter}\right)$$

WDW equation is the ``most useless" equation in theoretical physics?

V.G. Lapchinsky and V.A. Rubakov, Acta Phys. Polon. B10 (1979) 1041-1048

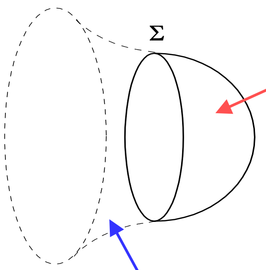
No-boundary (Hartle-Hawking) vs tunneling wavefunction

Hyperbolic nature of the Wheeler-DeWitt equation

$$\Psi_\pm\big(\varphi, \Phi(\mathbf{x})\big) = \exp\left(\mp\tfrac{1}{2}S_E(\varphi)\right)\Psi_{matter}\big(\varphi, \Phi(\mathbf{x})\big)$$

inflaton other fields

Euclidean action of quasi-de Sitter instanton with the effective Λ (slow roll):



Euclidean spacetime

no tunneling, really: "birth from nothing"

$$\Lambda \simeq \frac{V(\varphi)}{M_P^2}$$

FRW $ds^2 = N^2\,d\tau^2 + a^2\,d\Omega_{(3)}^2$

$$a_0(\tau) = \frac{1}{H}\sin(H\tau), \quad H = \sqrt{\frac{\Lambda}{3}}$$

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_P^4}{V(\varphi)} < 0$$

Analytic continuation – Lorentzian signature dS geometry:

$$\tau = \pi/2H + it$$

$$a_L(t) = \frac{1}{H}\cosh(Ht)$$

Hartle-Hawking no-boundary wavefunction

$$\Psi_{HH} \sim \exp(-S_E) = \exp\left(12\pi^2 \frac{M_P^4}{V(\varphi)}\right) \to \infty$$

$$\frac{V(\varphi)}{M_P^2} = \Lambda_{eff} \to 0$$

Most probable **at the minimum**
of inflaton potential **¤$_{eff} \to$ 0**
-- insufficient amount of inflation

Tunneling wavefunction

$$\Psi_T \sim \exp(+S_E)$$

Cosmology debate:
no-boundary vs tunneling

Questionable status of both states within unitarity approach to quantum gravity

Microcanonical density matrix of the Universe

$$\hat{\rho} = \sum_{\text{all } |\boldsymbol{\Psi}\rangle} w_\Psi |\,\boldsymbol{\Psi}\,\rangle\langle\,\boldsymbol{\Psi}\,|, \qquad w_\Psi = 1$$

sum over "everything" that satisfies the Wheeler-DeWitt equation $\hat{H}_\mu\,|\boldsymbol{\Psi}\rangle = 0$

Projector onto the subspace of quantum gravitational constraints

A.O.B., Phys. Rev. Lett. 99, 071301 (2007)

$$\hat{H}_\mu \equiv \underbrace{\hat{H}_{\perp\mathbf{x}},\ \hat{H}_{i\,\mathbf{x}}}$$

local operators of the Wheeler-DeWitt equations

$$\hat{\rho} = \frac{1}{Z}\prod_\mu \delta(\hat{H}_\mu), \qquad Z = \mathrm{Tr}\prod_\mu \delta(\hat{H}_\mu)$$

$$\mu = \big(\perp\mathbf{x},\ i\mathbf{x}\big)$$

Motivation:

A simplest analogy in unconstrained system with a conserved Hamiltonian $\hat{H}$ Is the microcanonical density matrix with a fixed energy E

$$\hat{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have freely specifiable constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

An ultimate equipartition in the full set of states of the theory --- "Sum over Everything". Creation of the Universe from *Everything* is conceptually more appealing than creation from *Nothing*, because the democracy of the microcanonical equipartition better fits the principle of Occam razor, preferring to drop redundant assumptions, than the selection of a concrete state.

Matrix element of the cosmological density matrix

Faddeev-Popov path integral measure

$$\boldsymbol{\rho}(\varphi_+,\varphi_-)=\frac{1}{Z}\int D[\,g_{\mu\nu},\varPhi\,]\;e^{iS[\,g_{\mu\nu},\varPhi\,]}\Big|_{g_{ij}(t_\pm)=g^\pm_{ij},\;\;\varPhi(t_\pm)=\varPhi_\pm}$$

$$\varphi_\pm=(g^\pm_{ij},\varPhi_\pm)$$

Lorentzian

$$Z=\int d\varphi\,\boldsymbol{\rho}(\varphi_+,\varphi_-)\Big|_{\varphi_\pm=\varphi}=\int_{\text{periodic}} D[\,g_{\mu\nu},\varPhi\,]\;e^{iS[\,g_{\mu\nu},\varPhi\,]}$$

Partition function

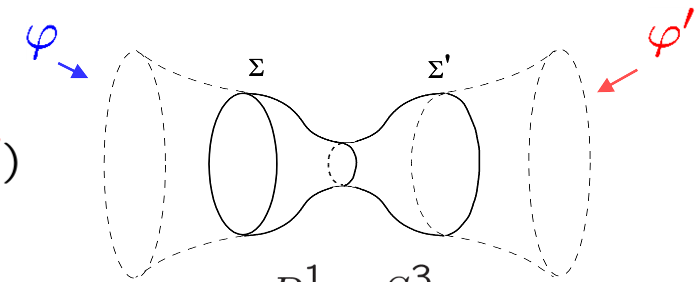
Absence of periodic Lorentzian histories and rotation of integration contours over fields and time

Euclidean path integral and its saddle points

$$Z=\int_{\text{periodic}} D[\,g_{\mu\nu},\varPhi\,]\;e^{-S[\,g_{\mu\nu},\varPhi\,]}$$

Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

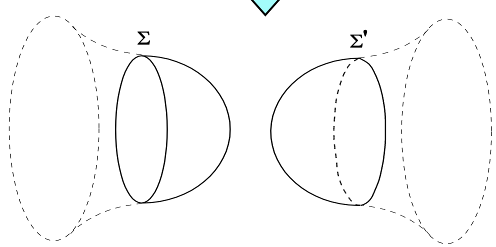
$$\hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}(\varphi, \varphi')$$



$R^1 \times S^3$

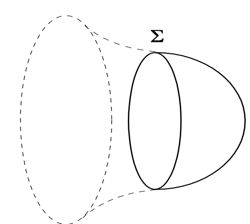
pinching a tubular spacetime

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}(\varphi, \varphi')$$



$D^4 \cup D'^4$

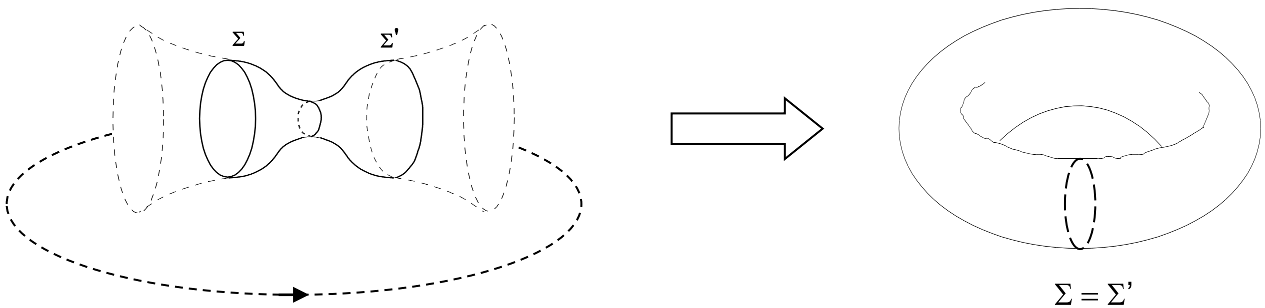
$$|\Psi_{HH}\rangle = \Psi_{HH}(\varphi)$$



D^4

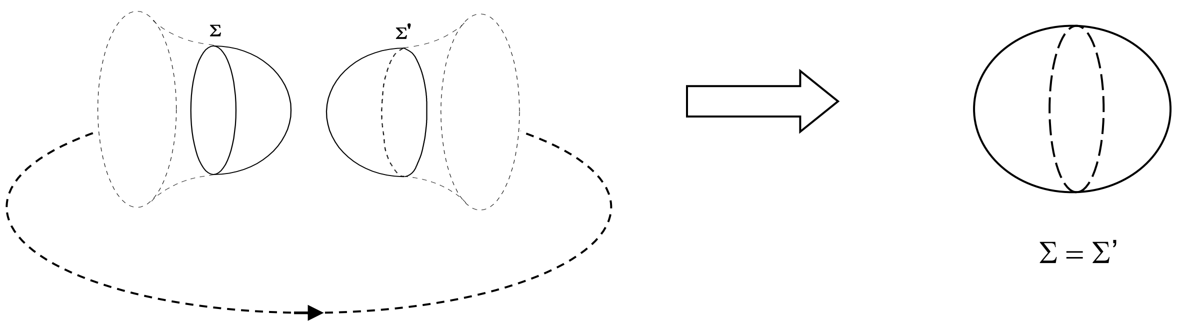
density matrix representation of a pure Hartle-Hawking state

Transition to statistical sums



$R^1 \times S^3$ $\qquad$ $S^1 \times S^3$

thermal instantons



$D^4 \cup D'^4$ $\qquad$ S^4

Hartle-Hawking (vacuum) instanton

Inflationary model driven by the trace anomaly of Weyl invariant fields --- CFT driven cosmology

$$S[\,g_{\mu\nu},\Phi\,] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}\,(R-2\Lambda) + S_{CFT}[\,g_{\mu\nu},\Phi\,]$$

Λ -- primordial cosmological constant

Omission of graviton loops

$$S_{\rm eff}[g_{\mu\nu}] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}(R-2\Lambda) + \Gamma_{CFT}[g_{\mu\nu}],$$

$$e^{-\Gamma_{CFT}[\,g_{\mu\nu}]} = \int D\Phi\, e^{-S_{CFT}[\,g_{\mu\nu},\Phi\,]}$$

Recovery of i_{CFT} from the conformal anomaly on a static Einstein Universe (anomaly, Casimir energy and free energy contributions)

$$g_{\mu\nu}\frac{\delta\Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{64\pi^2}g^{1/2}\left(\beta E + \alpha\Box R + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss –Bonnet term

Weyl

$\beta = \sum_s \beta_s \mathbb{N}_s,\quad \mathbb{N}_s$ − # of spin s fields, β_s − spin-dependent coefficients

β -- critically important parameter (overall coefficient of Gauss-Bonnet term in conformal anomaly)

Minisuperspace (FRW) ansatz
for the saddle point

$$g_{\mu\nu}dx^\mu dx^\nu = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)}$$

$$S_{\rm eff}[\,g_{\mu\nu}\,] = S_{\rm eff}[\,a,N\,]$$

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\rm eff}[\,a,N\,]}{\delta N(\tau)} = 0$$

$$\frac{1}{a^2} - \frac{\dot{a}^2}{a^2} = \frac{\varepsilon}{3M_\pm^2(\varepsilon)},$$

Friedmann equation

$$M_\pm^2(\varepsilon) = \frac{M_P^2}{2}\left(1 \pm \sqrt{1 - \frac{\beta}{6\pi^2 M_P^4}\,\varepsilon}\right),$$

Effective Planck mass

$$\varepsilon = M_P^2\Lambda + \frac{1}{2\pi^2 a^4}\sum_\omega \frac{\omega}{e^{\eta\,\omega} - 1},$$

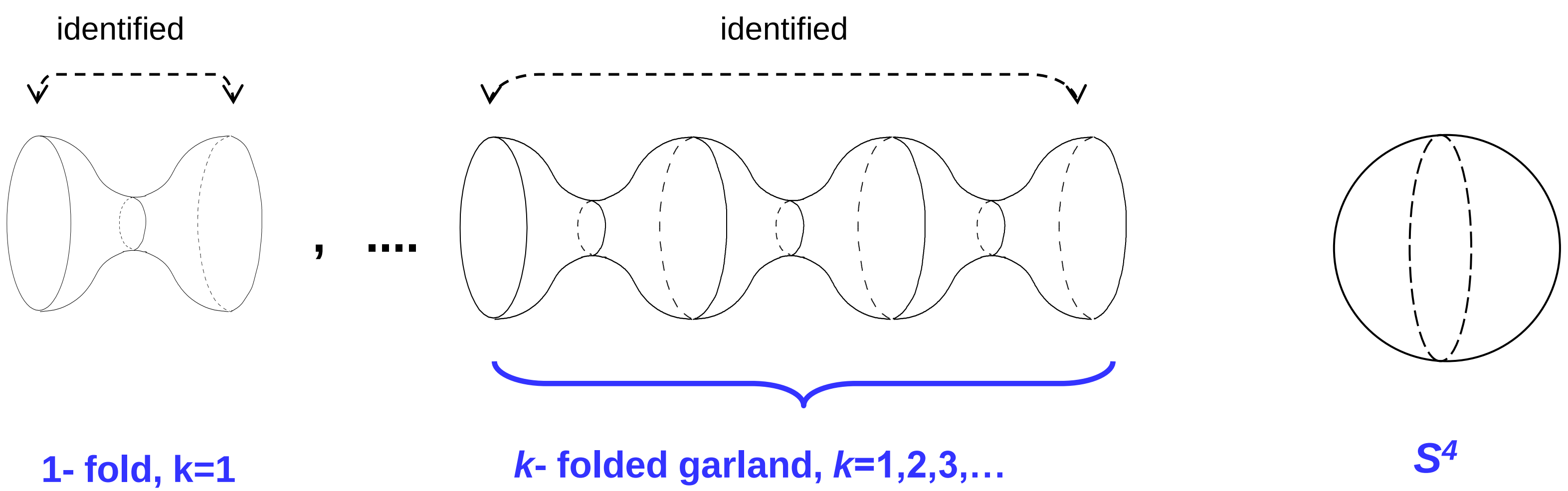
Energy density=¤ + radiation of CFT particles --
sum over field oscillators with frequencies !
(eigenvalues of Laplacian on S^3)

$$\eta = \int_{S^1} \frac{d\tau N}{a}$$

Inverse temperature in units of conformal
time period on S^1

Existence of the quasi-thermal stage preceding the inflation

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and the vacuum Hartle-Hawking instantons (S^4)



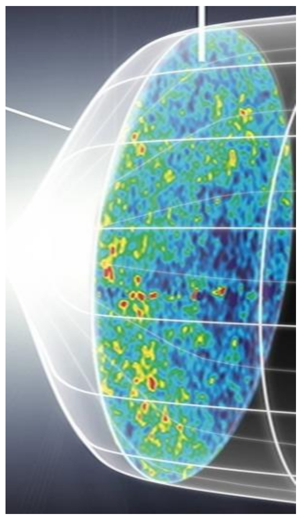
does not contribute: ruled out by *infinite positive* Euclidean action (effect of conformal anomaly)

UV bounded cosmological constant range:

$$\Lambda_{min} < \Lambda < \Lambda_{max} = \frac{12\pi^2}{\beta} M_P^2$$

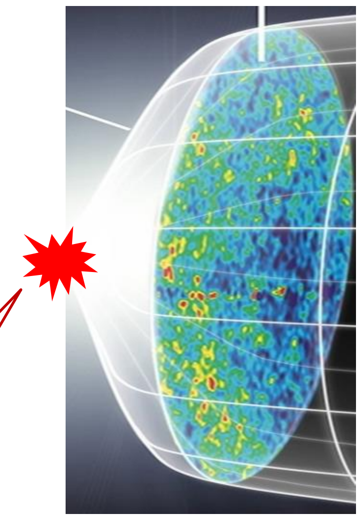
Initial thermal state with the primordial temperature T_{prim} of matter

Standard inflation scenario versus Density matrix scenario



Vacuum,
absolute zero
temperature

Inflation, hot
big-bang
$\rightarrow$ relic radiation



Thermal state,
primordial
temperature T_{prim}

Inflation, $T_{prim} \rightarrow 0$,
hot big-bang
$\rightarrow$ relic radiation

"SOME LIKE IT HOT" (SLIH) scenario

Known inflation paradigm retracted the BB concept by replacing it with the initial vacuum state.

"SOME LIKE IT HOT" (SLIH) scenario recovers a new incarnation of Hot Big Bang -- it incorporates effectively thermal state at the onset of the cosmological evolution.

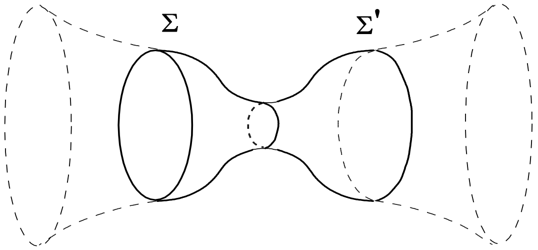
So how does SLIH scenario matches with inflation?

SLIH inflation

1) Generalization to Λ as a composite operator – inflaton potential in "slow roll" regime

$$\Lambda \to \frac{\rho_\phi}{M_P^2}, \quad \rho_\phi = V(\phi) - \frac{\dot{\phi}^2}{2} \simeq V(\phi)$$

2) Lorentzian Universe with initial conditions set by the instanton. Analytic continuation of the instanton solutions:



$$\tau = \tau_* + it, \; a_L(t) = a(\tau_* + it)$$

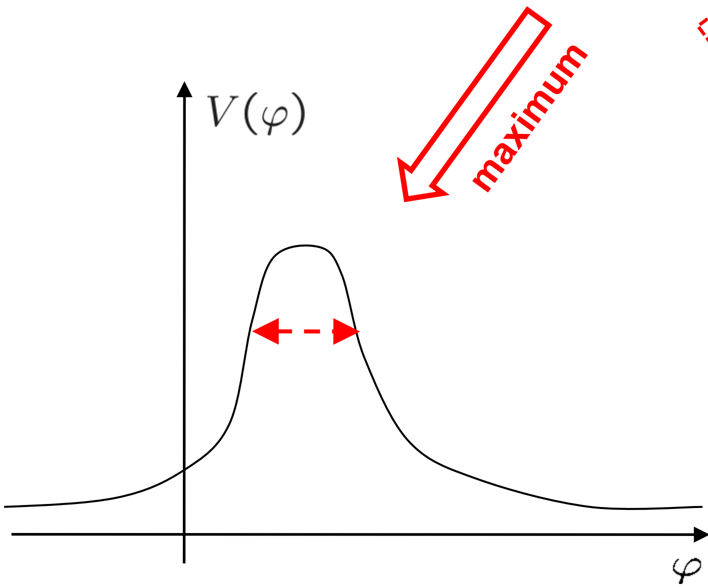
3) **Expansion and quick dilution of primordial radiation, decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter and its thermalization**

Selection of inflaton potential *maxima* as initial conditions for inflation

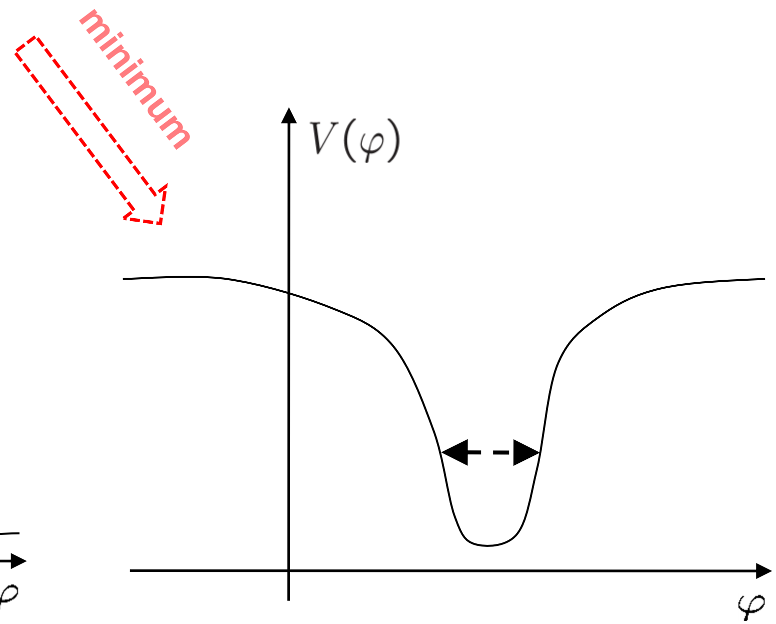
Critical feature:

$$\frac{d}{d\tau}a^3\dot{\phi} = a^3\frac{\partial V}{\partial \phi} \;\Rightarrow\; \oint d\tau\, a^3\frac{\partial V}{\partial \phi} = 0 \;\Rightarrow\; \frac{\partial V}{\partial \phi} \gtrless 0$$

Potential extremum ``inside" instanton

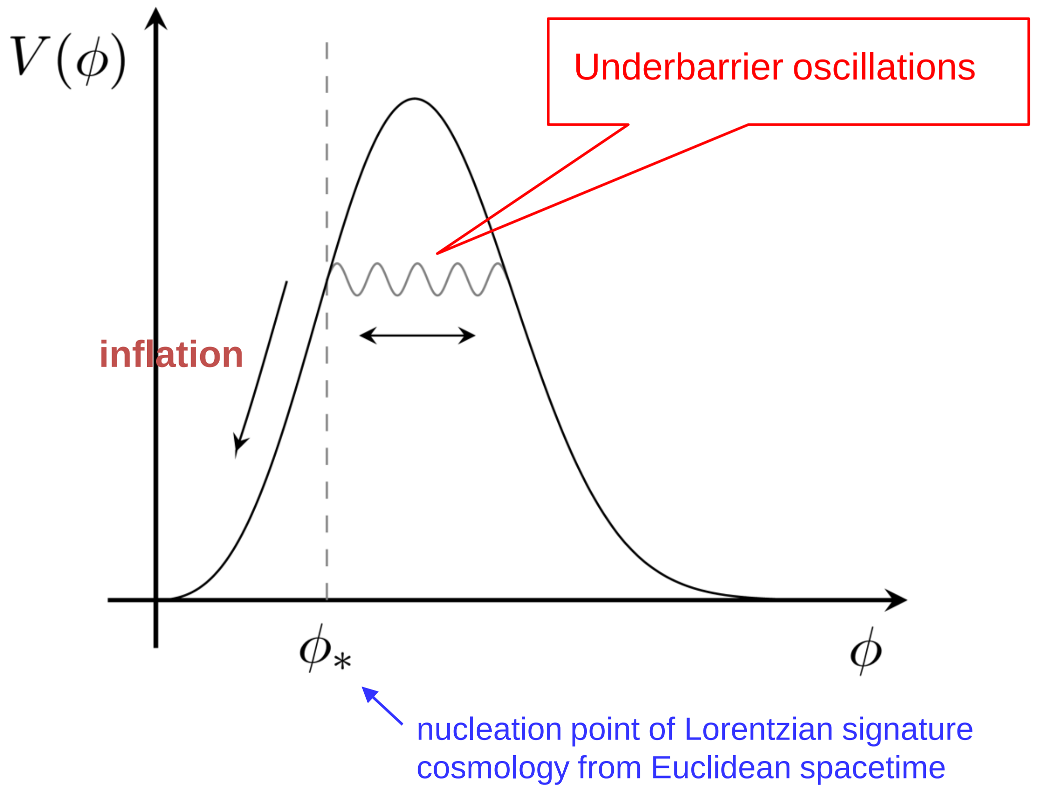


classically forbidden (underbarrier) oscillation



classically allowed (overbarrier) oscillation --- ruled out because of *underbarrier* oscillations of scale factor

Hill-top inflation



Approximation of two coupled oscillators → slow roll parameters typical of Higgs and R^2 inflation:

$$P_\zeta(k) = 2.2 \times 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}, \quad k_0 = 0.05 Mpc^{-1}, \quad n_s = 0.965 \pm 0.005$$

$$n_s = 1 - 6\epsilon + 2\eta, \quad \epsilon = \frac{1}{2}\left(M_P \frac{V_*'}{V_*}\right)^2, \quad \eta = M_P^2 \frac{V_*''}{V_*}$$

$$\epsilon \sim \eta^2 \ll |\eta|, \quad \eta < 0$$

Mechanism of hill-top potential: origin of non-minimal Higgs inflation and R^2 gravity

Higgs field *H* non-minimally coupled to curvature:

B.Spokoiny 1986, A.Kamenshchik & A.B 1991,
Bezrukov,Shaposhnikov 2008
A.Kamenshchik, A.Starobinsky & A.B 2008

$$\varphi^2 \equiv H^\dagger H$$

$$S_{EH+SM}[\,g_{\mu\nu}, H, \ldots] = \int d^4x\, g^{1/2} \left(\frac{\lambda \varphi^4}{4} - \frac{M_P^2 + \xi \varphi^2}{2} R + \frac{1}{2} (\nabla\varphi)^2 + \ldots \right)$$

Starobinsky model of R^2 gravity:

$$S_\xi^{\rm Star}[\,g_{\mu\nu}, \varphi\,] = \int d^4x\, g^{1/2} \left\{ -\frac{M_P^2}{2} R - \frac{\xi}{4} R^2 \right\}$$

$$S_\xi^{\rm Star}[\,g_{\mu\nu}, \varphi\,] = \int d^4x\, g^{1/2} \left\{ -\frac{M_P^2}{2} \left(1 + \xi \frac{\varphi^2}{M_P^2} \right) R + \frac{\xi \varphi^4}{4} \right\}$$

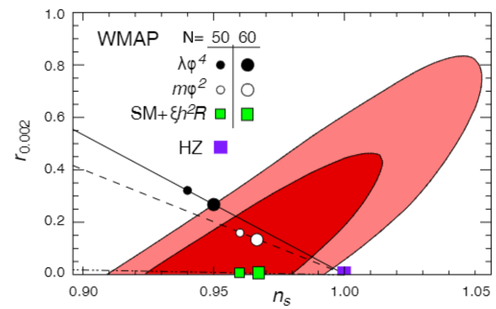


Fig. 2. The allowed WMAP region for inflationary parameters (r, n). The green boxes are our predictions supposing 50 and 60 e–foldings of inflation. Black and white dots are predictions of usual chaotic inflation with $\lambda\phi^4$ and $m^2\phi^2$ potentials, HZ is the Harrison-Zeldovich spectrum.

$\xi \sim 10^4 \gg 1$ ⟹ **Higgs inflation with** $\frac{\Delta T}{T} \sim 10^{-5},\ n_s \simeq 0.96,\ r \simeq 0.003$

$M_{\rm Higgs} \simeq 126 GeV$

Mechanism of hill-top inflaton potential– quantization in the Jordan frame and transition to Einstein frame:

Not in Einstein frame, no shift symmetry, IR instability and breakdown of grad. expansion!

non-minimal coupling

$$\Gamma[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2}\left(V(\varphi) - U(\varphi)\, R(g_{\mu\nu}) + \frac{1}{2} G(\varphi)\, (\nabla\varphi)^2 + \dots\right)$$

$$V_{\text{loop}}(\varphi) \sim \varphi^4 \ln\frac{\varphi^2}{\mu^2}, \qquad U_{\text{loop}}(\varphi) \sim \varphi^2 \ln\frac{\varphi^2}{\mu^2},$$

$$G_{\text{loop}}(\varphi) \sim \ln\frac{\varphi^2}{\mu^2}$$

Transition to the Einstein frame:

$$V(\varphi) \to V_{EF}(\phi) = \frac{M_P^4}{4}\frac{V(\varphi)}{U^2(\varphi)} \sim \frac{\ln\frac{\varphi}{\mu}}{\ln^2\frac{\varphi}{\mu}} \sim \frac{1}{\ln\frac{\varphi}{\mu}} \to 0, \qquad \varphi \to \infty$$

Any *l*-th loop order:

$$\frac{\ln^l\frac{\varphi}{\mu}}{\ln^{2l}\frac{\varphi}{\mu}} \sim \frac{1}{\ln^l\frac{\varphi}{\mu}} \to 0, \qquad \varphi \to \infty$$

Resummation by RG confirms this.

Justification of semiclassical expansion and hierarchy problem

Starobinsky R^2-model and non-minimal Higgs inflation model at $V(\phi)$ » ¤$_{max}$

$$10^{-11} M_P^4 \simeq V_{\text{inflation}} \sim \Lambda_{max} = \frac{12\pi^2}{\beta} M_P^2 \quad \Rightarrow \quad \beta \simeq 10^{13}$$

Impossible in Standard model with low spins $s=0,1/2,1$ and N_s » 100

$$\beta = \frac{1}{180}\left(\mathbb{N}_0 + 11\mathbb{N}_{1/2} + 62\mathbb{N}_1\right)$$

Hidden sector of conformal higher spin (CHS) fields

$$S_{CHS}^{(s)} = \int d^4x \left(h^{\mu_1\dots\mu_s}\Box^s h_{\mu_1\dots\mu_s} + \dots\right), \qquad \beta_s \sim s^6$$

Giombi, Klebanov, Pufu, Safdi, and Tarnopolsky 2013; Tseytlin 2013 arXiv:1309.0785

$$\beta = \sum_{s=1}^{S} \beta_s \simeq S^7$$

$$\mathbb{N} = \sum_{s=1}^{S} N_s \sim S^3 \text{ – total number of polarizations (species)}$$

1/N-expansion and effective field theory below the gravitational cutoff $\Lambda_{grav} = \frac{M_P}{\sqrt{\mathbb{N}}}$

$$\Lambda_{max} \sim \frac{M_P}{\sqrt{\beta}} \sim \frac{M_P}{S^3} \ll \Lambda_{grav} = \frac{M_P}{\sqrt{\mathbb{N}}} \sim \frac{M_P}{S^{3/2}}$$

Thermal corrections to primordial power spectrum

$$n_s(k) = n_s^{\text{vac}}(k) + \Delta n_s^{\text{thermal}}(k)$$

additional red tilt of the CMB spectrum

This number of hidden sector fields gives a red tilted thermal correction to CMB spectral index in the third (potentially observable) decimal order:

$$\Delta n_s^{\text{thermal}} \sim -0.001$$

A.B, arXiv:1308.4451
JCAP 1310 (2013) 059

Microcanonical state of CFT driven cosmology scenario works only for closed Universe with k=+1

99% C.L. evidence for positive spatial curvature (*k=+1*) of the closed Universe with Ω_k' -0.04 --- Hubble tension discordances

E. Di Valentino, A. Melchiorri and J. Silk, Nature Astron. 4, 196 (2019);
W. Handley, Phys. Rev. D 103 (2021) L041301, arXiv:1908.09139

Conclusions

Cosmological initial conditions: microcanonical density matrix of the Universe

CFT driven cosmology: suppression of no-boundary instantons; quasi-thermal stage preceding inflation and UV bounded range of its energy scale

New type of hill-top inflation, ¤$\to V(\phi)$ – selection of inflaton potential $V(\phi)$ maxima

Mechanism of hill-top potential: origin of non-minimal Higgs inflation and R^2 gravity

Conformal higher spin fields (CHS): solution of hierarchy problem -- origin of the Universe is the subplanckian phenomenon; justification of semiclassical expansion and *1/N*-expansion

Thermally corrected CMB spectrum: observable signature of the primordial thermal epoch