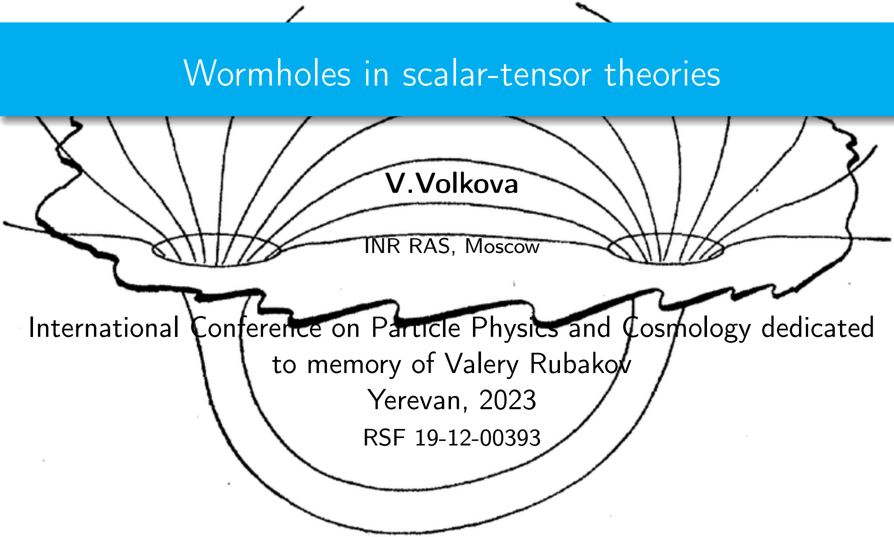
Wormholes in scalar-tensor theories



V.Volkova

INR RAS, Moscow

International Conference on Particle Physics and Cosmology dedicated to memory of Valery Rubakov
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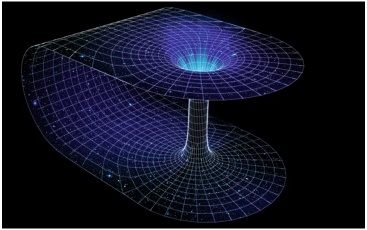
Traversable wormholes and their stability

Einstein, Rosen (1935), Wheeler (1962), Ellis (1973), Bronnikov (1973), Morris, Thorne (1988)



- The non-trivial feature of traversable wormholes: the necessity to fill the throat with matter, which violates the NEC/NCC
- Different options for supporting the throat: quantum effects (for microscopic wormholes), phantom scalar field, modified gravity
- One of the approaches to modifying gravity is to add extra DOFs, e.g. coupling GR to a scalar field → Scalar-tensor theories

Generalized Galileon a.k.a. Horndeski theory and beyond

Horndeski (1974)
Deffayet, Gao, Steer, Zahariade (2011)

Zumalacárregui, García-Bellido (2014)
Gleyzes, Langlois, Piazza, Vernizzi (2015)

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}.$$

π is a scalar field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{iX} = \partial G_i/\partial X$.

- Healthy NEC/NCC violation
- Stability issue: pathological DOFs may show up on the level of perturbations

Lorentzian wormholes in $\mathcal{L}_3$ and stability issues

- There is tension between stability conditions and NEC-violation requirement for wormholes in cubic Galileon theory:

$$\mathcal{L}_3 = -\frac{1}{2\kappa}R + F(\pi, X) + K(\pi, X)\Box\pi$$

Rubakov, 2015 (1509.08808)

- Wormholes in $\mathcal{L}_3$ are always plagued with ghost (no-go theorem in $\mathcal{L}_3$)

Rubakov, 2016 (1601.06566)

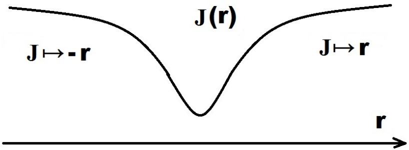
- No-go theorem is still valid for $\mathcal{L}_3$ + conventional scalar

Kolevatov, Mironov, 2016 (1607.04099)

- No-go theorem for wormholes in Horndeski theories: *static, spherically symmetric wormholes suffer from ghost instabilities in some region of space around them*

Evseev, Melichev, 2018 (1711.04152)

Wormhole: background setup



- Static, spherically-symmetric wormhole:

$$ds^2 = -A(r)\,dt^2 + \frac{dr^2}{B(r)} + J^2(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

where

$$A(r) \geq A_{min} > 0, \quad B(r) \geq B_{min} > 0, \quad J(r) \geq R_{min} > 0$$

- Asymptotically flat geometry
- Background Galileon field $\pi(r)$ – static and spherically-symmetric

Stable solutions are free from any kind of pathological DOFs among linear perturbations, i.e. ghosts, gradient instabilities, tachyons

Perturbations about a wormhole

- 2+1 DOFs: 1 odd-parity and 2 even-parity modes (w.r.t. 2D reflection)
- Odd-parity sector (Q):

$$S^{(2)}_{odd} = \int dt\, dr \sqrt{\frac{B}{A}} J^2 \cdot \left[\frac{\mathcal{H}^2}{A\mathcal{G}} \dot{Q}^2 - \frac{B\mathcal{H}^2}{\mathcal{F}} (Q')^2 - \frac{\ell(\ell+1)}{J^2} \cdot \mathcal{H} Q^2 - V(r) Q^2 \right]$$

- Even-parity sector (v_i, $i = 1, 2$):

$$S^{(2)}_{even} = \int dt\, dr \sqrt{\frac{A}{B}} J^2 \left(\frac{1}{2} \mathcal{K}_{ij} \dot{v}^i \dot{v}^j - \frac{1}{2} \mathcal{G}_{ij} v^{i\prime} v^{j\prime} - \mathcal{Q}_{ij} v^i v^{j\prime} - \frac{1}{2} \ell^2 \mathcal{M}_{(\ell^2)ij} v^i v^j - \frac{1}{2} \mathcal{M}_{ij} v^i v^j \right)$$

Stability conditions

No ghosts: $\mathcal{G} > 0, \quad \mathcal{K}_{11} > 0, \quad \det(\mathcal{K}) > 0,$

No radial gradient instabilities: $\mathcal{F} > 0, \quad \mathcal{G}_{11} > 0, \quad \det(\mathcal{G}) > 0,$

No angular gradient instabilities: $\mathcal{H} > 0, \quad \mathcal{M}_{(\ell^2)11} > 0, \quad \det(\mathcal{M}_{(\ell^2)}) > 0.$

No-go theorem and its circumvention

$$\begin{aligned} \text{No ghosts:} \quad & \mathcal{G} > 0, \quad \mathcal{K}_{11} > 0, \quad \det(\mathcal{K}) > 0, \\ \text{No radial gradient instabilities:} \quad & \mathcal{F} > 0, \quad \mathcal{G}_{11} > 0, \quad \det(\mathcal{G}) > 0, \\ \text{No angular gradient instabilities:} \quad & \mathcal{H} > 0, \quad \mathcal{M}_{(\ell^2)11} > 0, \quad \det(\mathcal{M}_{(\ell^2)}) > 0. \end{aligned}$$

The no-go theorem in Horndeski theory is based on the no-ghost constraint for even-parity sector:

$$\det \mathcal{K} \sim \mathcal{F}(2\frac{d\xi}{dr} - \mathcal{F}) > 0$$

$$\Rightarrow$$

$$\xi = \frac{(J\mathcal{H})^2}{\Theta}$$

- Key requirement: ξ has to cross zero

Franciolini, Hui, Santoni, Trincherini, 2019

Mironov, Rubakov, VV, 2019

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The no-go theorem in Horndeski theory is based on the no-ghost constraint for even-parity sector:

$$\det \mathcal{K} \sim \mathcal{F}(2\frac{d\xi}{dr} - \mathcal{F}) > 0 \qquad \Rightarrow \qquad \det \mathcal{K} \sim \mathcal{F}(2\frac{d\tilde{\xi}}{dr} - \mathcal{F}) > 0$$

$$\xi = \frac{(J\mathcal{H})^2}{\Theta} \qquad \qquad \tilde{\xi} = \frac{J^2\mathcal{H}(\mathcal{H} - \mathcal{D})}{\Theta}$$

- Key requirement: ξ has to cross zero
- One evades the no-go by going beyond Horndeski thanks to a new contribution $\mathcal{D}(F_4, F_5)$

Franciolini, Hui, Santoni, Trincherini, 2019

Mironov, Rubakov, VV, 2019

Stable wormhole: reverse engineering

- Choose a specific wormhole metric:

$$ds^2 = -A(r)\,dt^2 + \frac{dr^2}{B(r)} + J^2(r)\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)$$
$$A = 1, \quad B = 1 + 2\,\mathrm{sech}\left(\frac{r}{r_{min}}\right), \quad J = \ln\left[1 + 2\cosh\left(\frac{r}{r_{min}}\right)\right].$$

and the Galileon field profile:

$$\pi_0(r) = \tanh\left(\frac{r}{r_{min}}\right) - 1, \quad \rightarrow \quad X = -\frac{\mathrm{sech}\left(\frac{r}{r_{min}}\right)^4}{2\left(1 + \mathrm{sech}\left(\frac{r}{r_{min}}\right)\right)}$$

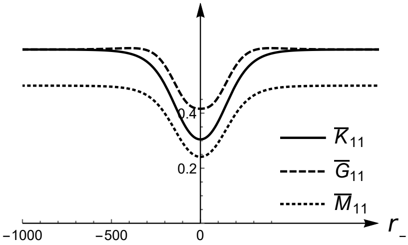
- Take an Ansatz for Lagrangian functions as a power series of X:

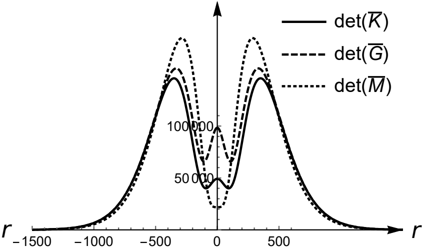
$$F(\pi, X) = f_0(\pi) + f_1(\pi)\cdot X + f_2(\pi)\cdot X^2,$$
$$G_4(\pi, X) = \frac{1}{2} + g_{40}(\pi) + g_{41}(\pi)\cdot X,$$
$$F_4(\pi, X) = f_{40}(\pi) + f_{41}(\pi)\cdot X$$

- Reconstruct the functions f_i, g_{4i} and f_{4i} by satisfying:
(a) background equations of motion (b) stability conditions

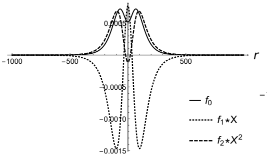
Semi-stable wormhole: an example

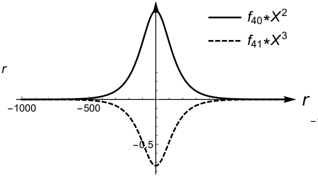
- Stability: no ghost, radial and angular gradient instabilities

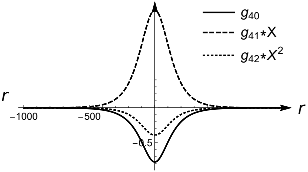




- Reconstructed Lagrangian functions:







There exists a wormhole solution that is stable w.r.t. high energy perturbations

Semi-stable wormhole: an example

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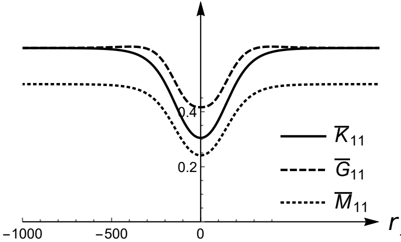

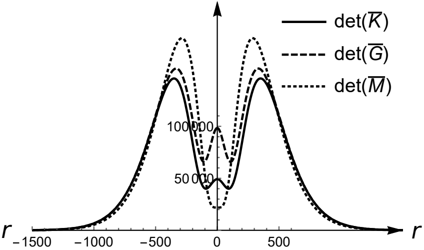

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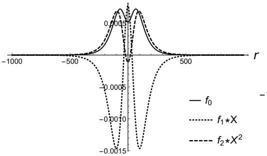

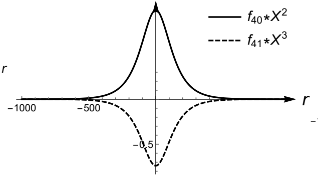

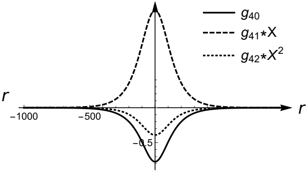

There exists a wormhole solution that is stable w.r.t. high energy perturbations $\rightarrow$ what about low energy modes?

Complete stability: tachyons

$$S^{(2)}_{even} = \int \mathrm{d}t\, \mathrm{d}r \sqrt{\frac{A}{B}} J^2 \left(\frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \frac{1}{2}\mathcal{G}_{ij}v^{i\prime}v^{j\prime} - \mathcal{Q}_{ij}v^i v^{j\prime} - \frac{1}{2}\ell^2\mathcal{M}_{(\ell^2)ij}v^i v^j - \frac{1}{2}\mathcal{M}_{ij}v^i v^j \right)$$

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$$\mathcal{M}_{11} = \frac{\sqrt{AB}}{4J^2}\frac{\mathcal{N}}{\mathcal{H}^2} - \frac{\sqrt{A}}{4\sqrt{B}J^2\,(\ell^2+\ell-2)}\frac{\Theta(\Theta+\mathcal{W})}{\mathcal{F}} + \frac{1}{4\ell(\ell+1)}\left((\ell^2+\ell-2)\frac{\sqrt{B}}{\sqrt{A}}\mathcal{H}\mathcal{T}^2 - \left[\sqrt{B}\frac{\Theta\mathcal{H}\mathcal{T}}{\mathcal{F}}\right]'\right),$$

$$\mathcal{M}_{22} = -\frac{\ell(\ell+1)}{(\ell^2+\ell-2)}\left(\frac{\sqrt{AB}}{J^2}\mathcal{H}\left[\ell(\ell+1)(\ell^2+\ell-2) - (2+J^2\mathcal{D}'-\ell(\ell+1))^2\right] + \right.$$
$$\left[\frac{\sqrt{AB}}{J}\frac{\mathcal{H}}{\Theta}\left(2(\ell^2+\ell-2)B\left[\ell(\ell+1)J^2\left(\mathcal{H}\left(\frac{A'}{A}+2\frac{J'}{J}\right)+\Gamma\pi'\right) - (2\mathcal{H}JJ'+\Xi\pi')(2+J^2\mathcal{D}')\right] + \right.\right.$$
$$\left.\left.\left. J\mathcal{D}\left[\ell(\ell+1)\mathcal{W} + \Theta(2+J^2\mathcal{D}')\right]\right)\right]'\right),$$

$$\mathcal{M}_{12} = \frac{1}{2(\ell^2+\ell-2)}\left(\frac{\Theta}{\mathcal{F}J}\left[\sqrt{AB}\,\mathcal{H}(2+J^2\mathcal{D}')\right]' + \frac{\sqrt{A}}{2\sqrt{B}J^2}\left(\ell(\ell+1)J\left[\mathcal{D}\mathcal{W} - B\frac{\Theta\mathcal{H}}{\mathcal{F}}\left(\frac{A'}{A}-2\frac{J'}{J}\right)\right]\right.\right.$$
$$\left.\left. +2(\ell^2+\ell-2)B\left[\ell(\ell+1)J^2\left(\mathcal{H}\left(\frac{A'}{A}+2\frac{J'}{J}\right)+\Gamma\pi'\right) - (2\mathcal{H}JJ'+\Xi\pi')(2+J^2\mathcal{D}')\right]\right)\right).$$

Superluminality problem

$$S_{odd}^{(2)} = \int dt\, dr\, \sqrt{\frac{B}{A}} J^2 \cdot \left[\frac{\mathcal{H}^2}{A\mathcal{G}} \dot{Q}^2 - \frac{B\mathcal{H}^2}{\mathcal{F}} (Q')^2 - \frac{\ell(\ell+1)}{J^2} \cdot \mathcal{H} Q^2 - V(r) Q^2 \right]$$

$$S_{even}^{(2)} = \int dt\, dr \sqrt{\frac{A}{B}} J^2 \left(\frac{1}{2} \mathcal{K}_{ij} \dot{v}^i \dot{v}^j - \frac{1}{2} \mathcal{G}_{ij} v^{i\prime} v^{j\prime} - \mathcal{Q}_{ij} v^i v^{j\prime} - \frac{1}{2} \ell^2 \mathcal{M}_{(\ell^2)ij} v^i v^j - \frac{1}{2} \mathcal{M}_{ij} v^i v^j \right)$$

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No tachyons: $\mathcal{M}_{\,22} > 0,\ \det \mathcal{M} > 0$

Odd-parity modes: $c_r^2 = \frac{\mathcal{G}}{\mathcal{F}}, \quad c_\theta^2 = \frac{\mathcal{G}}{\mathcal{H}},$

Even-parity modes (radial): $c_{r\,1,2}^2$ are eigenvalues of $\mathcal{K}^{-1}\mathcal{G}$,

Even-parity modes (angular): $c_{\theta,\,1,2}^2$ are eigenvalues of $\mathcal{K}^{-1}\mathcal{M}_{(\ell^2)}$

Superluminality problem

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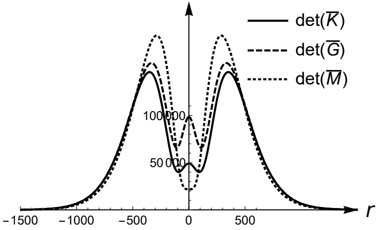
\text{Odd-parity modes:} \quad & c_r^2 = \frac{\mathcal{G}}{\mathcal{F}}, \quad c_\theta^2 = \frac{\mathcal{G}}{\mathcal{H}},\\
\text{Even-parity modes (radial):} \quad & c^2_{r\ 1,2} \text{ are eigenvalues of } \mathcal{K}^{-1}\mathcal{G},\\
\text{Even-parity modes (angular):} \quad & c^2_{\theta,\ 1,2} \text{ are eigenvalues of } \mathcal{K}^{-1}\mathcal{M}_{(\ell^2)}
\end{aligned}$$

$$c_{a1}^2 \cdot c_{a2}^2 = \frac{J^4}{A^2}\frac{\det\mathcal{M}_{(\ell^2)}}{\det\mathcal{K}}$$

One of the sound speeds $c_a^2 > 1$.



Conclusion and outlook

- There are no completely stable static, spherically symmetric wormholes in Horndeski theory
- It is possible in principle to construct a wormhole free from ghosts and gradient instabilities in beyond Horndeski theory (also in DHOST)
- Stability conditions for any static, spherically symmetric background were derived for beyond Horndeski class – useful tool for stability analysis
- A *completely* stable wormhole in beyond Horndeski theory is still to be constructed

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Thank you for your attention!

Types of instabilities

- Consider a spherically-symmetric scalar field $\pi = \pi_0(r) + \chi(t, r)$ in Minkowski space:

$$\delta^2 S = \int d^4x \left[\frac{1}{2}U(r)\dot{\chi}^2 - \frac{1}{2}V(r)(\partial_i\chi)^2 - \frac{1}{2}W(r)\chi^2\right].$$

- Dispersion relation and energy density for χ:

$$U\omega^2 = V\mathrm{p}^2 + W\ ,$$

$$T_{00} = \frac{1}{2}U\dot{\chi}^2 + \frac{1}{2}V(\partial_i\chi)^2 + \frac{1}{2}W\chi^2$$

- **Stable background:** $U > 0\ ,\quad V > 0\ ,\quad W \geq 0$
- Tachyonic instability (imaginary ω at low p): $U > 0\ ,\quad V > 0\ ,\quad W < 0$
- Gradient instability (imaginary ω at high p): $U > 0\ , V < 0$ or $U < 0\ , V > 0$
- Ghost instability: $U < 0\ , V < 0$ (quantum-mechanically unstable background)