Global stability in Horndeski theory and beyond

Dedicated to the memory of Valery Rubakov and based on his work

S. Mironov

INR RAS & ITMP MSU

Yerevan, YSU, October 7, 2023

based on the papers by
V. Rubakov and his group
Libanov,Osipov,Ramazanov,
SM,Kolevatov,Volkova,Sukhov,
Melichev,Evseev,Ageeva,Petrov
Shtennikova,Vallencia-Villegas

Plan:

- Conformal cosmological models
- NEC and Galileons models that violate it
- No-go Theorem
- Ways to circumvent it

Conformal cosmological model

V. Rubakov, 0906.3693

Harrison-Zeldovich spectrum from conformal invariance

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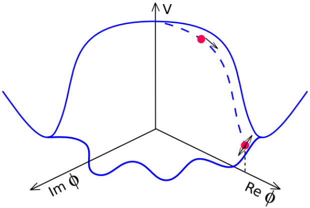
Harrison-Zeldovich spectrum from conformal invariance

$S_\chi = \int d^3x d\eta (\partial_\mu \chi^* \partial^\mu \chi + h^2 |\chi|^4)$



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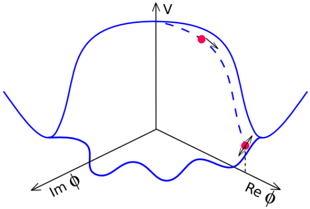

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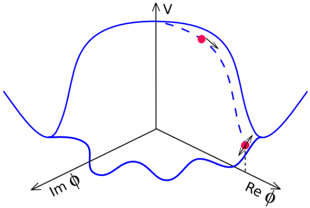

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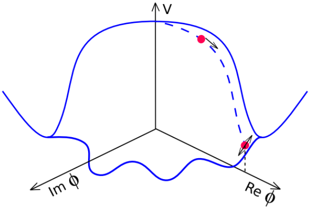

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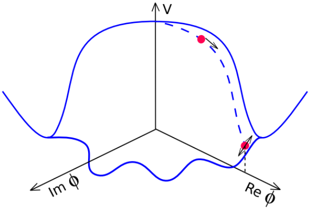

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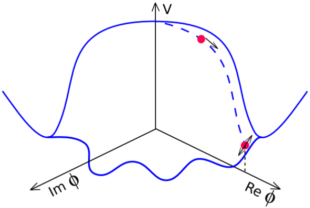

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$$\Downarrow$$

Flat spectrum

Further development of Conformal models..

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Statistical anisotropy of the spectrum:

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1012.5737, 1105.6230 (M.Libanov, SM, V.Rubakov)

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1107.1036 (M.Libanov, V.Rubakov), 1211.0262 (SM),

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1409.4363 (M.Libanov, V.Rubakov, S.Sibiryakov),
1502.05897 (M.Libanov, V.Rubakov),
1508.07728 (M.Libanov, V.Rubakov, G.Rubtsov)

Null Energy Condition $\iff$ Null Convergence Condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$ $\qquad$ $R_{\mu\nu} k^\mu k^\nu \geq 0$

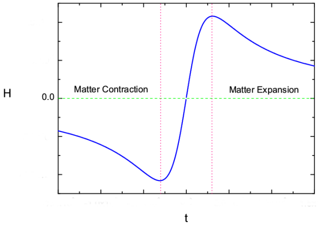
Friedmann equations

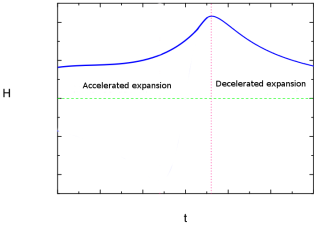
$\dot{H} = -4\pi G(p+\rho) \quad \leq 0$

NEC-violation

Penrose theorem

Absence of singularity requires





Bounce and genesis require NEC-violation

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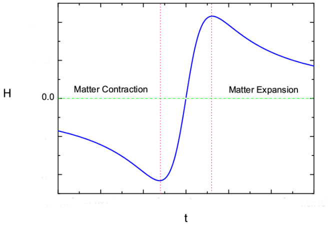

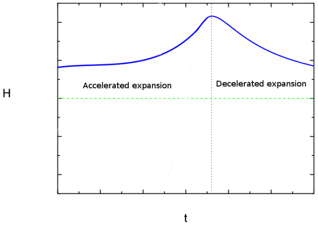

Bounce and genesis require NEC-violation

As well as wormhole-like solutions

Wormhole is similar to the bounce

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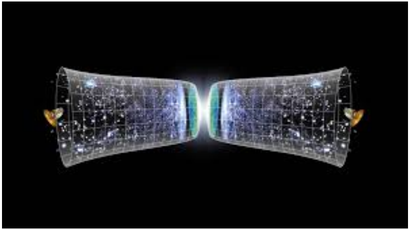
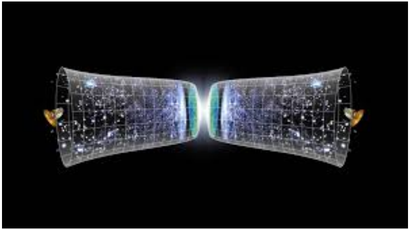
Radial axis $\longrightarrow$

Time axis $\longrightarrow$





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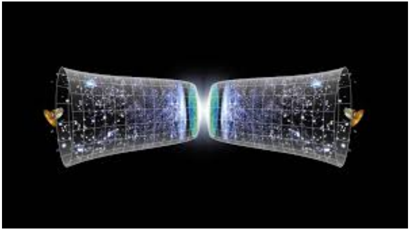

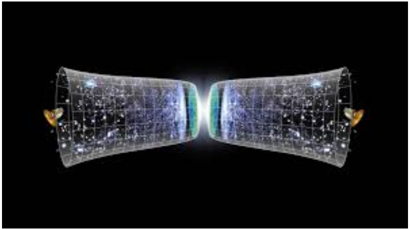

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2 \tag{1}$$

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - R(r)^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) . \tag{2}$$

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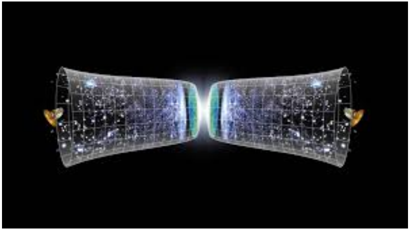

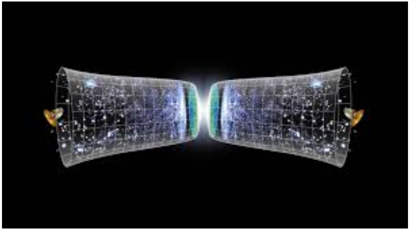

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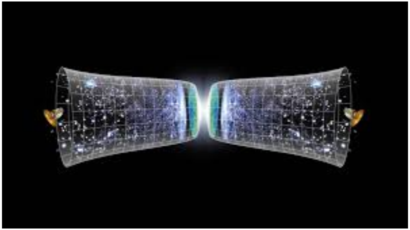

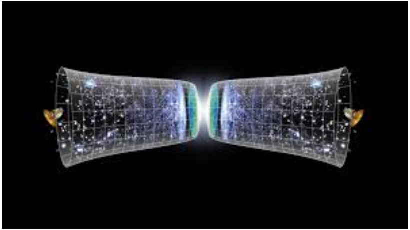

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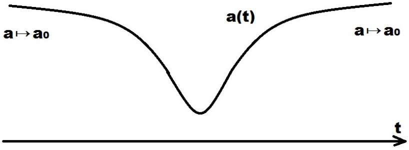

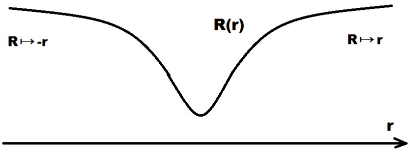

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Hence we need to consider Lagrangians with second derivatives:

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Hence we need to consider Lagrangians with second derivatives:

- Deal with higher derivative equations

NEC-violation

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Hence we need to consider Lagrangians with second derivatives:

- ~~Deal with higher derivative equations~~
- Get 2 derivatives equations only

$$\mathcal{L} = F(\pi, X) + K(\pi, X)\Box\pi$$

here $X = \partial_\mu\pi\partial^\mu\pi$

$$\delta\mathcal{L} = F_\pi \delta\pi + F_X \delta X + K_\pi \Box\pi \delta\pi + \underline{K_X \Box\pi \delta X + K \Box \delta\pi} =$$

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$= ...$only second derivatives

Generalized Galileons = Horndeski theory

$$
\begin{aligned}
S &= \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5\right),\\
\mathcal{L}_2 &= F(\pi,X),\\
\mathcal{L}_3 &= K(\pi,X)\Box\pi,\\
\mathcal{L}_4 &= -G_4(\pi,X)R+2G_{4X}(\pi,X)\left[(\Box\pi)^2-\pi_{;\mu\nu}\pi^{;\mu\nu}\right],\\
\mathcal{L}_5 &= G_5(\pi,X)G^{\mu\nu}\pi_{;\mu\nu}+\frac{1}{3}G_{5X}\left[(\Box\pi)^3-3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu}+2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right]
\end{aligned}
$$

where π is the Galileon field, $X=g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu}=\partial_\mu\pi$, $\pi_{;\mu\nu}=\nabla_\nu\nabla_\mu\pi$, $\Box\pi=g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X}=\partial G_4/\partial X$

Consistent NEC violation?

Galileon bounce: 1303.1221 (M.Osipov, V.Rubakov)

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The Null Energy Condition and its violation 1401.4024 (V.Rubakov)

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Further generalizations of no-go: 1607.04099 (R.Kolevatov, SM), 1607.01721 (O.Evseev, O.Melichev), 1711.04152 (O.Evseev, O.Melichev)

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Big[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_i h^T_{kl}\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G}_\mathcal{T} \geq \mathcal{F}_\mathcal{T} > 0, \quad \mathcal{G}_\mathcal{S} \geq \mathcal{F}_\mathcal{S} > 0$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned}
\mathcal{G}_\mathcal{S} &= \frac{\Sigma{\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\
\mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\
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\end{aligned} \qquad (4)$$

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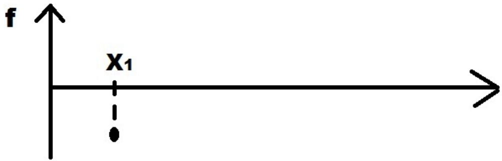

If $f(x_1) < 0$

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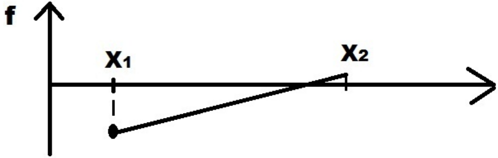

If $f(x_1) < 0$ Then $f(x_2) \geq 0$ $(x_2 = x_1 - \frac{f(x_1)}{\epsilon})$

Suppose we have "nice" function $f(x)$ defined for all x from $-\infty$ to ∞.

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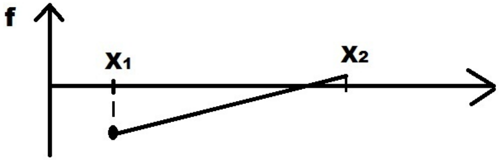

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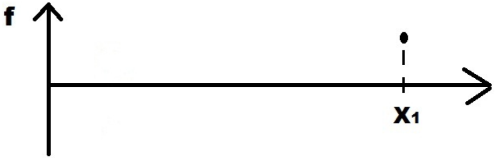

If $f(x_1) > 0$

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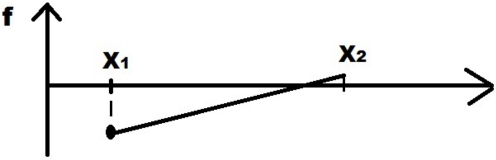

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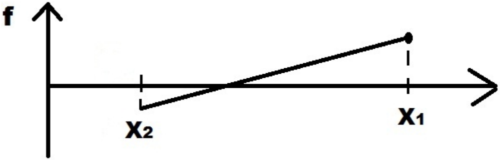

If $f(x_1) > 0$ Then $f(x_2) \leq 0$ $(x_2 = x_1 - \frac{f(x_1)}{\epsilon})$

$$S = \int \mathrm{d}t\mathrm{d}^3xa^3\left[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_i h^T_{kl}\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

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How to circumvent the no-go?

Go beyond Horndeski

How to circumvent the no-go?

Go beyond Horndeski

Allow strong gravity in the past

How to circumvent the no-go?

Go beyond Horndeski

Allow strong gravity in the past

$\Theta = 0$, adding torsion,..

Going beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}})\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

$$S = \int \mathrm{d}t\mathrm{d}^3xa^3\Big[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_i h^T_{kl}\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

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Going beyond Horndeski

Cosmological solutions:

Going beyond Horndeski

Cosmological solutions:

Bounce and genesis: 1705.06626 (R.Kolevatov, SM, N.Sukhov, V.Volkova)

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Assorted properties of (beyond) Horndeski theories:

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1906.12139, 2005.12626, 2011.14912 (SM, V.Rubakov, V.Volkova)

1712.09909, 2204.05889, 2306.17791 (SM, V.Volkova)

2305.19171 (SM, A.Shtennikova)

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2305.19171 (SM, A.Shtennikova)

Wormhole solutions:

1811.05832, 1812.07022 (SM, V.Rubakov, V.Volkova)

2212.05969 (SM, V.Rubakov, V.Volkova)

Strong gravity in the past

$$S = \int \mathrm{d}t\mathrm{d}^3 x a^3 \Big[\frac{\mathcal{G_T}}{8} \left(\dot{h}_{ik}^T\right)^2 - \frac{\mathcal{F_T}}{8a^2} \left(\partial_i h_{kl}^T\right)^2 + \mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2} \Big]$$

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Strong gravity in the past

1810.00465, 2003.01202 (Y.Ageeva, O.Evseev, O.Melichev, V.Rubakov)

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2009.05071, 2104.13412, 2207.04071 (Y.Ageeva, P.Petrov, V.Rubakov)

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2206.10646 (Y.Ageeva, P.Petrov)

2310.xxxxx (Y.Ageeva, P.Petrov)

$\Theta \equiv 0$ loophole in Horndeski:

2212.03285 (SM, A.Shtennikova)

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2212.03285 (SM, A.Shtennikova)

Hondeski-Cartan and stable solutions there:

2304.04722, 2307.06929 (SM, M.Valencia-Villegas)




THANK YOU FOR YOUR ATTENTION!




$$
\begin{aligned}
&\mathcal{G}_{\mathcal{T}} = 2G_4 - 4G_{4X}X + G_{5\pi}X - 2HG_{5X}X\dot{\pi},\\
&\mathcal{F}_{\mathcal{T}} = 2G_4 - 2G_{5X}X\ddot{\pi} - G_{5\pi}X,\\
&\mathcal{D} = 2F_4X\dot{\pi} + 6HF_5X^2,\\
&\hat{\mathcal{G}}_{\mathcal{T}} = \mathcal{G}_{\mathcal{T}} + \mathcal{D}\dot{\pi},\\
&\Theta = -K_XX\dot{\pi} + 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}-\\
&\ - 5H^2G_{5X}X\dot{\pi} - 2H^2G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2+\\
&\ + 10HF_4X^2 + 4HF_{4X}X^3 + 21H^2F_5X^2\dot{\pi} + 6H^2F_{5X}X^3\dot{\pi},\\
&\Sigma = F_XX + 2F_{XX}X^2 + 12HK_XX\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2-\\
&\ - 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3-\\
&\ - 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 30H^3G_{5X}X\dot{\pi}+\\
&\ + 26H^3G_{5XX}X^2\dot{\pi} + 4H^3G_{5XXX}X^3\dot{\pi} - 18H^2G_{5\pi}X - 27H^2G_{5\pi X}X^2-\\
&\ - 6H^2G_{5\pi XX}X^3 - 90H^2F_4X^2 - 78H^2F_{4X}X^3 - 12H^2F_{4XX}X^4-\\
&\ - 168H^3F_5X^2\dot{\pi} - 102H^3F_{5X}X^3\dot{\pi} - 12H^3F_{5XX}X^4\dot{\pi}.
\end{aligned}
$$