Effectively flat potential in the Friedberg-Lee-Sirlin model.

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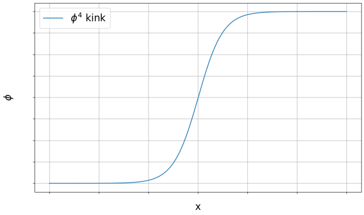
based on e-Print: 2309.09661

Outline

Single field models

Static solutions

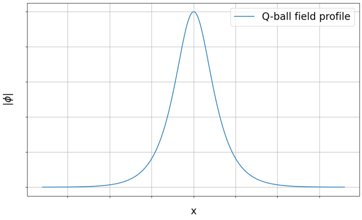
- ϕ^4-kink in $(1+1)$ dimensions;



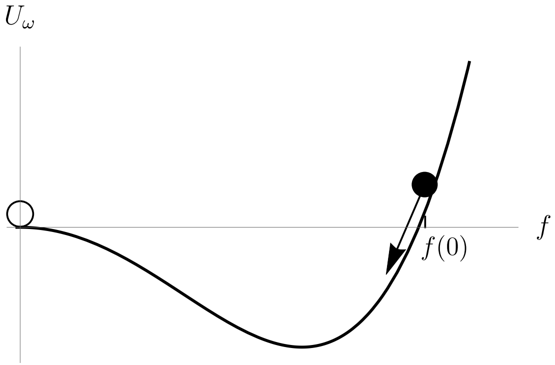
Kink profile.
$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{2}(\phi^2 - v^2)^2$

Stationary solutions

- Q-balls in any dimensions;



Q-ball profile in one spatial dimension. $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$



Rosen(1968), Coleman's condidions (1985). Overshoot and undershoot. Unrenormalizable potential. UV compleetion is needed?

True theory (FLS)

Friedberg-Lee-Sirlin (FLS) model (1976) provides non-topological solitons in $(3+1)$ dimensions. Renormalizable theory in $(3+1)$: quartic couplings. Very good for semiclassical consideration.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V(\phi^*\phi, \chi)$$

$$V(\phi^*\phi, \chi) = h^2 |\phi|^2 \chi^2 + \frac{m^2}{2}(\chi^2 - v^2)^2$$

Symmetries:
global $U(1)$ for complex field ϕ, $\phi \to \mathrm{e}^{i\alpha}\phi$,
discrete $\mathcal{Z}_2$ for real χ, $\chi \to -\chi$.

Single field from FLS

Ansatz $\phi(t,\vec{x}) = e^{-i\omega t} f(\vec{x})$, $\chi(t,\vec{x}) = \chi(\vec{x})$, Eq. of motion for FLS

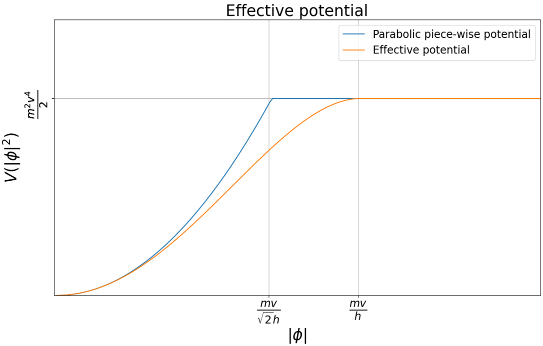
$$\begin{aligned} \nabla^2 f &= h^2\chi^2 f - \omega^2 f, \\ \nabla^2 \chi &= 2h^2 f^2 \chi + 2m^2(\chi^2 - v^2)\chi \end{aligned} \tag{1}$$

Heavy χ: the mass $m_\chi = mv$ is large.
Omit gradient terms!

$$\begin{cases} \chi = 0, \\ \chi^2 = v^2 - \frac{h^2}{m^2}|\phi|^2 \end{cases} \tag{2}$$

Two branches for solution. The matching is needed!

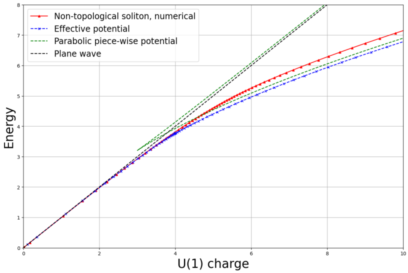
$$V_{eff} = \left(m_\phi^2|\phi|^2 - \frac{h^4}{2m^2}|\phi|^4\right)\theta\left(\frac{mv}{h} - |\phi|\right) + \frac{m^2v^4}{2}\theta\left(|\phi| - \frac{mv}{h}\right) \quad (3)$$



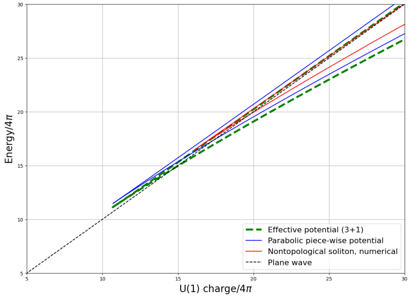
Effective potential. Just compare with parabolic piece-wise potential of the Rosen (1968) model, which is integrable in any dimensions.

Comparison of integral characteristics

In $(1+1)$ dimensions

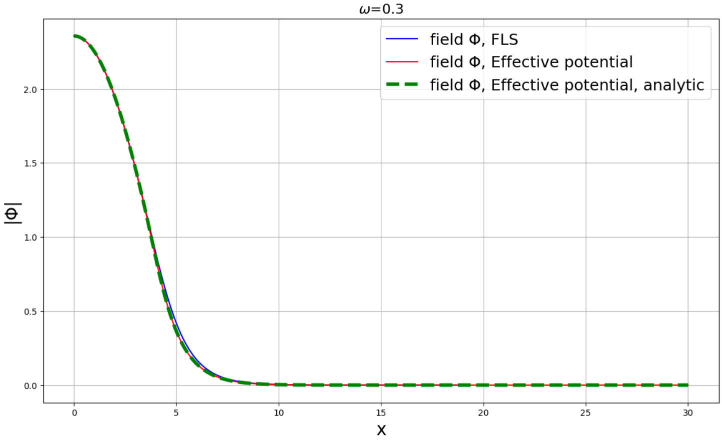


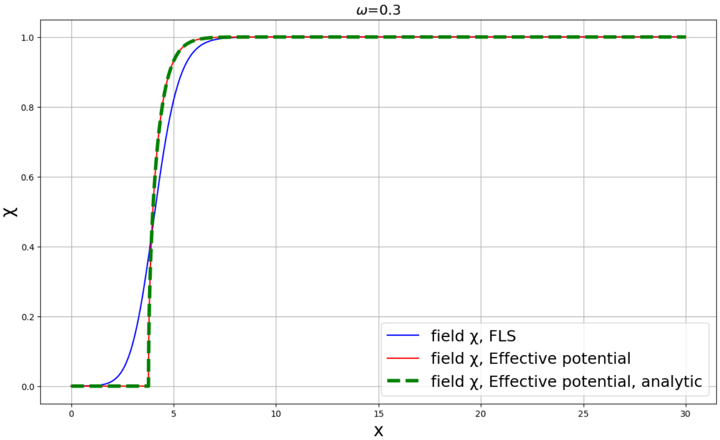
In $(3+1)$ dimensions



Energy as function of global charge for FLS non-topological solitons, for Q-balls in effective and parabolic piece-wise potential.

Comparison (numerical cartoons) for profiles




ω=0.3
1.0
0.8
0.6
0.4
0.2
0.0
χ
0
5
10
15
20
25
30
x
field χ, FLS
field χ, Effective potential
field χ, Effective potential, analytic

Q-ball on domain wall

One can use full EOM for the field χ (with gradient terms) and put in Lagrangian $\partial_\mu\chi\partial^\mu\chi \to -\chi\partial_\mu\partial^\mu\chi$, $\partial_\mu\partial^\mu\chi \to -\frac{\partial V(|\phi|^2,\chi)}{\partial\chi}$

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi + \frac{m^2}{2}\left(\chi^4(|\phi|^2) - v^4\right) \tag{4}$$

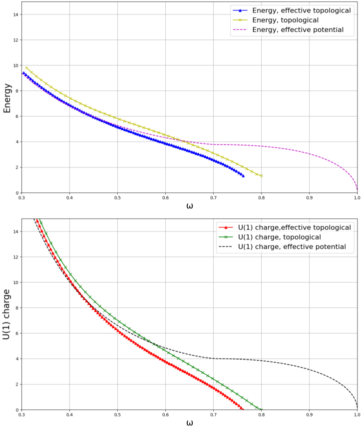
The same profile for the effective potential (using constant solutions). But one can use kink solution for χ, assuming constant $|\phi|$, $|\phi| \leq \frac{mv}{h}$:

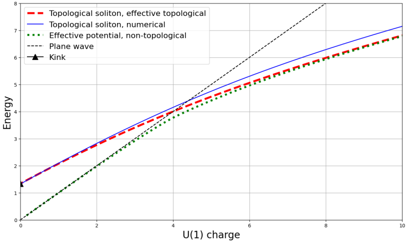
$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi + \frac{m^2}{2}\left((v^2 - \frac{h^2}{m^2}|\phi|^2)^2 \tanh^4\left(mx\sqrt{v^2 - \frac{h^2}{m^2}|\phi|^2}\right) - v^4\right)$$

Matching at $|\phi| = \frac{mv}{h}$

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - \frac{m^2v^4}{2}$$

for large values of the complex field





Improved method works better than effective potential! Result can be checked perturbatively.

Results and outlook

- FLS model is very good lab for EFT methods (both classical and quantum);
- Effective potential is quite effective for inhomogeneous configurations;
- Vacuum rearrangement on the domain wall can be studied by easy improvement in non-perturbative regime.

THANK YOU

This work was supported by the grant RSF-22-12-00215