

# Self-similar growth of Bose stars

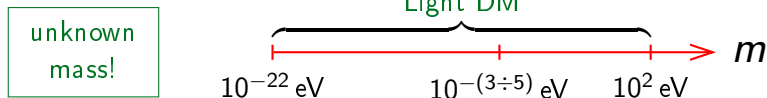
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INR RAS

International Conference  
on  
**Particle Physics and Cosmology**  
October 02-07, 2023  
Yerevan, Armenia


AD, D. Levkov, A. Panin, I. Tkachev, [arXiv:2305.01005](https://arxiv.org/abs/2305.01005)



## String axions (Fuzzy DM)

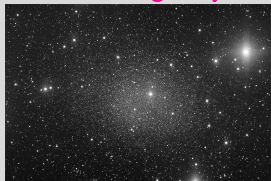
- Predicted by string theory
  - Any mass  $m$ 
    - Arvanitaki et al '10*
  - Fuzzy DM:  $m \sim 10^{-22}$  eV
    - e.g. Schive et al '04*
- ⇒ Quantum at galaxy scales!
- $\lambda_4 \sim 10^{-100}$  — only gravity!

## QCD axions

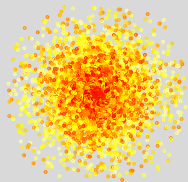
- Solve strong CP problem
  - Peccei, Quinn '77*
- Dark Matter:
  - $m \sim 10^{-5} - 10^{-3}$  eV
  - Klaer, Moore '17, Gorghetto et al '20*
- Form miniclusters:
  -   $M \sim 10^{-17} - 10^{-12} M_{\odot}$
  - Kolb, Tkachev '93; Pierobon et al '23*
- $\lambda_4 \sim 10^{-50}$  — only gravity!

Any small structure

a dwarf galaxy



or axion minicluster



$\rho, \mathbf{v}$  — known!  
density, velocity

- Large phase-space density!

$$m \ll 10^2 \text{ eV} \Rightarrow f \sim \frac{\rho/m}{(m\mathbf{v})^3} \gg 1$$

$\Rightarrow$  classical field  $\psi(t, \mathbf{x})$   
or collective wave function

- Nonrelativistic approximation

$$v \ll 1 \Rightarrow$$

Schrödinger-Poisson (SP) eqs

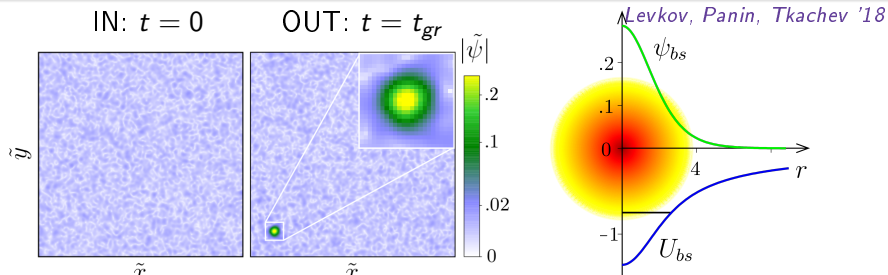
$$i\partial_t \psi = -\Delta \psi / 2m + m\mathbf{U}\psi$$

$$\Delta \mathbf{U} = 4\pi G m |\psi|^2$$

grav. potential  $U(t, \mathbf{x})$

field  $\psi(t, \mathbf{x})$

# Light DM Bose-condensates by gravitational scattering!



- **Bose star** = Bose-condensate on a single level of  $U_{bs}$
- **Gravitational kinetic relaxation:**

$$t_{gr} = \frac{b\sqrt{2}}{12\pi^3} \frac{m^3 v^6}{G^2 \Lambda \rho^2}, \quad b \approx 0.9$$

Coulomb logarithm:  
 $\Lambda \equiv \log(mvR)$

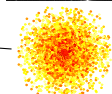
- Fuzzy DM in dwarf galaxies:

$$t_{gr} \gtrsim 10^3 \text{ yr}$$



- QCD axions in miniclusters:

$$t_{gr} \gtrsim \text{hr}$$



- DM is light  $\Rightarrow$  **The Universe is packed with Bose stars!**

## How do the Bose stars grow?

- A difficult problem
- Numerical simulations: conflicting results

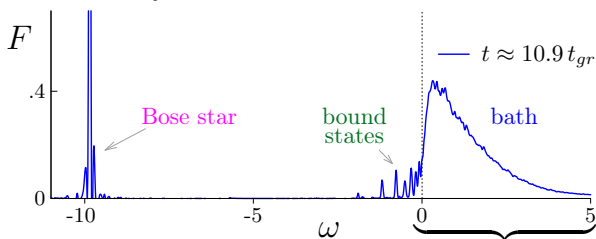
$$M_{bs} \propto t^{1/2}, t^{1/4}, t^{1/8}$$

e.g. *Levkov et al '18*, *Eggemeier et al '19*, *Chan et al '22*

**But the solution is simple and analytical!**

# Distribution of particles over energies

$$F(t, \omega) \equiv \frac{1}{N} \frac{dN}{d\omega} = \int \frac{dt_1 d^3\mathbf{x}}{2\pi N} \psi(t, \mathbf{x}) \psi^*(t + t_1, \mathbf{x}) e^{i\omega t_1 - t_1^2/\Delta t^2}$$

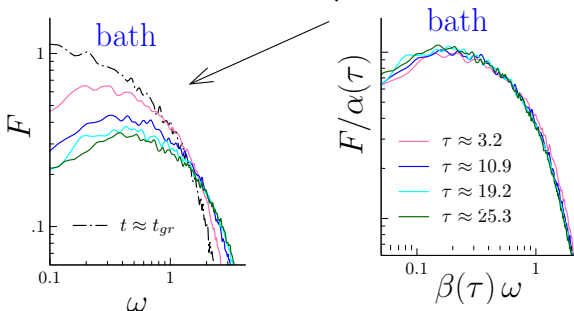


## Scaling symmetry

$$F = \alpha F_s(\beta\omega)$$

$$\alpha = \tau^{-1/D} \quad \beta = \tau^{2/D-1}$$

$$\tau \equiv t/t_{gr}, \quad D = 2.8$$



# Self-similar bath

- Consider the bath  $\omega > 0$
- Ignore the Bose star potential  $U_{bs}(r)$

$$\partial_t F = \text{St} F$$

bath kinetic eq.

Ansatz passes the equation!

$$F(\tau, \omega) = \alpha(\tau) \underbrace{F_s}_{\text{self-similar profile}}(\beta(\tau)\omega) \quad \begin{array}{l} \alpha = \tau^{-1/D} \\ \beta = \tau^{2/D-1} \end{array}$$

$$\tau \equiv t/t_{gr}$$

⇒ Profile Eq:

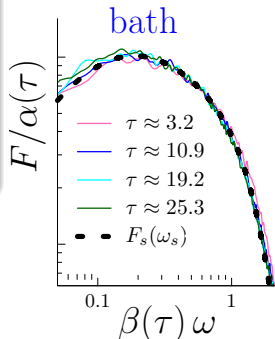
$$(2/D - 1)\omega_s \partial_{\omega_s} F_s - F_s/D = \text{St} F_s$$

⇒ Power-law bath mass & energy

$$M_b \propto \tau^{1-3/D}, \quad E_b \propto \tau^{2-5/D}$$

$$E_b^3/M_b^5 \propto \tau$$

non-autonomous system!



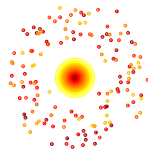
- Nontrivial BC: Particle Flux  $\neq 0$  at  $\omega = 0$

Then the solution exists  
And it is a kinetic attractor

## Assumptions:

- Quasi-stationary & self-similar bath:

$$D = D(t) \quad \text{but still} \quad E_b^3/M_b^5 \propto \tau$$



- Energy & mass conservation:

$$E_b = E - E_{bs} - E_e, \quad M_b = M - M_{bs} - M_e$$

Bose star    ↑    ex. states                      Bose star    ↑    ex. states

- Bose star energy:

$$E_{bs} = -\gamma M_{bs}^3, \quad \gamma \approx 0.0542 m^2 G^2$$

constant

- Low occupancies of bound states:

$$x_e \equiv M_e/M \approx \text{const}, \quad E_e \approx 0 \quad \text{— confirmed by simulations}$$

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x_e - x)^5} = \frac{\tau - \tau_i}{\tau_*}$$

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

A simple and predictive law!



# Compare with simulations

## Equation

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x_e - x)^5} = \frac{\tau - \tau_i}{\tau_*}$$

## Variables

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

## Parameters

Known:

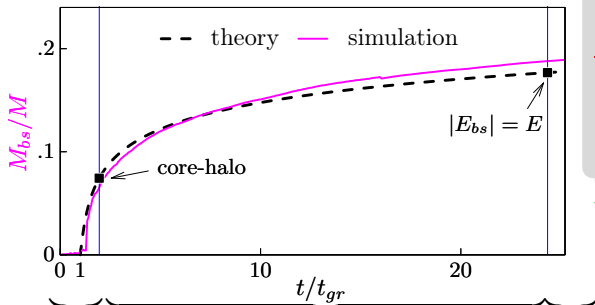
- $\epsilon^2 \equiv E/\gamma M^3$
- $\tau_* = \frac{1-\tau_i}{(1-x_e)^5}$   
 $\leftrightarrow M_{bs}(1) = 0$

To fit:

- $x_e \equiv M_e/M$  — small
- $\tau_i \approx -0.1$  — universal

←  $\epsilon = 0.074$ ,  $x_e = 0.043$ ,  
 $\tau_i = -0.1$

Comparison:



$M_{bs} \propto t^{1/2}$

$t^{1/4}$

$t^{1/8}$  — like in simulations!

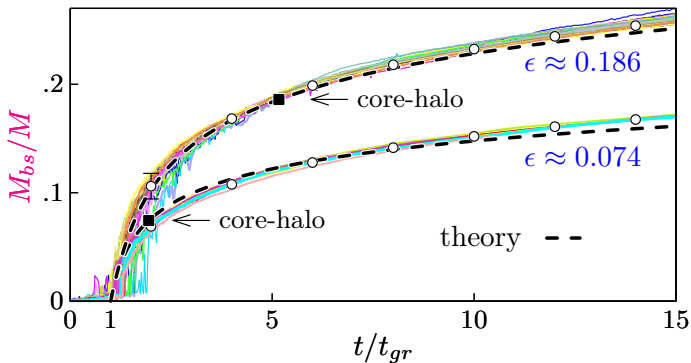
cf. *Levkov et al '18*, *Eggemeier et al '19*, *Chan et al '22*

core-halo:  $|E_{bs}/M_{bs}| = E/M$  — stars in simulations “stop growing”

*Schive et al '14*

# Statistical test

- 33 simulations
- Essentially different  $t_{gr}$  & two values of  $\epsilon$



## Other performed tests

- + Nonzero self-interaction  $\lambda_4 \neq 0$
- + Condensation in miniclusters

# Nonzero self-interaction $\lambda\psi^4$

## Exactly the same!

- Gravitational kinetic relaxation only:
- But Bose star energy:

$$E_{bs} = -\gamma M_{bs}^3 \mathcal{E}(M_{bs}/M_\lambda),$$
$$\gamma \approx 0.0542 m^2 G^2$$

$$\mathcal{E}(M_{bs}/M_\lambda) \approx 1; M_{bs} - \text{small}$$

$$\mathcal{E}(M_{bs}/M_\lambda) \neq 1; M_{bs} - \text{big}$$

## Equation

$$\frac{(1 + x^3 \mathcal{E}/\epsilon^2)^3}{(1 - x_e - x)^5} = \frac{\tau - \tau_i}{\tau_*}$$

## Variables

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

## Gross-Pitaevskii-Poisson (GPP) eqs

$$i\partial_t \psi = -\Delta \psi / 2m + m \mathbf{U} \psi + \lambda |\psi|^2 \psi / (8m^2)$$

$$\Delta \mathbf{U} = 4\pi G m |\psi|^2$$

## Parameters

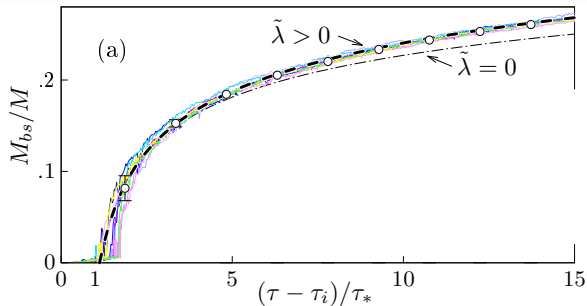
To fit:

- $x_e \equiv M_e/M$  — small
- $\tau_i$  — universal

Known:

- $\epsilon^2 \equiv E/\gamma M^3$
- $\tau_* = \frac{1 - \tau_i}{(1 - x_e)^5}$   
 $\leftrightarrow M_{bs}(1) = 0$
- $\mathcal{E} \equiv \mathcal{E}(xM/M_\lambda)$

# Nonzero self-interaction



## Repulsion

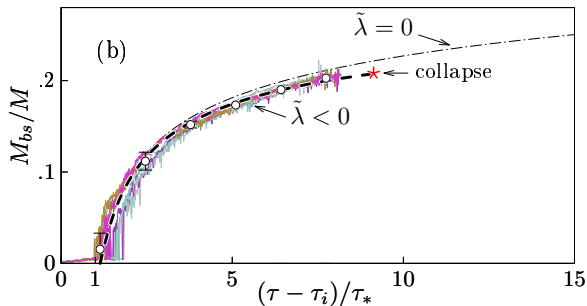
$$\tilde{\lambda} = \lambda v_0^2 / (m^2 G) = 15$$

From fit:

- $x_e \equiv M_e/M = 0.022$
- $\tau_i \approx -0.51$

Faster!

Chen et al '21



## Attraction

$$\tilde{\lambda} = \lambda v_0^2 / (m^2 G) = -6$$

From fit:

- $x_e \equiv M_e/M = 0.025$
- $\tau_i \approx 0.14$

Slower!

Chen et al '21

Collapse!

Chen et al '21

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x)^5} \sim \frac{t}{t_{gr}}$$

$$x(t) \equiv \frac{M_{bs}}{M}$$

## Core of Fornax dwarf



$$M \sim 10^8 M_{\odot}$$

$$v_{vir} \sim 20 \text{ km/s}$$

- Parameters:

$$t_{gr} \sim 0.05 \frac{m^3 (GM)^4}{\Lambda v_{vir}^6}$$

$$\epsilon \sim 3 \frac{v_{vir}}{GmM}$$

- Time to “core-halo” slowdown:

$$t_{c/h} \sim 9\epsilon t_{gr} \sim \Lambda^{-1} m^2 (GM)^3 / v_{vir}^5$$

- Fuzzy DM in Fornax Dwarf ( $m \sim 10^{-22} \text{ eV}$ )

$$t_{c/h} \sim 10^7 \text{ yr} \text{ — form \& grow}$$

- Experimental bound:  $m \gtrsim 2 \cdot 10^{-20} \text{ eV}$

$$t_{c/h} \gtrsim 10^{11} \text{ yr} \text{ — do not grow! (in Fornax)}$$

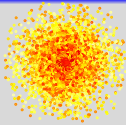
Larger galaxies are worse!

Only small Bose stars exist in the Universe!

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x)^5} \sim \frac{t}{t_{gr}}$$

$$x(t) \equiv \frac{M_{bs}}{M}$$

## Axion minicluster



$$M \sim 10^{-17 \div 12} M_{\odot}$$

$$\Phi = \delta\rho_a / \bar{\rho}_a |_{RD}$$

$$= 0 \div 10^3$$

Hogan, Rees '88; Kolb, Tkachev '93

- Parameters:

$$t_{gr} \sim \frac{5 \cdot 10^8 \text{ yr}}{\Phi^4} \left( \frac{M}{10^{-14} M_{\odot}} \right)^2 \left( \frac{m}{10^{-4} \text{ eV}} \right)^3$$

$$\epsilon \sim 0.02 \Phi^{2/3} \left( \frac{M}{10^{-14} M_{\odot}} \right)^{-2/3} \left( \frac{m}{10^{-4} \text{ eV}} \right)^{-1}$$

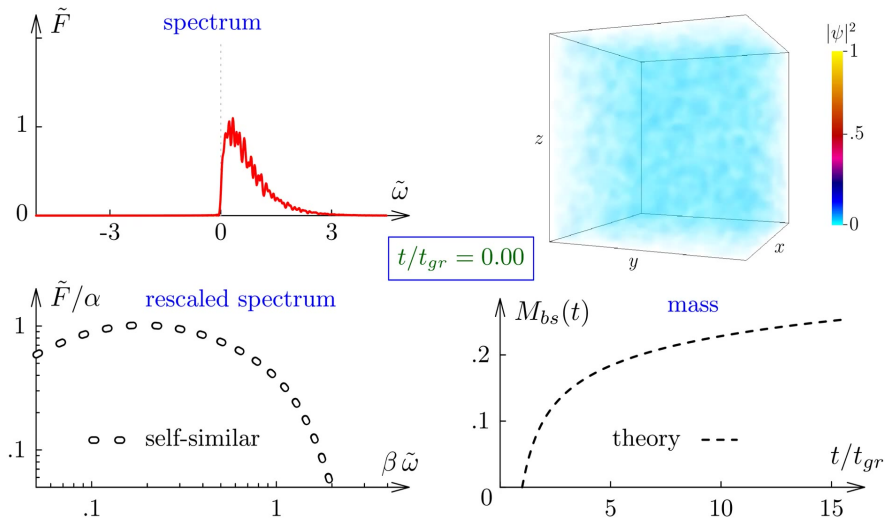
- Time to eat 10% of minicluster:

$$t_{10} \sim t_{gr} \frac{(10\%)^9}{\epsilon^6} < 10^{10} \text{ yr} \quad \text{if} \quad \boxed{\Phi \gtrsim 1}$$

All denser miniclusters turn into

The Universe is packed with grown-up axion stars!

# Conclusions I: the movie



- Less diffuse DM  $\Rightarrow$  weaker signals in DM detectorts
- Gravitational lensing

*Kolb, Tkachev '96; Fairbairn et al '17*

- Parametric resonance: radio explosions of heavy stars — explain FRB?

*Levkov, Panin, Tkachev '20; Chung-Jukko et al '22*

- **Bosenovas**: heavy stars collapse & emit relativistic axions

*Levkov, Panin, Tkachev '17; Eby et al '22*

## THANK YOU FOR ATTENTION!

This work was supported by the RSF grant 22-12-00215