Testing universal dark-matter caustic rings with galactic rotation curves

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7 Oct 2023

Introduction

- Caustics are concentric structures of enhanced dark-matter density;

- These overdensities should reveal themselves as features in
the rotation curves of galaxies.

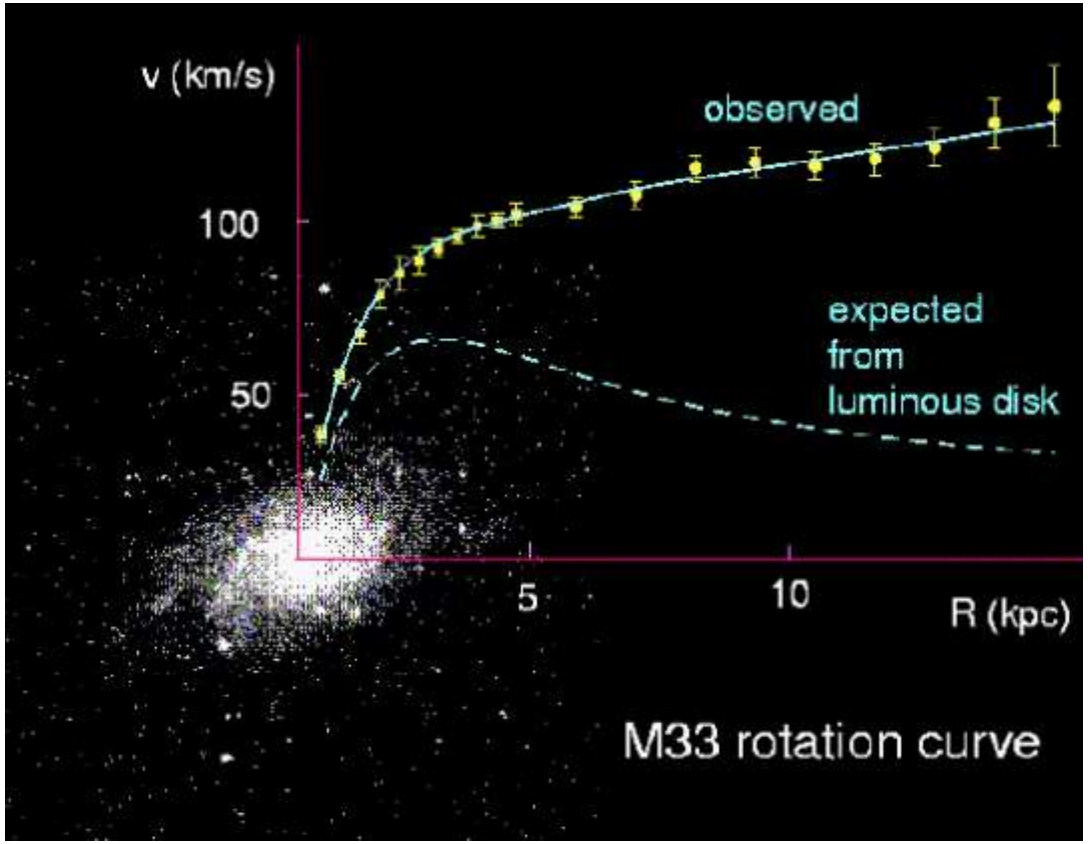

Fig. 1 M33 rotation curve

Introduction

• Positions of the caustics are governed by the distribution of angular momenta of infalling dark-matter particles;

• The authors proposed [1] that this distribution is universal and therefore positions of the caustic rings in different galaxies should coincide up to a well-determined rescaling.

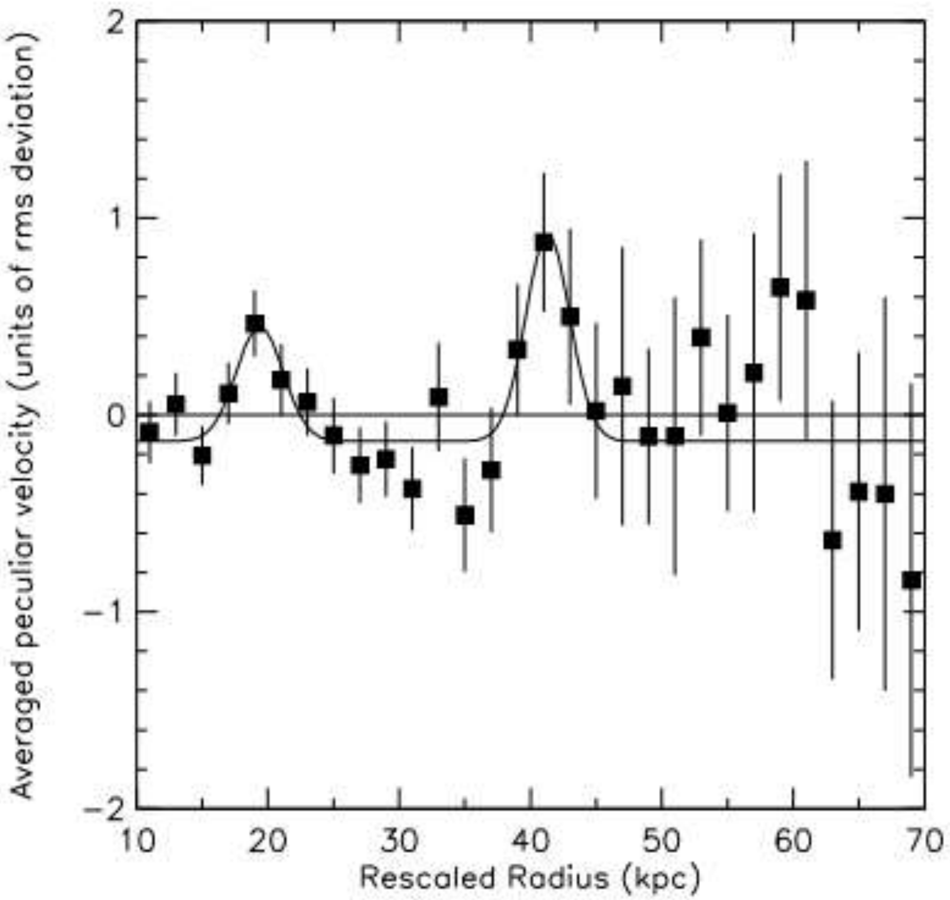

Fig. 2 Binned data for 32 galaxy sample according to [1]

[1] - W. H. Kinney, P. Sikivie, Evidence for universal structure in galactic halos, Phys. Rev. D 61 (2000) 087305. arXiv:astro-ph/9906049, doi:10.1103/PhysRevD.61.087305

Motivation and purpose

- To our best knowledge, these results have never been tested with newer data;

- A statistical analysis of possible universal caustic rings was performed only in Ref. [1];

- The **purpose** of the present work is to **test** the same hypothesis of **universal** caustic rings [1] with a large independent set of rotation curves published since then;

Data on rotation curves

- We use the Spitzer Photometry and Accurate Rotation Curves (SPARC) database of rotation curves for 175 galaxies [2].
- The previous study [1] used a sample of 32 rotation curves taken from [3, 4]. Of these 32 galaxies, 29 are present in the SPARC database.
- The main, independent from Ref. [1], sample we use contains 121 rotation curves.

[2] - F. Lelli, S. S. McGaugh, J. M. Schombert, Sparc: Mass models for 175 disk galaxies with spitzer photometry and accurate rotation curves, The Astronomical Journal 152 (6) (2016) 157.

[3] K. G. Begeman, A. H. Broeils, R. H. Sanders, Extended rotation curves of spiral galaxies : dark haloes and modified dynamics., Mon. Not. Roy. Astron. Soc. 249 (1991) 523.

[4] - R. H. Sanders, The published extended rotation curves of spiral galaxies: Confrontation with modified dynamics, The Astrophysical Journal 473 (1) (1996) 117–129

Data analysis

Individual rotation curves

- Each rotation curve is a set of measurements $(r_i, v_i \pm \Delta v_i)$;
- The hypothesis of Ref. [1] predicts caustic rings at

$$\tilde{r} = a_n \frac{j_{\max}}{0.25} \frac{0.7}{h} \qquad (1)$$

where the rescaled radius is defined as

$$\tilde{r} \equiv r \left(\frac{220 \text{ km/s}}{v_{\text{rot}}} \right) \qquad (2)$$

- The universal rescaled positions of these caustics are:

$$a_n = \{39, 19.5, 13, 10, 8, \ldots\} \text{ kpc}$$

Data analysis

Individual rotation curves

1.)The data points are fitted by a smooth curve (a line or a polynomial was used in [1]).

2.) approximately flat part of the rotation curve ⬄ $\tilde{r}_i \gtrsim 10$ kpc

3.) We determine the fitting parabola:

$$\bar{v}(\tilde{r}) = c_0 + c_1\tilde{r} + c_2\tilde{r}^2$$

4.) Then the average value

$$v_{\mathrm{rot}} = \langle v(\tilde{r}) \rangle_{\tilde{r} \geq 10 \text{ kpc}}$$

used in eq. (2)

The rescaled rotation curve is determined **iteratively**. As a result, we obtain a set of rescaled data points $(\tilde{r}_i, v_i \pm \Delta v_i)$ and the corresponding fitting parabola $\bar{v}(\tilde{r})$.

Data analysis

Individual rotation curves

Next step

1.) The output of this step for each galaxy is a set of $(\tilde{r}_i, \sigma_i)$, where we defined:

$$\sigma_i \equiv \left(v_i - \bar{v}(\tilde{r}_i)\right)/\Delta v_i$$

2.) We require at least four data points to search for bumps against the background of parabola $\bar{v}(\tilde{r})$.

3.) This quality cut removes **25 galaxies** from the main data set (and none from the set of 32 galaxies used in [3,4]).

Data analysis

Ensemble of rotation curves

Next step

- For each σ_i we assign a p_i value $(0 \leqslant p_i \leqslant 1)$ with the meaning of the probability of a random deviation to $\geq \sigma_i$ for the Gaussian distribution with the mean 0 and variance 1:

$$p_i = 1 - \mathrm{CDF}(\sigma_i) = \frac{1}{2}\left(1 - \mathrm{erf}(\frac{\sigma_i}{\sqrt{2}})\right)$$

$$\downarrow$$

a function $p_j(\tilde{r})$ by the linear interpolation of the corresponding points $(\tilde{r}_i,\ p_i)$ for this galaxy.

Data analysis

Ensemble of rotation curves

3.) We are now ready to construct the averaged likelihood function as:

$$L(\tilde{r}) = -\sum_{j=1}^{N(\tilde{r})} \frac{\log[p_j(\tilde{r})]}{N(\tilde{r})},$$

where $N(\tilde{r})$ is the number of galaxies, for which p_j is determined at the point $\tilde{r}$.

Data analysis

Monte-Carlo estimate of significance

To simulate artificial rotation curves without universal features, we assume that:

- the fitted functions $\bar{v}(\tilde{r})$ represent the true smooth rotation curves;
- the measurements were done at the same sets of r_i as in the real data;
- the measurement errors are Gaussian with the same widths as quoted for the real data.

Within these assumptions, we obtain a **simulated** data set and process it in the same way as the real data.

Data analysis

Monte-Carlo estimate of significance

- We use the maximal value $L_{\max}$ of $L(\tilde{r})$ over the interval of interest, 10 kpc ≤ $\tilde{r}$ ≤ 75 kpc, which covers well the first two caustic rings discussed in Ref. [1].

- We repeat *M* times the same procedure with simulated rotation-curve measurements and obtain a set of simulated $L_{\max}^{(k)}$, $k = 1, \ldots, M$.

- The significance of the strongest universal feature in the set of rescaled rotation curves is determined by the p-value.

Results and discussion

Results for the main sample
(contains 121 rotation curves)

The vertical dashed lines indicate the expected positions of the n = 1, 2 caustic rings claimed in Ref. [1].

No significant peaks of $L(\tilde{r})$ are observed.

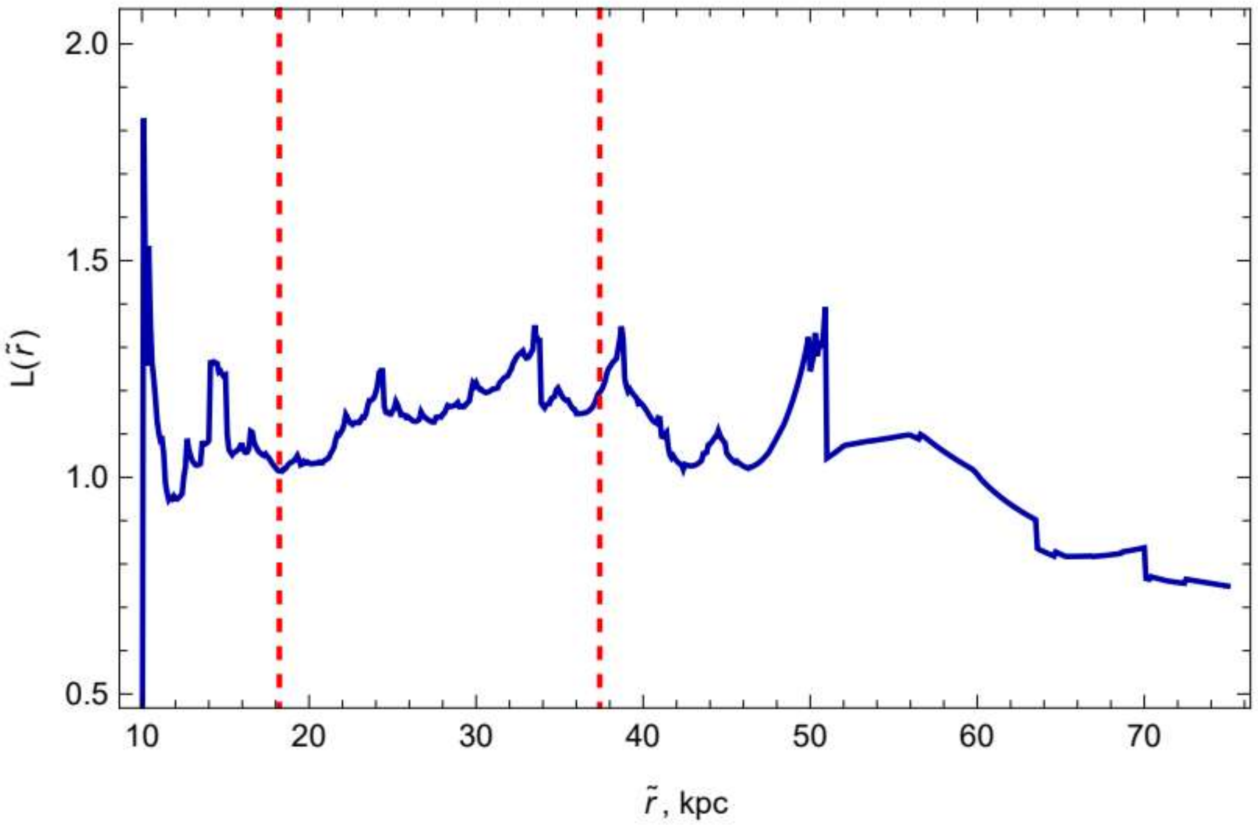

Fig. 3 The function $L(\tilde{r})$ for the main sample of 121 galaxies which does not include those studied in Ref. [1].

Results and discussion

Results for the main sample
(contains 121 rotation curves)

The observed $L_{\max} \approx 1.83$ or larger was found 202 times out of 1000, resulting in the p-value of 0.2 for the null hypothesis of the absence of universal caustic rings.

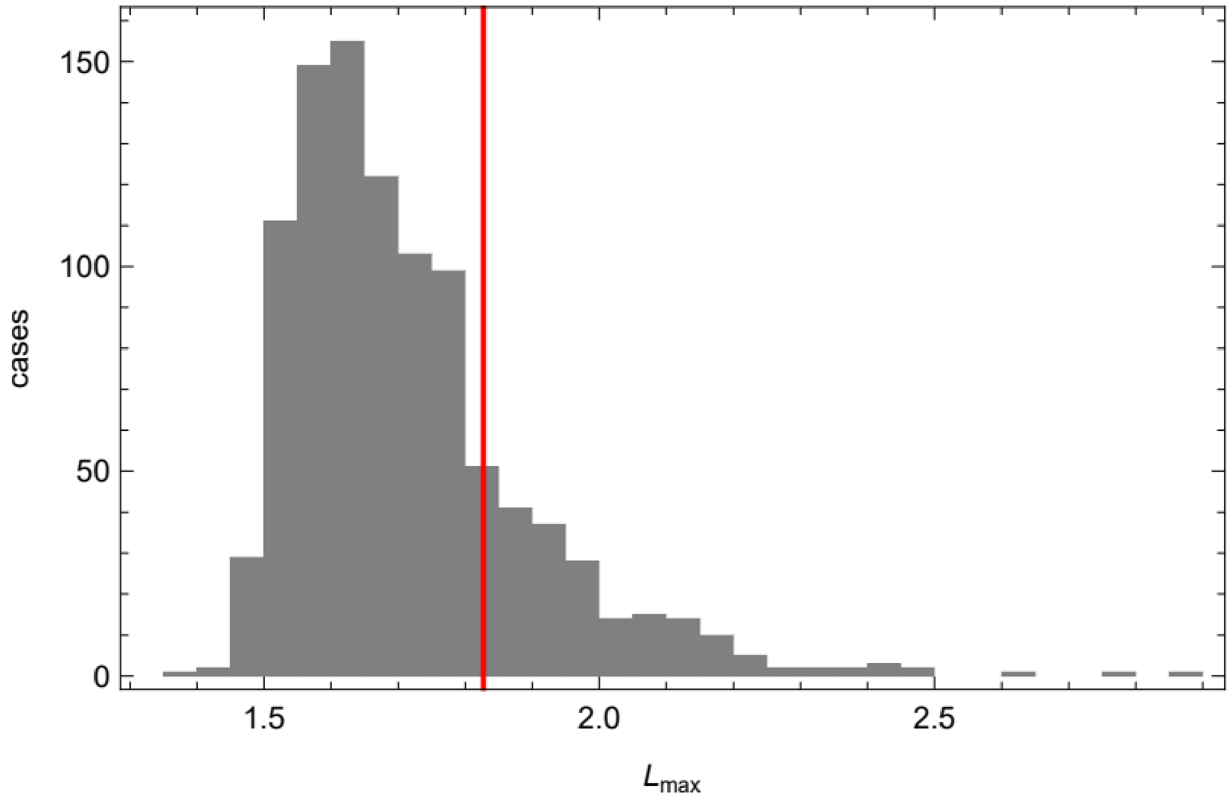

Fig. 4 The distribution of the maxima of 1000 Monte-Carlo simulated $L(\tilde{r})$ functions for the main sample of rotation curves.

Results and discussion

Results for the previously used sample

(contains 32 rotation curves)

The p-value of 0.98 for the null hypothesis.

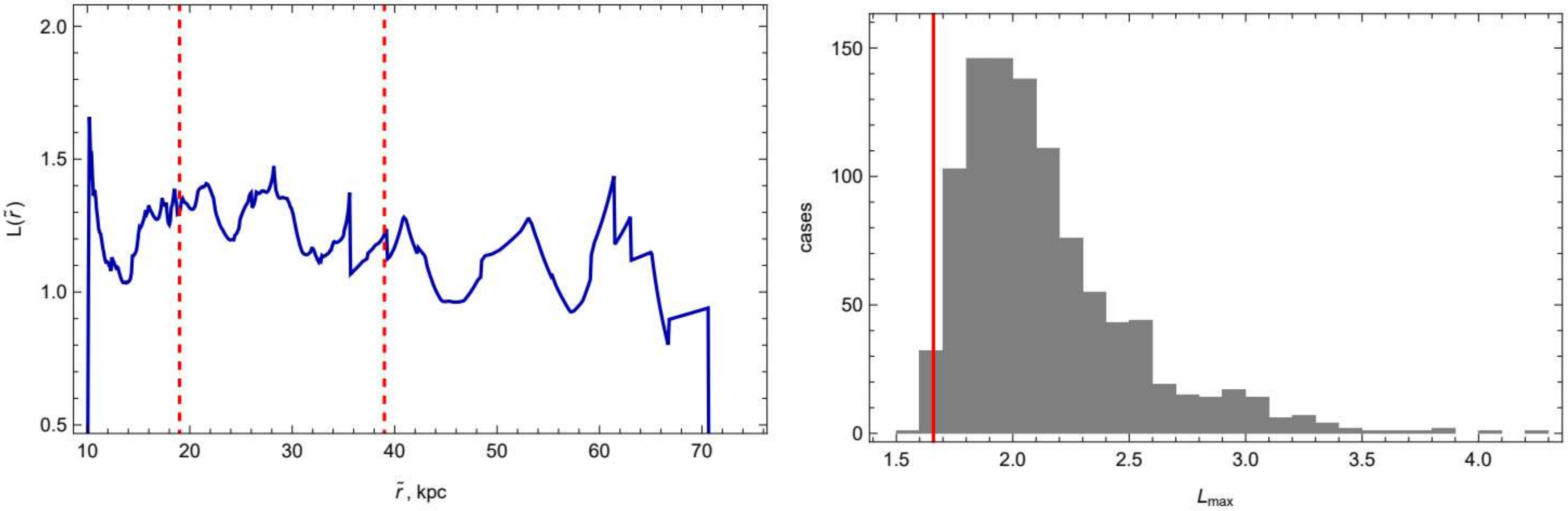

Fig. 5 Results for the sample of 32 rotation curves studied in Ref. [1].

Results and discussion

Discussion: comparison with the previous result

- There are two main differences between the statistical analysis used in Ref. [1] and that of the present work:

  - We use the unbinned likelihood while the analysis of Ref. [1] was based on binning.

  - we use Monte-Carlo simulations based on the null hypothesis to estimate the significance, while Ref. [1] assigned statistical errors to the binned data by hand.

Results and discussion

Discussion: comparison with the previous result

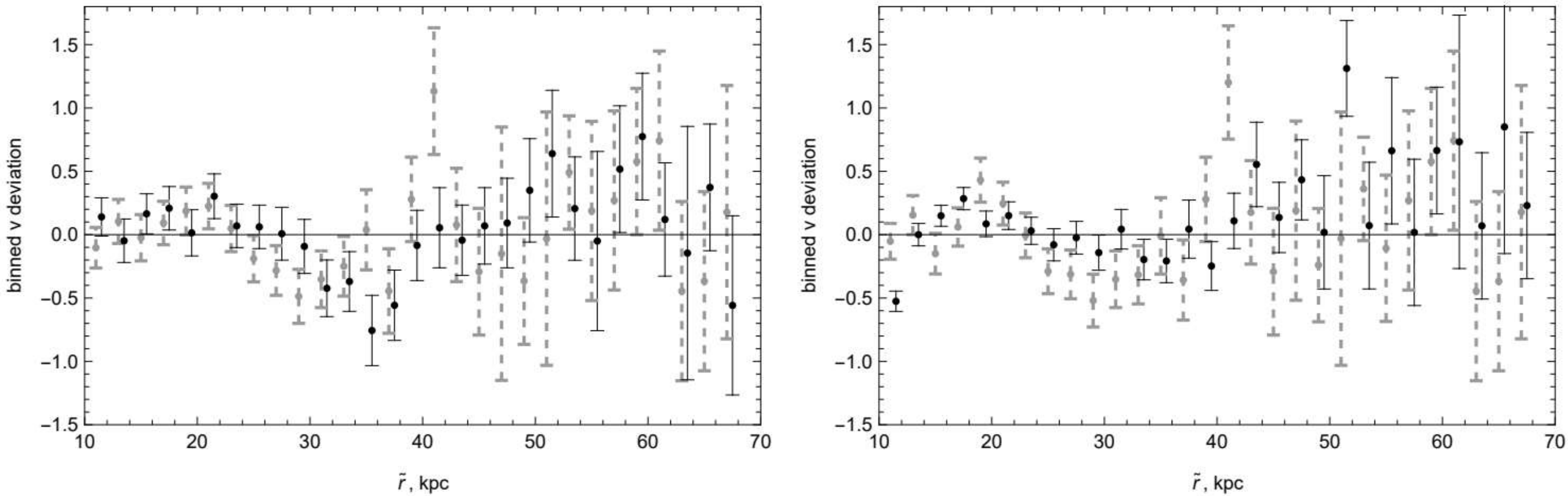

Fig. 6 Comparison of the new results with those of Ref. [1] by the binned method of Ref. [1]. Grey points with dashed error bars: the rotation curves used in Ref. [1]; black points with full error bars: new rotation curves from the SPARC database. Left panel: 29 galaxies present in both samples. Right panel: 32 galaxies in the old sample and 121 other galaxies in the new sample.

Conclusions

- The hypothesis of the universality of caustic rings in galaxies, proposed in Ref. [1], was tested.

- We find no indication of the presence of universal caustic rings: the data agrees with the expectations of the null hypothesis with the p-value of 0.20.

- We demonstrate in addition that the new data do not support the conclusions of Ref. [1] even if their method is used.

- However, the stacking method, used in Ref. [1] and in the present work, tests only the universality of the caustics.

Thank you for your attention!

This work is supported by the Russian Science Foundation, grant 22–12–00215 (DD).

Запасные слайды

Статистический анализ

- Входные данные: ($\tilde{r}_i$, $\delta v_i \pm \Delta v_i$). Дальше строим точки : ($\tilde{r}_i$, σ_i), где

$\sigma_i = \dfrac{\delta v_i}{\Delta v_i}$

Каждому σ_i ставится в соответствие p_i, имеющая смысл вероятности случайного отклонения от нуля на величину δv_i (и более) для нормального распределения:

$$p_i = \frac{1}{2\boldsymbol{\pi}} \int_{\sigma_i}^{\infty} \exp\left[-\frac{\sigma_i^2}{2}\right] d\sigma$$ (получили набор ($\tilde{r}_i$, p_i))

Дальше составляем $L(\tilde{r}) = -\sum_{i=1}^{N(\tilde{r})} \frac{\ln[pi(\tilde{r})]}{N(\tilde{r})}$