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Gravitational waves as waveguides

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On 17 August 2017, the Advanced LIGO1 and Virgo2 detectors observed the gravitational-wave event GW170817—a strong signal from the merger of a binary neutron-star system. Less than two seconds after the merger, a γ-ray burst (GRB 170817A) was detected within a region of the sky consistent with the LIGO-Virgo-derived location of the gravitational-wave source. This sky region was subsequently observed by optical astronomy facilities, resulting in the identification of an optical transient signal within about ten arcseconds of the galaxy NGC 4993. This detection of GW170817 in both gravitational waves and electromagnetic waves represents the first 'multi-messenger' astronomical observation.
Such observations enable GW170817 to be used as a 'standard siren' (meaning that the absolute distance to the source can be determined directly from the gravitational-wave measurements) to measure the Hubble constant.

Troja, E. and others, “The X-ray counterpart to the gravitational wave event GW 170817”, 1710.05433, “Nature”

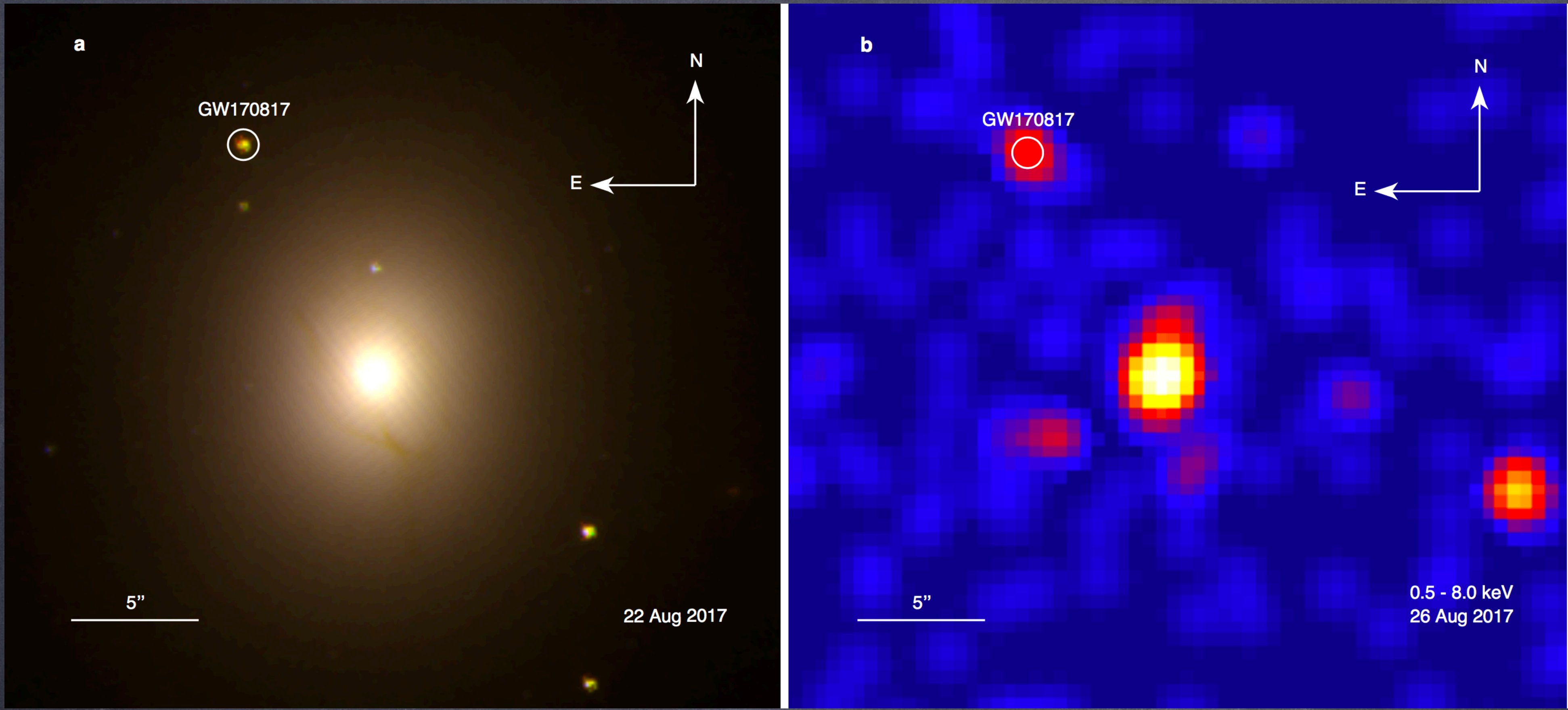


Figure 1: Optical/Infrared and X-ray images of the counterpart of GW170817

a Hubble Space Telescope observations show a bright and red transient in the early-type galaxy NGC 4993, at a projected physical offset of ~2 kpc from its nucleus. A similar small offset is observed in some (~25%) short GRBs. Dust lanes are visible in the inner regions, suggestive of a past merger activity. b Chandra observations revealed a faint X-ray source at the position of the optical/IR transient. X-ray emission from the galaxy nucleus is also visible.

Gravitational waves can act as waveguides for electromagnetic radiation, i.e. if the latter is initially aligned with the gravitational waves then the alignment will survive during the propagation

- Jacobi equation for null geodesics
- Hamiltonian formalism

Deviation of null geodesics is described by the Jacobi equation

$$\frac{d^2n^a}{dt^2} + R^a{}_{bcd}u^bu^dn^c = 0$$

u is the velocity of geodesics

n is the deviation of geodesics

R_{abcd} is the Riemannian tensor

Behavior of deviation vector n^a depends on Riemmanian tensor R_{abcd}

$$
\begin{aligned}
\dot{\theta} &= -R_{ab}n^a n^b + 2\omega^2 - \sigma_1^2 - \sigma_2^2 - \frac{1}{2}\theta^2, \\
\dot{\omega} &= -\theta\,\omega, \\
\dot{\sigma}_1 &= -\theta\,\sigma_1 - C_{1010}, \\
\dot{\sigma}_2 &= -\theta\,\sigma_2 - C_{1020},
\end{aligned}
$$

$$
\theta = (\det A)^{-1}\frac{d}{ds}(\det A), \qquad \hat{\omega}_{mn} = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}, \qquad \hat{\sigma}_{mn} = \begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_2 & -\sigma_1 \end{pmatrix}
$$

C_{abcd} -> Weyl tensor

θ -> expansion scalar as the trace of the expansion tensor

ω_{mn} -> vorticity tensor

σ_{mn} -> shear tensor

matrix A is defining the shape and orientation of the fluid element

S. W. Hawking, G. F. R. Ellis, The large scale structure of space-time, Cambridge University Press (1973)

- We take $\omega_{mn} = 0$, no vorticity, i.e. no centrifugal forces
- We consider gravitational waves in empty space-time

$$
\begin{aligned}
R_{ab} &= 0, \\
C_{abcd} &= R_{abcd} .
\end{aligned}
$$

- Then

$$
\begin{aligned}
\dot{\theta} &= 2\omega^2 - \sigma_1^2 - \sigma_2^2 - \tfrac{1}{2}\theta^2, \\
\dot{\omega} &= -\,\theta\ \omega, \\
\dot{\sigma}_1 &= -\,\theta\ \sigma_1 - a_1, \\
\dot{\sigma}_2 &= -\,\theta\ \sigma_2 - a_2 .
\end{aligned}
$$

where $\quad a_1 = R_{1010} \qquad a_2 = R_{1020}$

- We take $\omega = 0$. Equations that we have to solve are

$$\dot{\sigma}_i + \theta\ \sigma_i = -\ a_i$$

$$\dot{\theta} + \frac{1}{2}\theta^2 + \sigma_1^2 + \sigma_2^2 = 0,$$

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi \sin\varphi \\ -\sin\varphi \cos\varphi \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}, \quad \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi \sin\varphi \\ -\sin\varphi \cos\varphi \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$\tan\varphi = \dfrac{a_2 - a_1}{a_2 + a_1}$ in the case $\tilde{a}_1 = \tilde{a}_2$.

In terms of s_i and a , our equations become

$$\dot{s}_i + \theta s_i = -a,$$

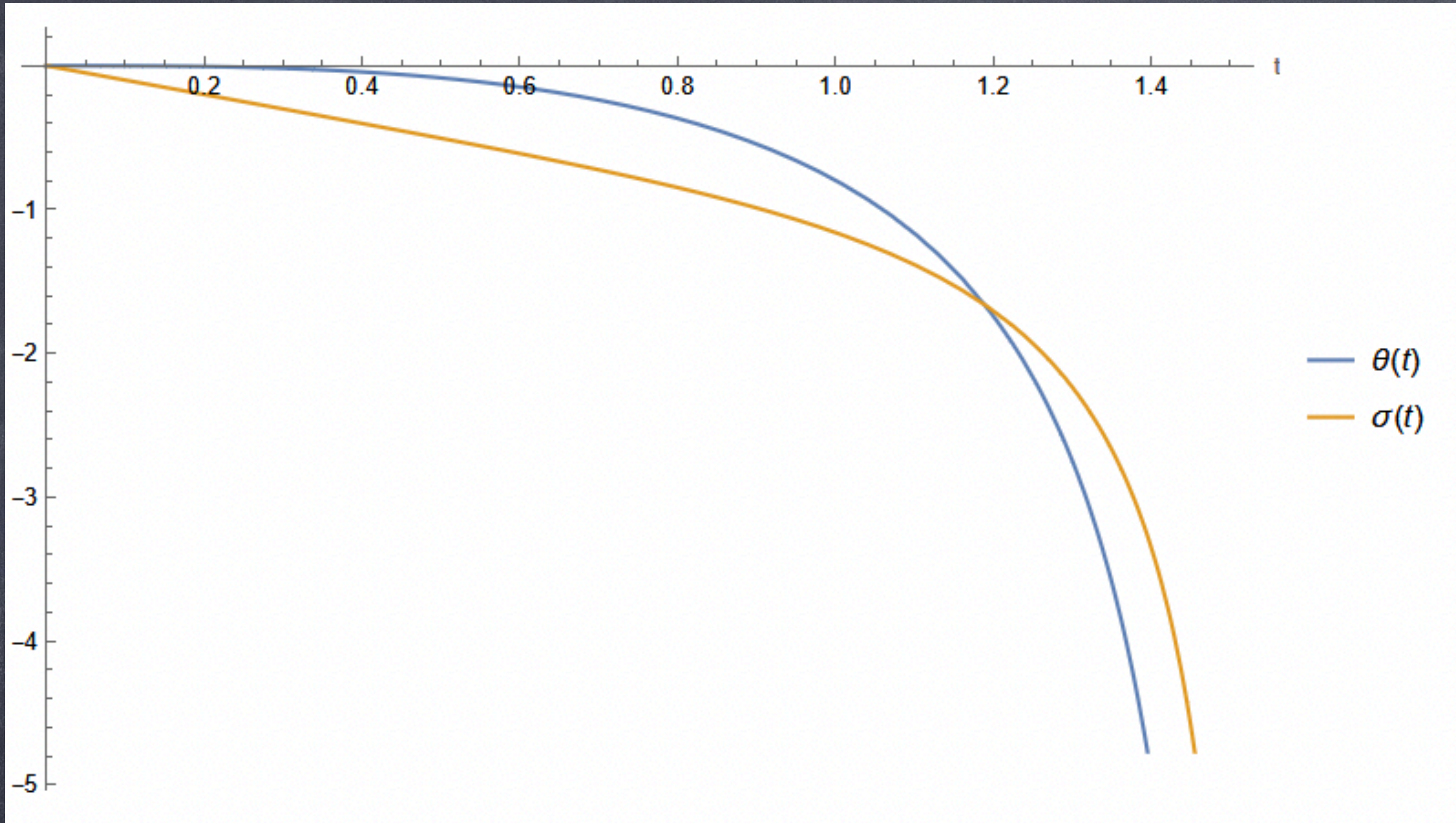
$$\dot{\theta} + \frac{1}{2}\theta^2 + s_1^2 + s_2^2 = 0$$

$s_1 = s_2 \rightarrow a_1 = a_2$ with the solution

$$s(t) = -\frac{\sqrt{a}}{2}\left[\tanh(\sqrt{a}(t - t_1)) + \tan(\sqrt{a}(t - t_0))\right]$$

$$\theta(t) = \sqrt{a}\left[\tanh(\sqrt{a}(t - t_1)) - \tan(\sqrt{a}(t - t_0))\right]$$

$$\sigma_1 = (\cos\varphi - \sin\varphi)s, \quad \sigma_2 = (\cos\varphi + \sin\varphi)s; \quad \sigma_1 = \frac{a_1}{\alpha}s, \quad \sigma_2 = \frac{a_2}{\alpha}s$$



Photon trapping in gravitational waves indicated by the mutual behavior of θ and σ during the propagation. The scalar θ determines the rate of the change of volume element in time measured by a comoving observer, while the shear σ determines the distortion of the shape of an initial ball.

Using the Hamiltonian Formalism to determine the behavior of the photon beams within the gravitational waves defined by the linearized metric (weak-field approximation)

$$\mathbf{g} = -\mathbf{d}t^2 + \mathbf{d}z^2 + \gamma_{ij}\mathbf{d}x^i\mathbf{d}x^j = -\mathbf{d}u\mathbf{d}v + (\delta_{ij} + h_{ij}(u))\mathbf{d}x^i\mathbf{d}x^j \qquad (h_{ij}(u) = A(u)e_{ij}),$$

$$\mathbf{g} = \begin{pmatrix} 0 & -1/2 & 0 & 0 \\ -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 + h_{11}(u) & h_{12}(u) \\ 0 & 0 & h_{21}(u) & 1 + h_{22}(u) \end{pmatrix}.$$

$$\mathscr{H} = -2p_u p_v + \frac{1}{2}(\delta^{ij} + A(u)e^{ij})p_i p_j = 0$$

$$A(u) = A_0 \cos(\omega u) \qquad \text{then} \qquad \begin{aligned} h_{11} &= -h_{22} = Re(A_+ e^{-i\omega u}) \\ h_{21} &= h_{12} = Re(A_\times e^{-i\omega u}) \end{aligned}$$

e^{ij} is the polarization tensor of the gravitational waves, and $u = t - z, v = t + z$

The Hamiltonian equations then have the form, with the defined momentum p_i

$$\begin{cases} \dfrac{du}{d\lambda} = \dfrac{\partial \mathscr{H}}{\partial p_u} = -2p_v \\ \dfrac{dv}{d\lambda} = \dfrac{\partial \mathscr{H}}{\partial p_v} = -2p_u \\ \dfrac{dx^i}{d\lambda} = \dfrac{\partial \mathscr{H}}{\partial p_i} = \gamma^{ij} p_j \end{cases} \quad \text{and} \quad \begin{cases} \dfrac{dp_u}{d\lambda} = -\dfrac{\partial \mathscr{H}}{\partial u} = -\dfrac{1}{2} A'(u) e^{ij} p_i p_j \\ \dfrac{dp_v}{d\lambda} = -\dfrac{\partial \mathscr{H}}{\partial v} = 0 \\ \dfrac{dp_i}{d\lambda} = -\dfrac{\partial \mathscr{H}}{\partial x^i} = 0 \end{cases}$$

thus $p_v = p_v(0) \neq 0, \; p_i = p_i(0), \; du = -2p_v(0) d\lambda$

$$\begin{cases} \dfrac{dv}{du} = \dfrac{p_u}{p_v(0)} \\ \dfrac{dx^i}{du} = -\dfrac{\gamma^{ij}(u) p_j(0)}{2p_u(0)} \\ \dfrac{dp_u}{du} = \dfrac{1}{4} A'(u) e^{ij} \dfrac{p_i(0) p_j(0)}{p_v(0)} \end{cases}$$

$$\mathscr{H} = -2p_u p_v + \frac{1}{2}(\delta^{ij} + A(u) e^{ij}) p_i p_j = 0,$$

$$p_u = \frac{1}{4}(\delta^{ij} + A(u) e^{ij}) \frac{p_i(0) p_j(0)}{p_v(0)},$$

$$\Longrightarrow \begin{cases} u = u(0) - 2p_v(0)\lambda, \\ \dfrac{dv}{du} = \dfrac{p_u}{p_v(0)}, \\ \dfrac{dx^i}{du} = -\dfrac{1}{2}(\delta^{ij} + A(u)e^{ij})\dfrac{p_j(0)}{p_v(0)}, \\ p_u = \dfrac{1}{4}(\delta^{ij} + A(u)e^{ij})\dfrac{p_i(0)p_j(0)}{p_v(0)}, \end{cases}$$

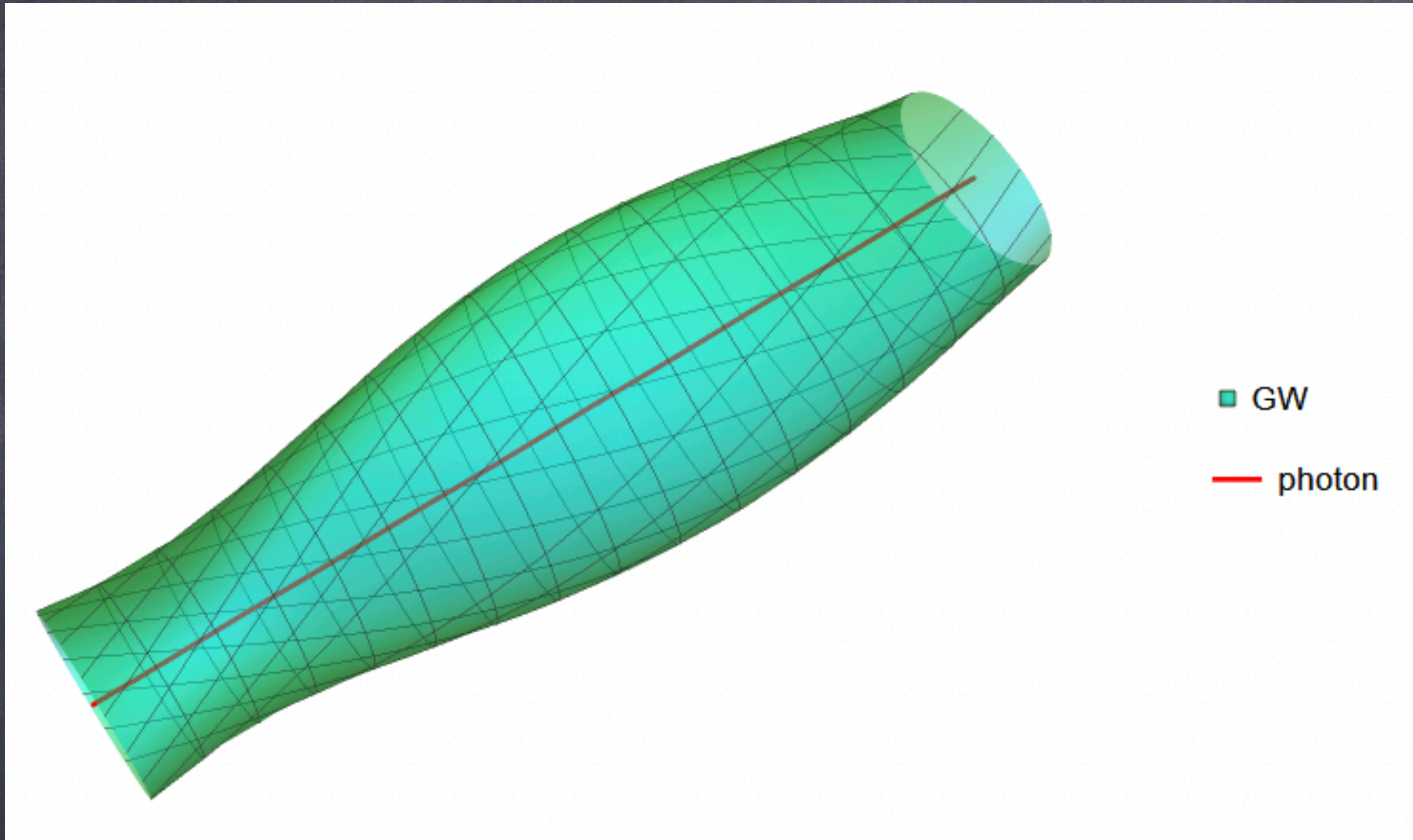
$$\begin{cases} u = u(0) - 2p_v(0)\lambda, \\ v(u) = v(0) + \dfrac{1}{4}(\delta^{ij}u + B(u)e^{ij})\dfrac{p_i(0)p_j(0)}{p_v(0)p_v(0)}, \\ x^i(u) = x^i(0) - \dfrac{1}{2}(\delta^{ij}u + B(u)e^{ij})\dfrac{p_j(0)}{p_v(0)}, \\ p_u = \dfrac{1}{4}(\delta^{ij} + A(u)e^{ij})\dfrac{p_i(0)p_j(0)}{p_v(0)}, \\ B(u) = \int_0^u A(s)ds\,. \end{cases}$$

From $p_u = \frac{1}{2}(p_t - p_z)$, $p_v = \frac{1}{2}(p_t + p_z)$ for photons with $p_t(0) = p_z(0)$, $p_1(0) = p_2(0) = 0$ we have $p_t(\lambda) = p_z(\lambda)$ for all $\lambda \geq 0$

Therefore, if photons are aligned with gravitational waves initially, then they will stay aligned during the propagation, trapped by the gravitational waves.

Interesting to note that for small $p_i(0)$, $p(t) \approx p(z)$ and

$$p_t - p_z = \left(\delta^{ij} + A(u)e^{ij}\right)\frac{p_i(0)p_j(0)}{p_v(0)}$$



Schematic view of the gravitational waves acting as waveguides for the aligned photons.

Summary

- Using the Hamiltonian formalism and the Jacobi equation of the divergence of null geodesics (analog of Landau--Raychaudhuri equation), we analysed the propagation of electromagnetic radiation in the metric of the gravitational waves. Particular cases of parameters of both waves are considered, and the conditions are obtained when the photons aligned with the gravitational wave initially, will continue to remain aligned, i.e. being trapped by the gravitational wave metric.
- If the gravitational waves act as waveguides to transmit the electromagnetic waves, then the intensity of the latter will not decay by distance according to the inverse square law.
- This effect can be important especially at the interpretation of the optical or X-ray counterparts associated to the gravitational wave signals. Namely, then the detected intensity of electromagnetic radiation can indicate a lower integral power of its pulse, as compared to the power evaluated by the inverse square law. The increasing number of associations of the gravitational wave events with electromagnetic pulses will enable to evaluate empirically the efficiency of the gravitational waves as of waveguides.

THANK YOU!