Unitarity bounds on effective field theories: positivity and causality in photon EFT

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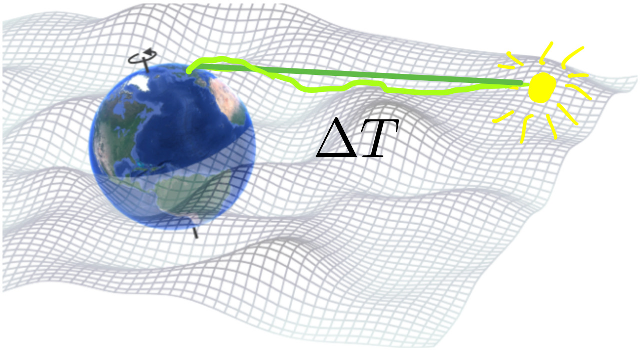
Definitions of causality

No time machine - what does it mean?



ΔT - time delay

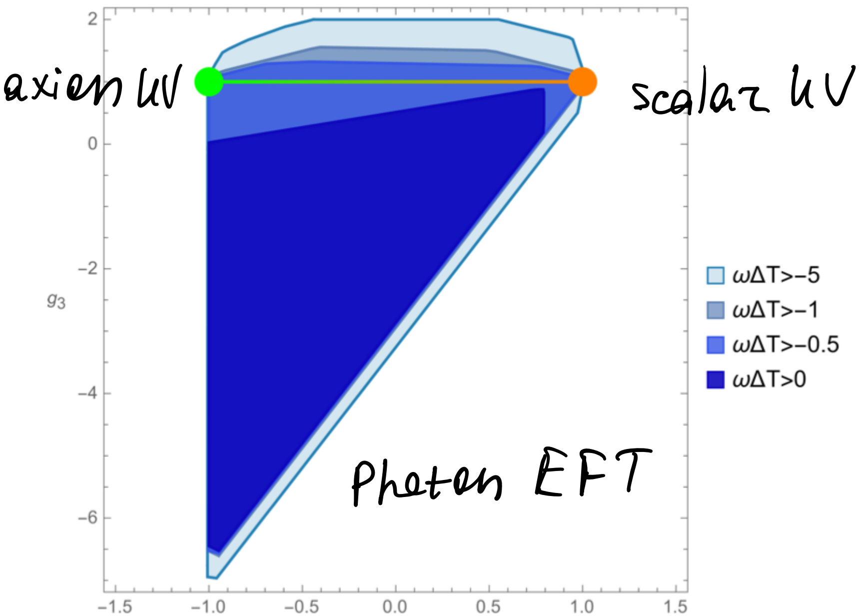
$\Delta T > 0$ - strict causality condition

rules out all higher derivative terms

Weaker condition

$$\Delta T > -\frac{O(1)}{\omega}$$

Unresolvable time advance



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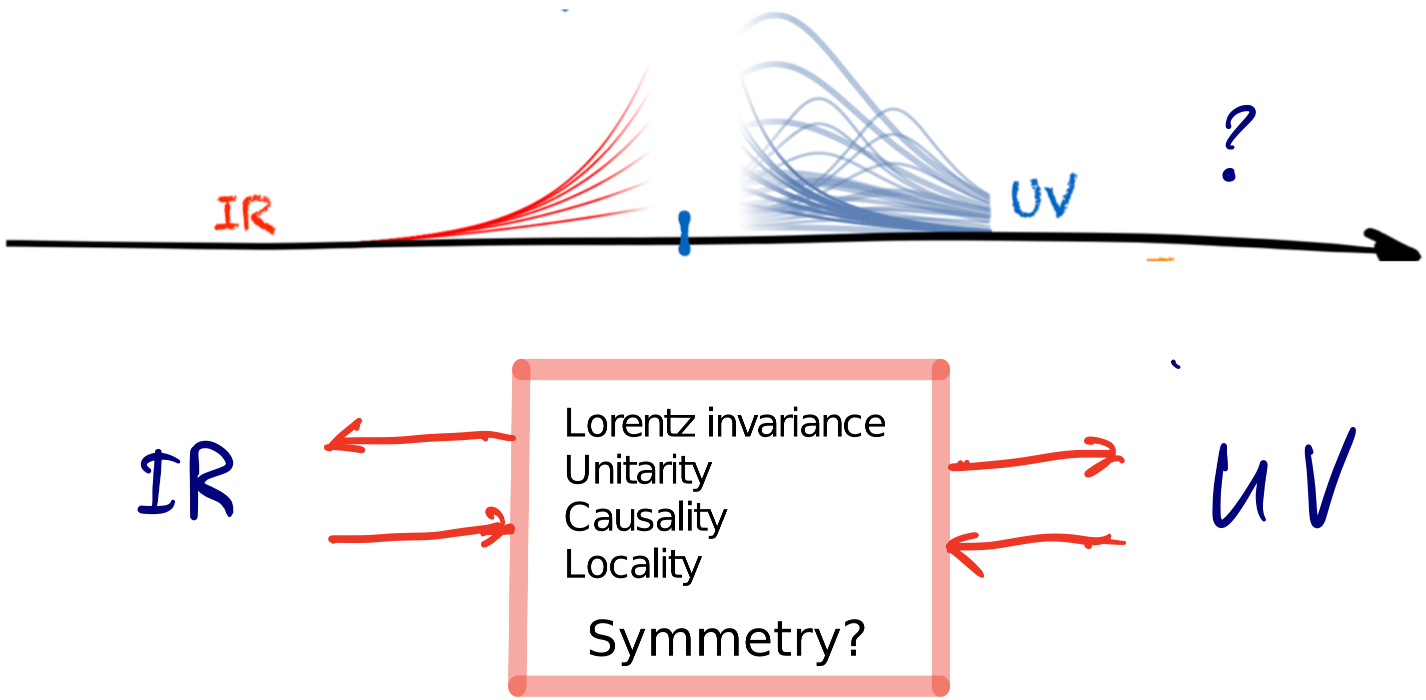
EFT framework: UV - IR connections



- Assumptions about UV constraints on IR (positivity bounds)
- IR results may require special UV properties for consistency
- The symmetry working in UV and IR can constrain the structure of IR EFT

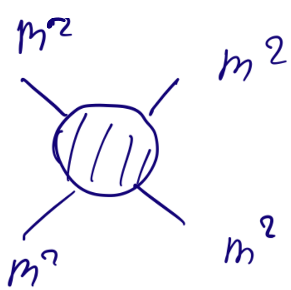
A 'good' UV completion

What do we mean by 'good'?

- Lorenz-invariant $\Rightarrow \mathcal{A} = \mathcal{A}(s, t, u)$
- unitary $\Rightarrow Im\,\mathcal{A} > 0$
- satisfying causality $\Rightarrow \mathcal{A}(s, t, u)$ is analytic everywhere except real axes
- local $\Rightarrow$ polynomial boundedness (Froissart-Martin bound)

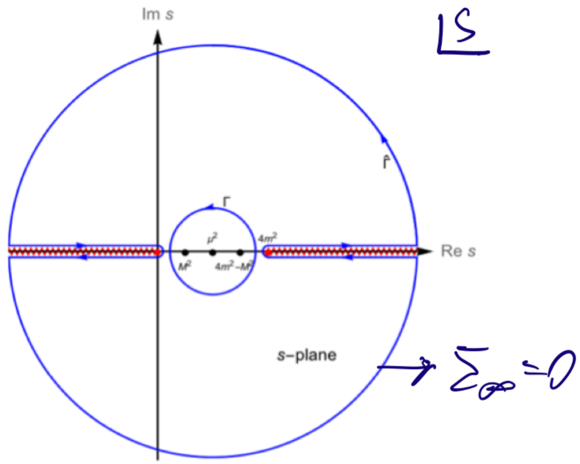
$$A(s) < s \log^2 s$$

2 → 2 amplitude



What is positive in positivity bounds?

Example: forward limit $t = 0$



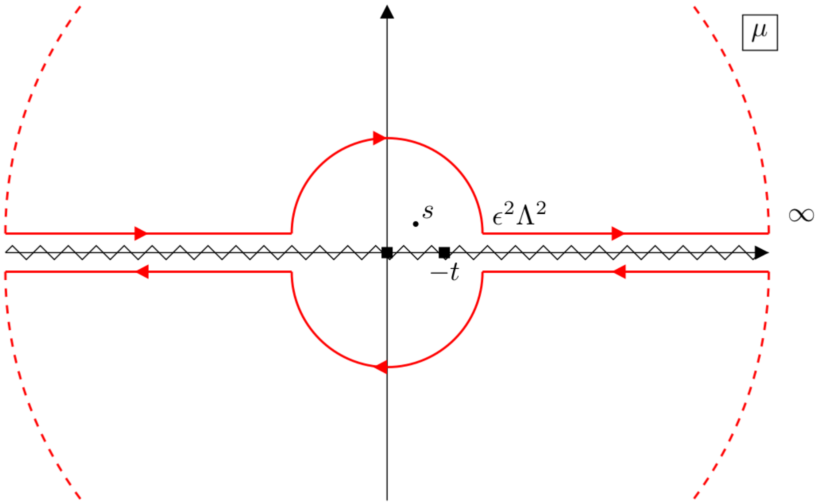
Singularities:

- poles on real axes
- branch cuts

$$\Sigma_{IR} = \frac{1}{2\pi i}\int_{\Gamma} ds \frac{\mathcal{A}(s)}{(s-\mu^2)^3} = \int_{4m^2}^{\infty} \frac{ds}{\pi}\left(\frac{Im\mathcal{A}(s)}{(s-\mu^2)^3} + \frac{Im\mathcal{A}^{+}(s)}{(s-4m^2+\mu^2)^3}\right)$$

$$\Sigma_{IR} = \frac{1}{2}\mathcal{A}''(s) > 0$$

Advance further: non-linear bounds



$$C(n,m) \equiv \frac{n!/2}{2\pi i}\int_{\text{arcs}} d\mu\, \frac{\partial_t^m \mathcal{A}(\mu,0)}{\mu^{n+1}} = \frac{\partial^m}{\partial t^n}\frac{\partial^n}{\partial s^n}\mathcal{A}(s,t)$$

$$I_{s,u}(n,m) \equiv \int_{\epsilon^2\Lambda^2}^{\infty} \frac{d\mu}{\pi}\frac{\partial_t^m \mathrm{Disc}_s \mathcal{A}_{s,u}(\mu,0)}{\mu^{n+1}} > 0$$

Cauchy–Schwarz inequality

$$\left(\sum_{i=1}^n u_i v_i\right)^2 \leq \left(\sum_{i=1}^n u_i^2\right)\left(\sum_{i=1}^n v_i^2\right)$$

$$I_{s,u}(3,0)^2 < I_{s,u}(2,0)I_{s,u}(4,0)$$

$$\frac{4}{3}C(3,0)^2 < C(2,0)C(4,0) \quad \Rightarrow \quad \frac{4}{3}\mathcal{A}_{sss}^2 < \mathcal{A}_{ss}\mathcal{A}_{ssss}$$

many inequalities can be derived!

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M. Carrillo González, C. de Rham, S. Jaitly, V. Pozsgay, and A. Tokareva, (2023), arXiv:2307.04784 [hep-th].

Photon EFT and amplitudes

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ &+\frac{c_1}{\Lambda^4}F^{\mu\nu}F_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}+\frac{c_2}{\Lambda^4}F^{\mu\nu}F^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}\\ &+\frac{c_3}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\mu F_{\beta\gamma}\partial_\nu F_\alpha{}^\gamma+\frac{c_4}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta F_{\mu\gamma}\partial^\gamma F_{\alpha\nu}+\frac{c_5}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta F_{\nu\gamma}\partial^\gamma F_{\alpha\mu}\\ &+\frac{c_6}{\Lambda^8}F^{\mu\nu}\partial_\mu F_{\nu\rho}\partial^\rho\partial^\alpha F^{\beta\gamma}\partial_\alpha F_{\beta\gamma}+\frac{c_7}{\Lambda^8}F^\mu{}_\gamma\partial_\mu F_{\nu\rho}\partial^\nu F_{\alpha\beta}\partial^\rho\partial^\gamma F^{\alpha\beta}\\ &+\frac{c_8}{\Lambda^8}F^{\mu\gamma}\partial_\mu F_{\nu\rho}\partial^\rho\partial^\beta F_{\alpha\gamma}\partial^\alpha F^\nu{}_\beta\,.\end{aligned}$$

$$\begin{aligned}\mathcal{A}_{++++} &= f_2\left(s^2+t^2+u^2\right)+f_3 stu+f_4\left(s^2+t^2+u^2\right)^2\\ \mathcal{A}_{++--} &= g_2 s^2+g_3 s^3+g_4 s^4+g_4' s^2 tu\\ \mathcal{A}_{+++-} &= h_3 stu\end{aligned}$$

$$f_2 = 2\left(4c_1+c_2\right)\,,\quad g_2 = 2\left(4c_1+3c_2\right)$$

$$f_3 = -3\left(c_3+c_4+c_5\right)\,,\quad g_3=-c_5\,,\quad h_3=-\frac{3}{2}c_3\,,$$

$$f_4=\frac{1}{4}c_6\,,\quad g_4=\frac{1}{2}(c_6-c_8)+c_7\,,\quad g_4'=-\frac{1}{2}(c_7+c_8)\,.$$

$$\mathcal{A}_u(s,t,u) = \sum_{h_i}\alpha_{h_1}\beta_{h_2}\alpha^*_{-h_3}\beta^*_{-h_4}\mathcal{A}_{h_1h_4h_3h_2}(s,t,u) = \sum_{h_i}\alpha_{h_1}\beta^*_{-h_2}\alpha^*_{-h_3}\beta_{h_4}\mathcal{A}_{h_1h_2h_3h_4}(s,t,u)$$

$$\alpha_+=\cos\theta\,,\quad \alpha_-=\sin\theta\, e^{i\phi}\,,\quad \beta_+=\cos\chi\,,\quad \beta_-=\sin\chi\, e^{i\psi}\,.$$

Indefinite polarisation scattering

$$\begin{aligned}\mathcal{A}_{\mathrm{ih}} = &\frac{1}{2}(\cos(2\theta)(\mathcal{A}_{++--}-\mathcal{A}_{+--+})\cos(2\chi)+\mathcal{A}_{++--}+4\mathcal{A}_{+---}\sin(\chi)\cos(\chi)\cos(\psi)+\mathcal{A}_{+--+}\\ &+\sin(2\theta)\sin(2\chi)(\mathcal{A}_{++++}\cos(\psi+\phi)+\mathcal{A}_{+-+-}\cos(\phi-\psi))+4\mathcal{A}_{+---}\sin(\theta)\cos(\theta)\cos(\phi))\,.\end{aligned}$$

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Our assumptions

Property	Causality Bounds	Positivity Bounds
Lorentz invariance	• Lorentz invariant EFT	• Invariant EFT and UV completion • Crossing symmetry
Unitarity	• Hermitian Hamiltonian: real Wilson coefficients	• Positive discontinuity of the EFT and UV amplitude
Causality	• No resolvable time advance	• Analyticity of amplitude in the complex s plane for fixed t
Locality	• IR theory is local	• IR and UV theories are local • Froissart-like bound in the UV
Other assumptions	• EFT and WKB expansions under control • Background generated by localized external source	• IR EFT is under perturbative control

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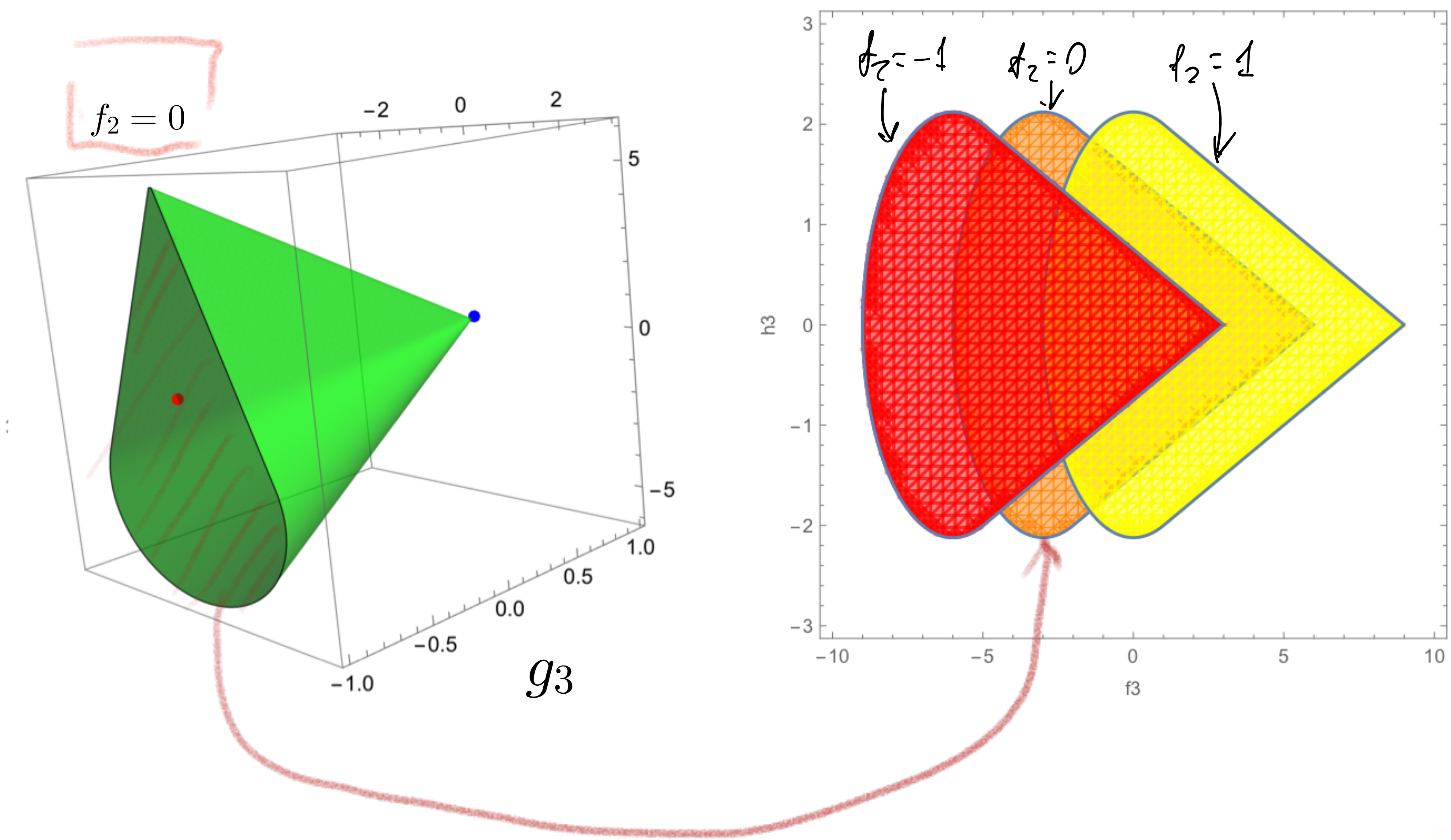
Positivity bounds



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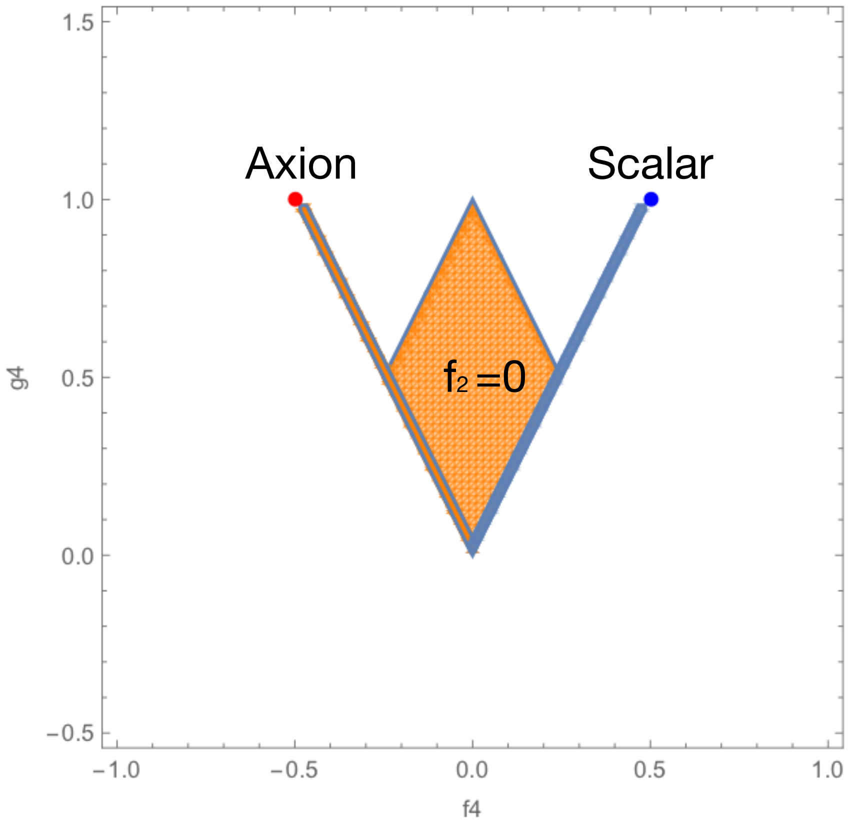
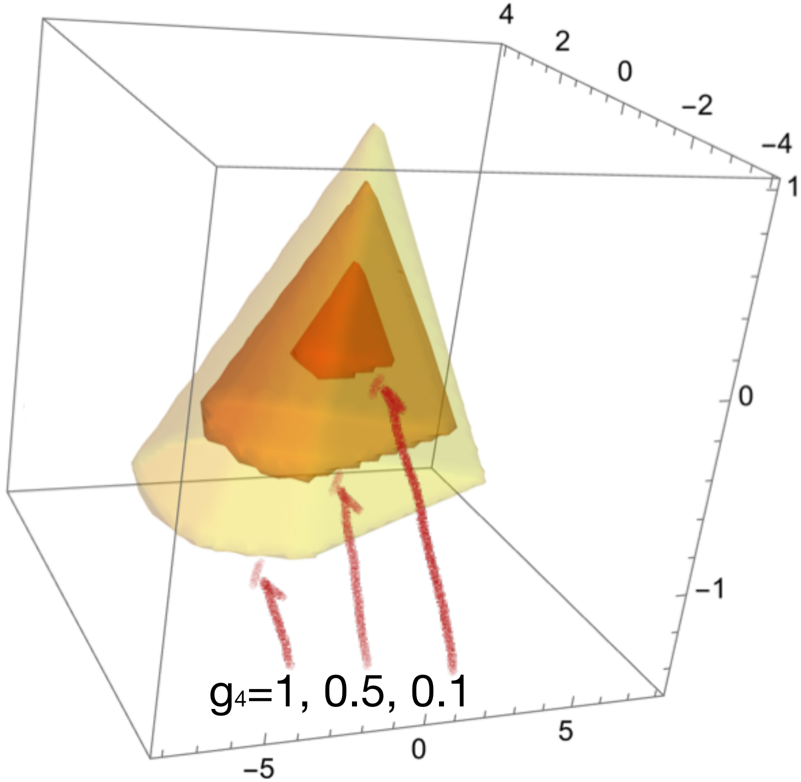
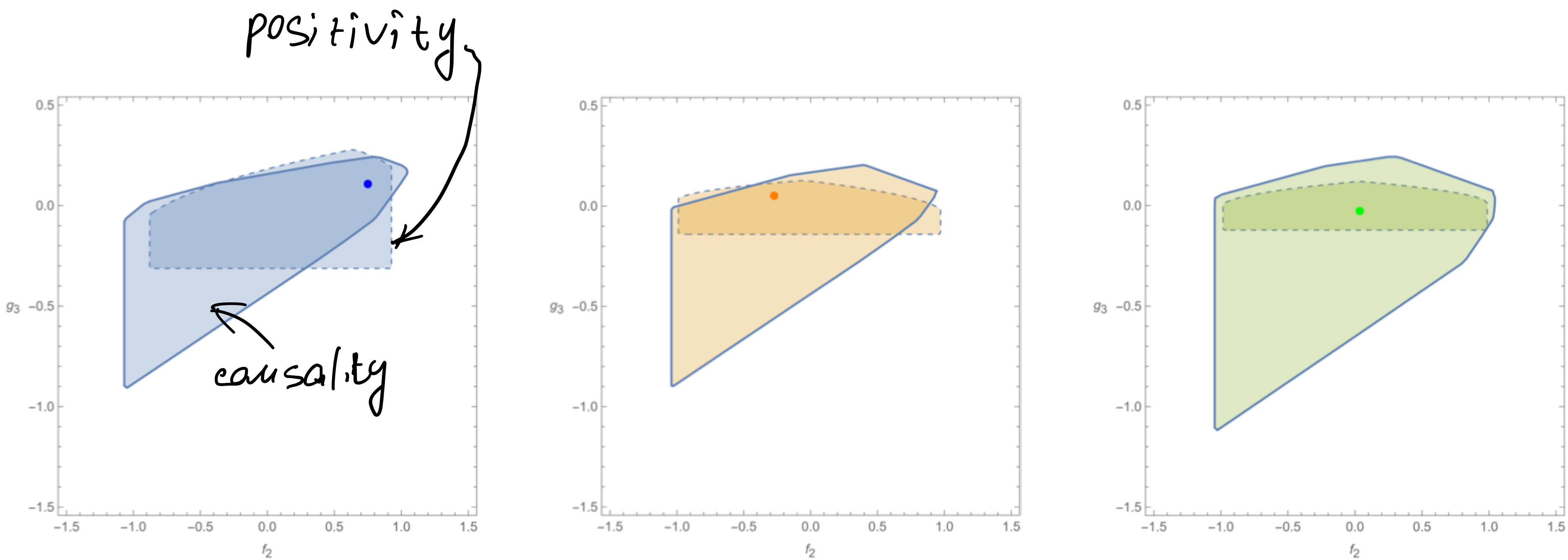
Positivity bounds





Dim 6 operators are squeezed between dim 4 and dim 8

Causality vs positivity plots



(a) Scalar (b) Spinor (c) Vector

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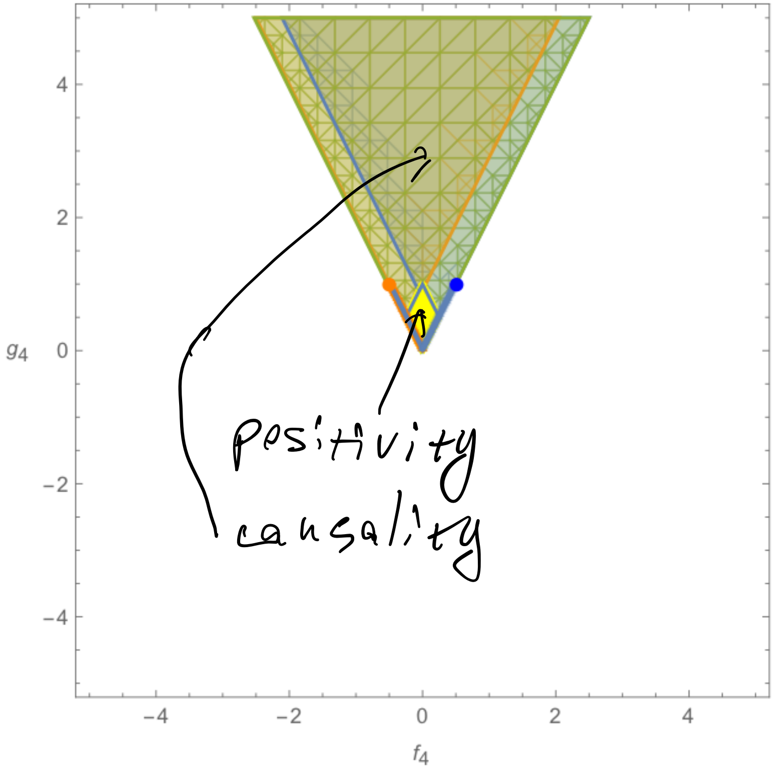
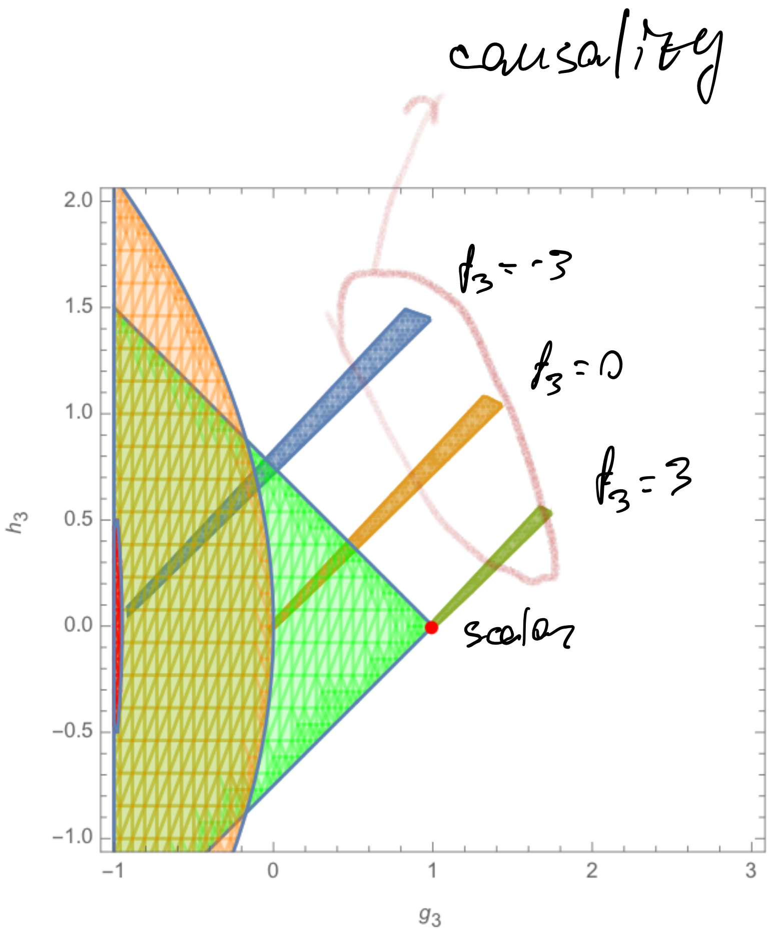
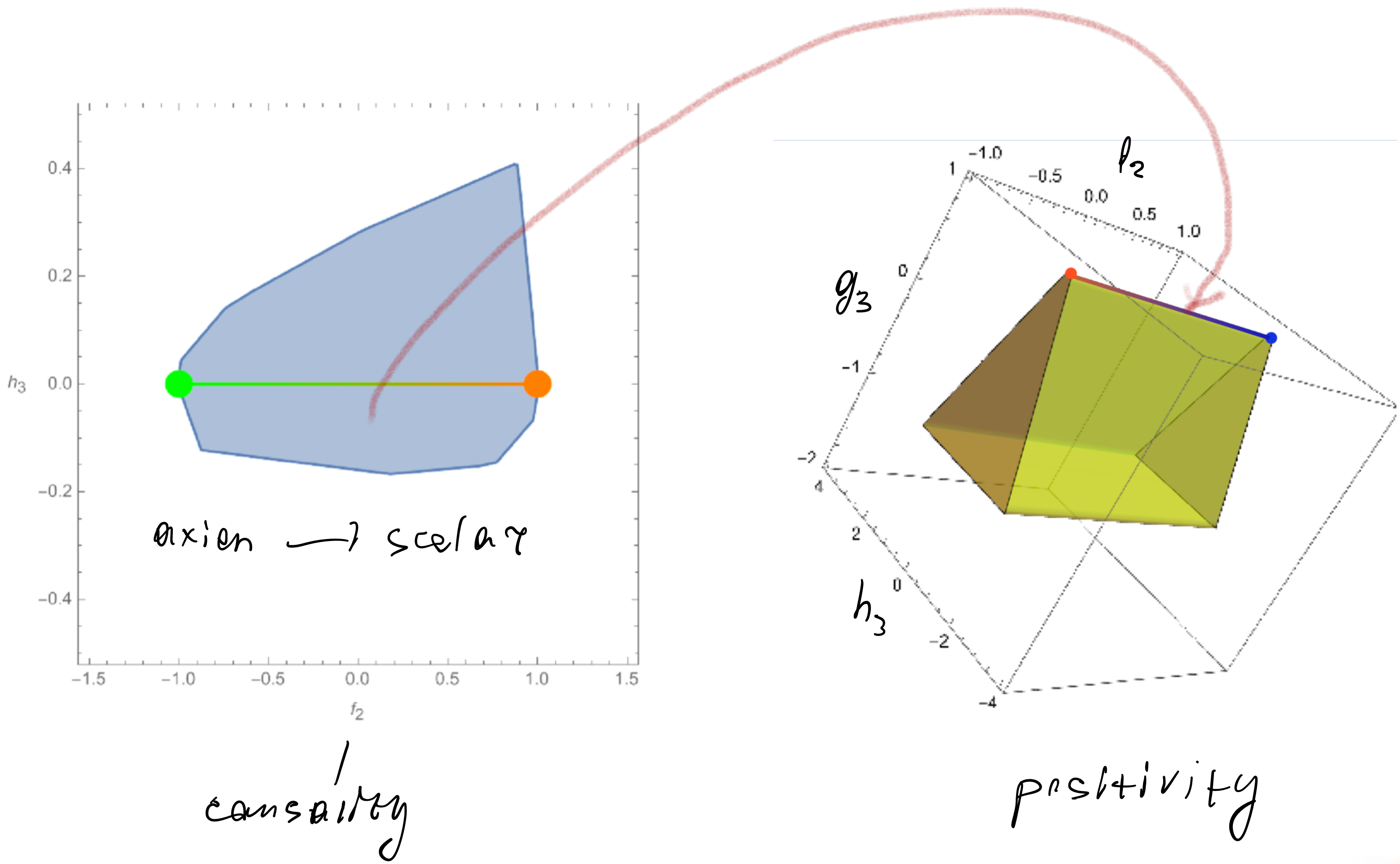
Causality vs positivity plots



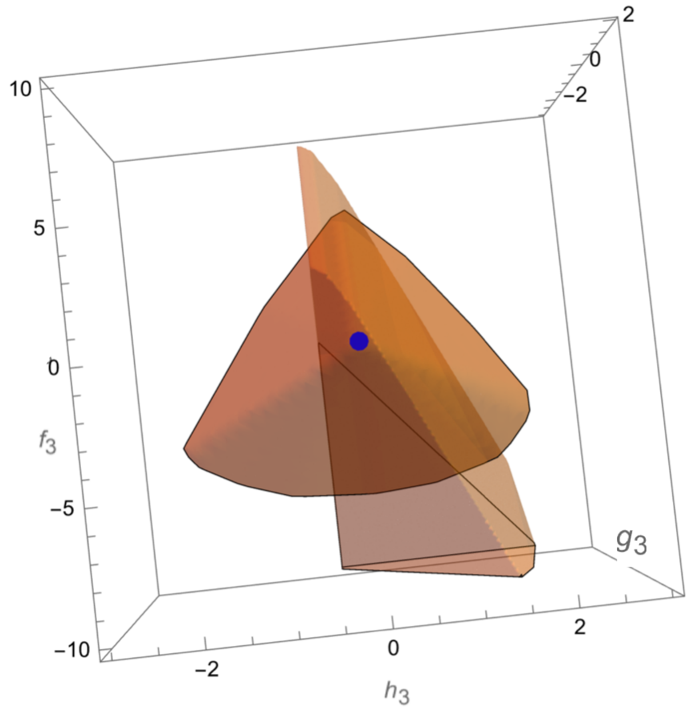


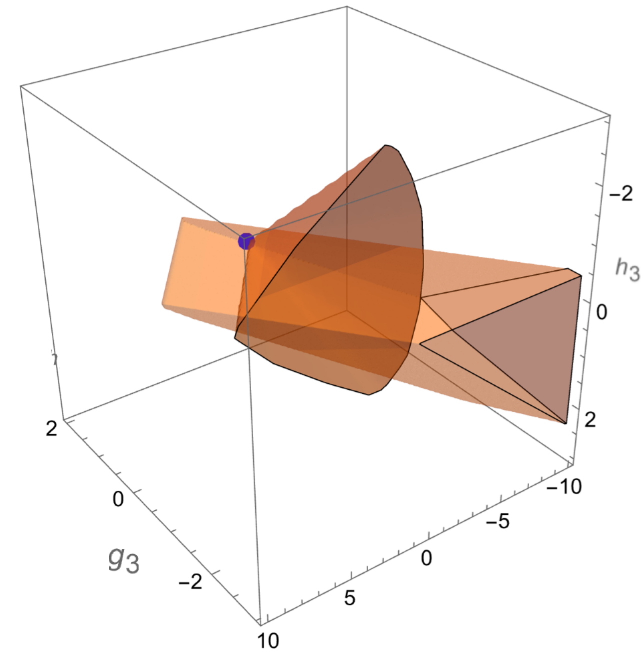
Causality vs positivity plots

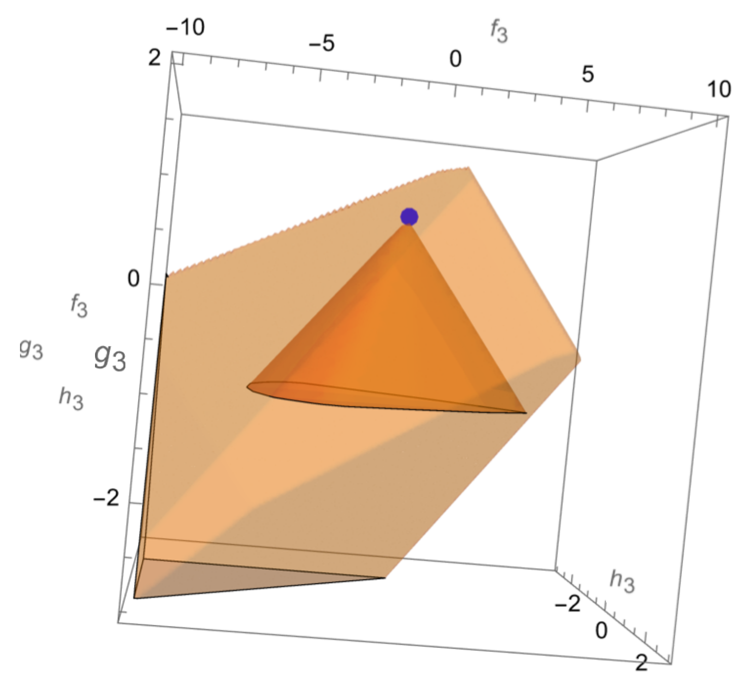


Causality vs positivity plots

$f_2 = f_4 = 0, \; g_4 = 1$







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Conclusions

- Indefinite helicity scalttering provides stronger bounds on EFT of photons. The optimal choice of polarization state may depend on the EFT couplings.
- Causality of the photon propagation is a condition independent of the assumptions about the UV completion - expected to be weaker than unitarity
- For $g_4 - f_4$ couplings positivity is stronger. Causality fails to give a compact bound.
- For $g_3 - h_3 - f_3$ couplings positivity and causality are complementary
- Some regions naively allowed by unitarity correspond to acausal propagation - positivity bounds can be improved further.

Thank you!

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