Real-time path integral for semiclassical description of evaporating black holes

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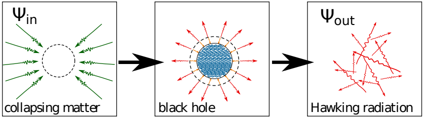
International Conference on Particle Physics and Cosmology

2023 October 2, Yerevan

The Goal: Information loss problem

- Apparent violation of unitarity:



$$\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \;\mapsto\; \hat{\rho}_{out} = \mathrm{Tr}_{BH}\left(|\Psi_{ext}\rangle|\Psi_{BH}\rangle\langle\Psi_{BH}|\langle\Psi_{ext}|\right),$$

$$\mathrm{Tr}(\hat{\rho}^2_{out}) < 1$$

S. W. Hawking, 1976

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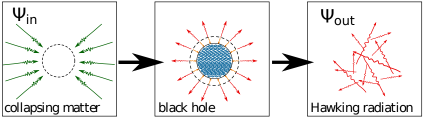

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- Pro-unitary arguments:
  - Holography: gauge/string duality (AdS/CFT)
  - Islands: unitary Page curve.

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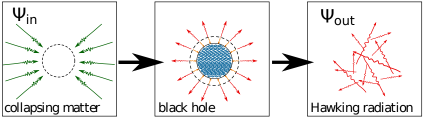

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- Problems?
  - AMPS-firewall: unitarity vs equivalence principle.
  - Dynamics: S-matrix derivation.

ArXiv:gr-qc/9607022 't Hooft

Models

Models w/ linear dilaton vacuum

$$S_{\mathrm{LDV}} = \int d^2x\sqrt{-g}\left(W(\phi)R + W''(\phi)\left((\nabla\phi)^2 + \lambda^2\right)\right) + S^{\mathrm{m}}$$

ArXiv:2005.09479 [hep-th] Banks

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Field equations

$$W'(\phi)R = 2W''(\phi)\Box\phi + W'''(\phi)\left((\nabla\phi)^2 - \lambda^2\right) ,$$
$$g_{\mu\nu}\left(W''(\phi)((\nabla\phi)^2 - \lambda^2) + 2W'(\phi)\Box\phi\right) - 2W'(\phi)\nabla_\mu\nabla_\nu\phi = T^{\mathrm{m}}{}_{\mu\nu} ,$$

where $T^{\mathrm{m}}{}_{\mu\nu} = (-2/\sqrt{-g})\delta S^{\mathrm{m}}/\delta g^{\mu\nu}$.

Vacuum solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} , \qquad \phi = -\lambda r , \qquad f(r) = 1 + \frac{M}{\lambda W'(\phi)}$$

Models

CGHS model

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}(R+4(\nabla\phi)^2+4\lambda^2) - \frac{1}{2}(\nabla f)^2\right]$$

ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

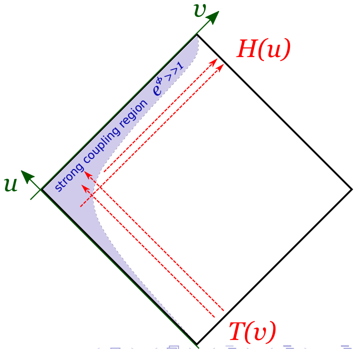
In the bulk:

$ds^2 = -e^{2\phi}dvdu,$

$f(v,u) = f_{out}(u) + f_{in}(v)$

$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$

$\partial_v^2\mathcal{T} = (\partial_v f_{in})^2/2$, $\partial_u^2\mathcal{H} = (\partial_u f_{out})^2/2$



Models

CGHS model w/ dynamical boundary $\phi = \phi_0$

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}(R+4(\nabla\phi)^2+4\lambda^2) - \frac{1}{2}(\nabla f)^2\right] + 2\int_{\partial\mathcal{M}} d\tau e^{-2\phi}(K+2\lambda)$$

ArXiv:1702.02576 [hep-th] M.F., D. Levkov, Y. Zenkevich, 2017

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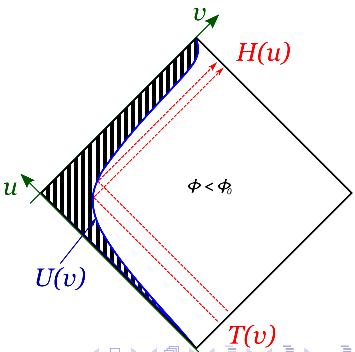
$\partial_v^2\mathcal{T} = (\partial_v f_{in})^2/2,\ \partial_u^2\mathcal{H} = (\partial_u f_{out})^2/2$

On the boundary:

Reflecting condition $f_{out}(U(v)) = f_{in}(v)$

Weak coupling: $g_{gr} = e^{\phi} \leq e^{\phi_0} \ll 1$

Minimal BH mass $M_{cr} = 2\lambda e^{-2\phi_0}$



Models

Sinh-CGHS model

$$S_{\sinh} = -2 \int d^2x \sqrt{-g}\, \sinh(2\phi) \left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)$$

ArXiv:2202.00023 [gr-qc] M.F.

Vacuum solution w/ metric function (fig. a)

$$f(r) = 1 - \frac{M}{4\lambda \cosh(2\lambda r)}$$

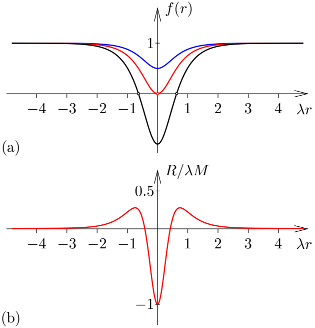
Ricci scalar (fig. b) $R = -\partial_r^2 f(r)$ is finite everywhere. Non-singular black holes:

- Limiting curvature ${R_{\mu\nu\rho\sigma}}^2 < \Lambda^2$.

  Markov, 2111.14318 [gr-qc] Frolov ...

- Other models: Bardeen's black hole, black bounces, planck stars...

  1812.07114 [gr-qc] Visser, 1802.04264 [gr-qc] Rovelli...



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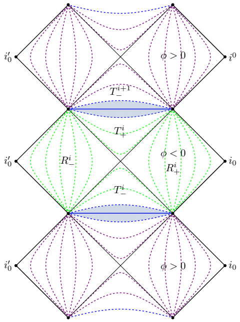
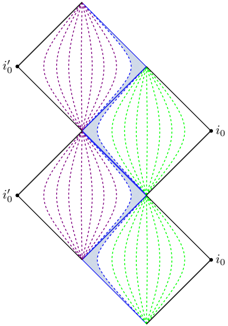
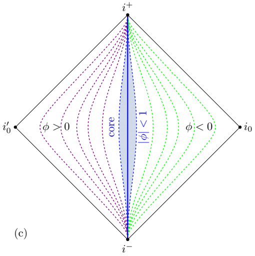
Non-extremal black hole
$M > M_{ext}$



Extremal black hole
$M = M_{ext}$



Gravitational kink
$M < M_{ext}$



Thermodynamic properties

Euclidean solution

$$ds_E{}^2 = f(r)dt_E{}^2 + \frac{dr^2}{f(r)}\,, \qquad 0 \le t_E < \beta_H\,,$$

has imaginary time period

$$\beta_H = T_H{}^{-1} = 4\pi / f'(r_\mathrm{h})$$

$\Leftarrow$ no conifold singularity at $r = r_\mathrm{h}$.
Derive black hole temperature and entropy

$$\boxed{T_H = \frac{\lambda^2 W''(\phi_\mathrm{h})}{4\pi M}}$$

$$\boxed{S_\mathrm{BH}(M) = 4\pi W(\phi_\mathrm{h}) - 4\pi W(\phi_\mathrm{h,\,ext})}$$

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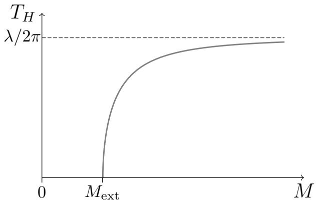
$$S_{BH} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M_\mathrm{ext}^2}{M^2}}$$

$$T_H = \frac{\lambda}{2\pi} \sqrt{1 - \frac{M_\mathrm{ext}^2}{M^2}}$$

Sinh-CGHS reduces to CGHS in $M_\mathrm{ext} \to 0$.

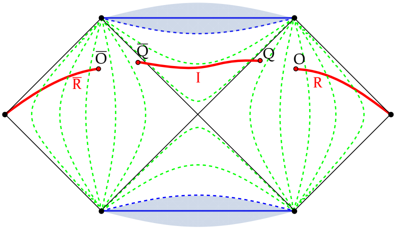


Entropy from entanglement island

Island formula for black hole entropy

$$S_{\mathrm{gen}}[R] = \min_{I} \operatorname*{ext}_{\partial I} \left(S_{\mathrm{grav}}[\partial I] + S_{\mathrm{ent}}[R \cup I] \right)$$

Ref. D. Ageev's talk at Friday.



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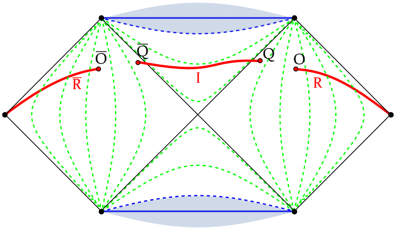

For linear dilaton gravity

$$S_{\rm gen} = 8\pi W(-\lambda r_Q) + \frac{N}{3}\log(\epsilon^{-2}(v_O - v_Q)(u_Q - u_O)) + \frac{N}{3}(\rho_O + \rho_Q)$$

where ρ is conformal factor: $ds^2 = -e^{2\rho} dv du$. Vary $S_{\rm gen}$ with respect to t_Q and r_Q.

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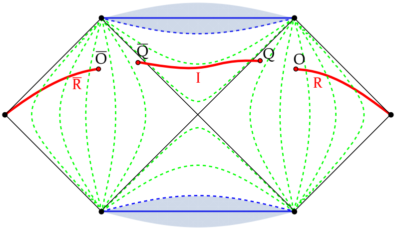

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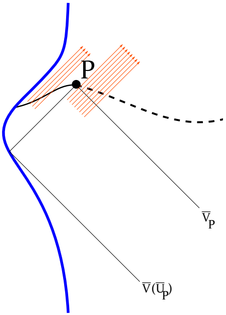
Numerically for sinh-CGHS:

$$S_{gen} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M_{\rm ext}^2}{M^2}} + O\left(N \log \frac{M - M_{ext}}{\lambda}\right)$$

Diverges at $M \to M_{ext}$.

Entanglement entropy on regular geometry

Consider CGHS w/ boundary; coordinates in $ds^2 = -e^{2\rho} d\bar{v}\bar{u}$ are flat at infinity: $\rho(\mathcal{I}^{\pm}) = 0$.



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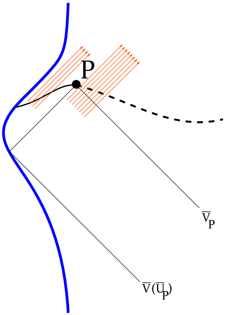
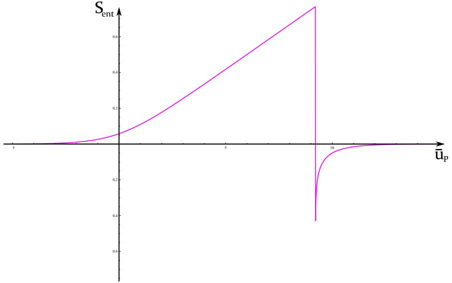


$$S_{ent} = \frac{N}{6}\rho_P + \frac{N}{6}\log(\epsilon^{-1}(\bar{v}_P - \bar{v}(\bar{u}_P))) - \frac{N}{12}\log\left(\frac{d\bar{v}}{d\bar{u}}(\bar{u}_P)\right)$$



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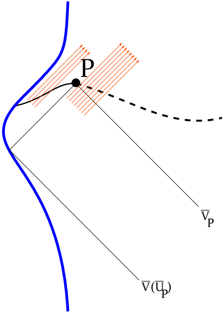

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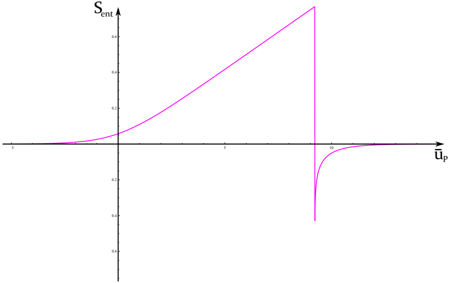

Unitarity entropy bound $S \leq O(\mathrm{Area})$

ArXiv:2003.05546 [hep-th] G. Dvali, 2020 (also plenary talk tomorrow)

Law of evaporation

2D Stefan–Boltzmann

$$\frac{dM}{dt} = -\frac{\pi}{12} T_H^2(M)$$

$\Rightarrow$ averaged mass function

$$M(t) + \frac{M_{\rm ext}}{2} \log\left(\frac{M(t) - M_{\rm ext}}{M(t) + M_{\rm ext}}\right) = M_0 - \frac{\lambda^2 t}{48\pi}$$

with initial value $M_0 \gg M_{\rm ext}$.

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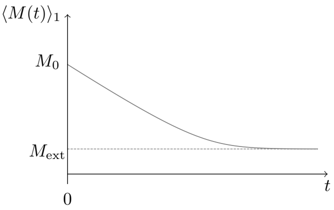

Remnants formation

2D Stefan–Boltzmann

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$\Rightarrow$ asymptotically

$$M \simeq M_{\text{ext}} \left(1 + \exp\left(-\frac{\lambda^2 t}{24\pi M_{\text{ext}}}\right)\right)$$

i.e. remnant is formed.

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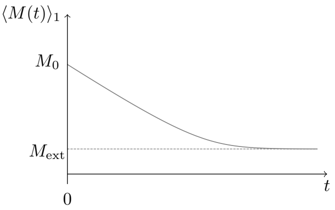

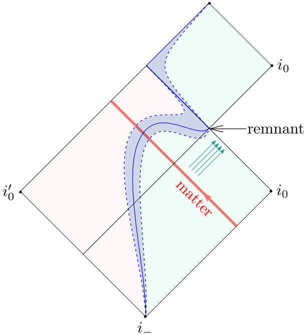

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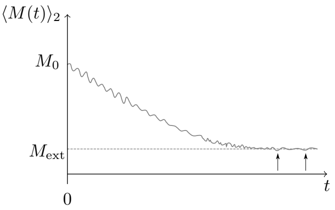
Fluctuations of Hawking flux

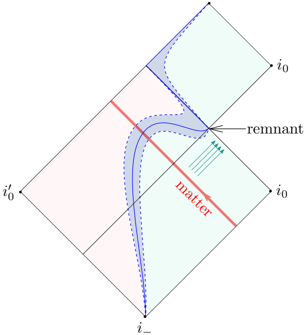
$$\langle : \Delta \hat{T}_{tr} : \rangle = O(1) \langle : \hat{T}_{tr} : \rangle$$

on timescale $O(M)$

gr-qc/9905012 Wu, Ford

w/ q/thermal noise:





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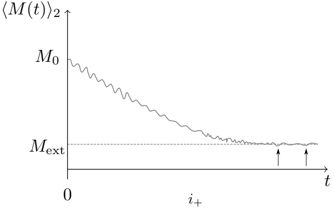

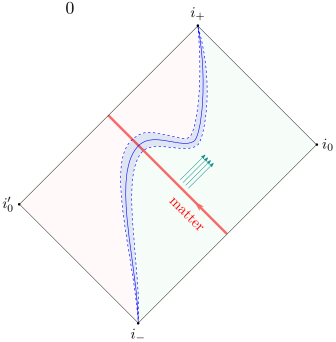

Remnants ~~formation~~ decay

From fluctuations theory

$$\langle (\Delta M)^2 \rangle = -\frac{\partial \langle E \rangle}{\partial \beta} \simeq \frac{\lambda^2}{M_{\rm ext}} O(1)$$

assuming $\Delta M \ll M_{\rm ext}$.

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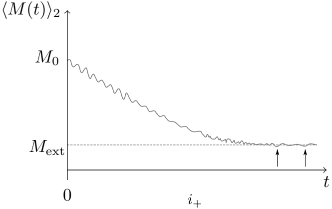

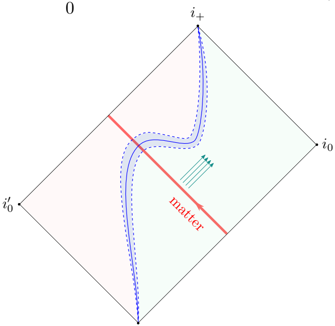

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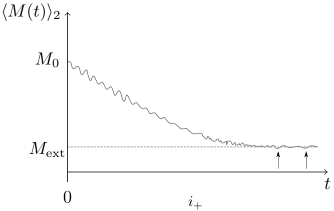
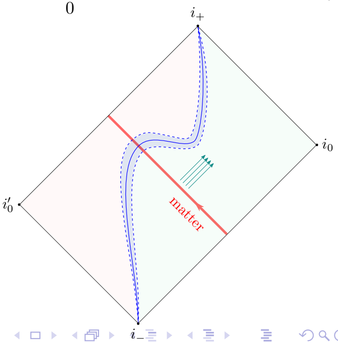
Thermal estimate

$$t_{\rm dec} \simeq 48\pi\frac{M_{\rm ext}}{\lambda^2}\log\left(\frac{M_{\rm ext}}{\lambda}\right)$$

w/ q/thermal noise:





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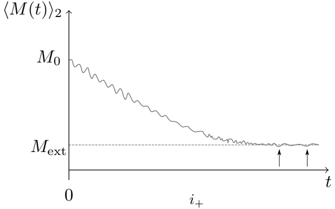
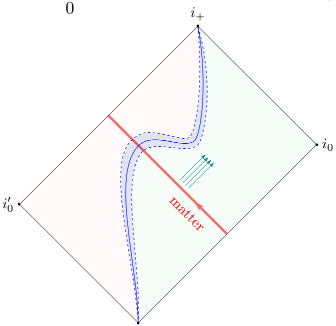
Adiabaticity condition

$$T\frac{\partial T}{\partial M} \ll T \;\Rightarrow\; T\frac{\partial S}{\partial T} \gg 1 \;\Rightarrow\; T \gg \frac{\lambda^2}{4\pi^2 M_{ext}}$$

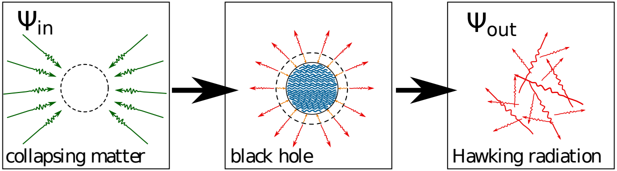
We need quantum treatment of remnant

w/ q/thermal noise:





S-matrix from path integral



$$\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle = \int \mathcal{D}\Phi \, \Psi^*_{out} \Psi_{in} \exp\{\frac{i}{\hbar} S[\Phi]\} \,, \quad \Phi = \{g_{\mu\nu}, \, \phi, \, f\}$$

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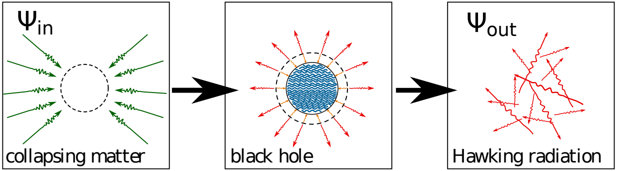

$$\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle = \int \mathcal{D}\Phi \, \Psi^*_{out} \Psi_{in} \exp\{\frac{i}{\hbar} S[\Phi]\} \,, \quad \Phi = \{g_{\mu\nu}, \phi, f\}$$

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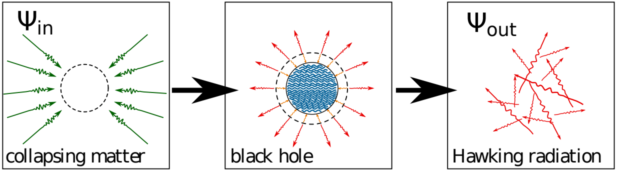

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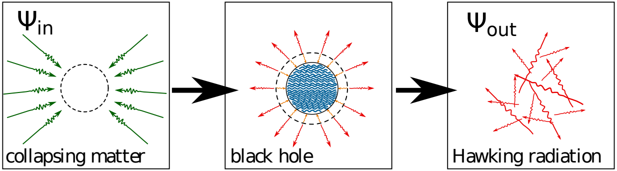

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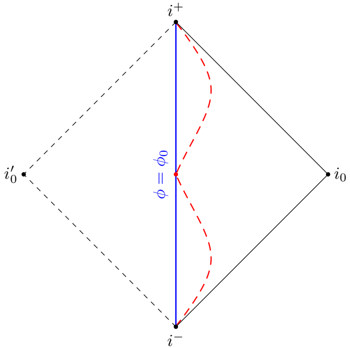
  - Problem: complexification of spacetime is ambiguous.
- Non-singular model can help!

Example: Point-particle scattering amplitude

CGHS w/ boundary ϕ_0 and matter action $S_m = -m \int d\tau$. EOM follows from Israel condition

$$\left(\frac{dr}{d\tau}\right)^2 + V_{eff}(r) = 0$$

where $V_{eff}(r) = 1 - \left(\frac{M}{m} + \frac{m}{8\lambda} e^{-2\lambda r}\right)^2$



ArXiv: 2006.03606 M.F., D. Levkov, S. Sibiryakov, 2020

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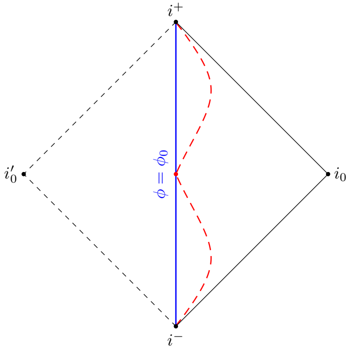

Semiclassical scattering amplitude

$$\mathcal{A}_{fi} = \langle \Psi_f | \hat{U} | \Psi_i \rangle = \int \mathcal{D}\Phi \, \Psi_f^*[\Phi] \Psi_i[\Phi] e^{iS'[\Phi]} \simeq F \cdot e^{iS_{\text{tot}}}$$

where $S_{tot} = S(t_f, t_i) + S_0(t_i, 0_-) + S_0(0_+, t_f) - i \ln \Psi_i - i \ln \Psi_f^*$

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How to finds complex trajectory. Let functional T_{int} has properties:

- diff. invariant
- positive–definite for real solutions
- diverges for solutions with eternal black hole and finite for asy. flat

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Inserting into path integral a unity

$$1 = \int_0^{+\infty} dT_0 \, \delta(T_{\rm int}[\Phi] - T_0) = \int_0^{+\infty} dT_0 \int_{-i\infty}^{+i\infty} \frac{d\varepsilon}{2\pi i} \, e^{-\varepsilon(T_{\rm int} - T_0)}$$

is equivalent to complexifying the acton:

$$S_\varepsilon[\Phi] = S[\Phi] + i\varepsilon \, T_{\rm int}[\Phi] - i\varepsilon \, T_0$$

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Result: T-functional shifts mass $\boxed{M \;\mapsto\; M + i\varepsilon}$.

Tested on models with collapsing shells.

ArXiv:1503.07181, F. Bezrukov, D. Levkov, S. Sibiryakov, 2015

Consistent with Hamiltonian methods.

ArXiv: 9907001 [hep-th] M. Parikh, F. Wilczek

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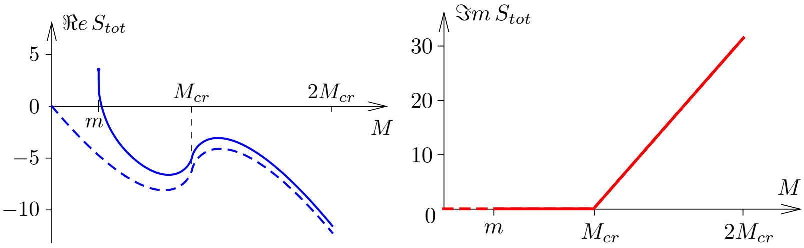

For massless particle

$$S_{\text{tot}} = -\frac{M - M_{\text{cr}}}{\lambda} \log\left(1 - \frac{M + i\varepsilon}{M_{\text{cr}}}\right) + \frac{M}{\lambda}\left(1 - \log\frac{M_{\text{cr}}}{2\lambda}\right)$$

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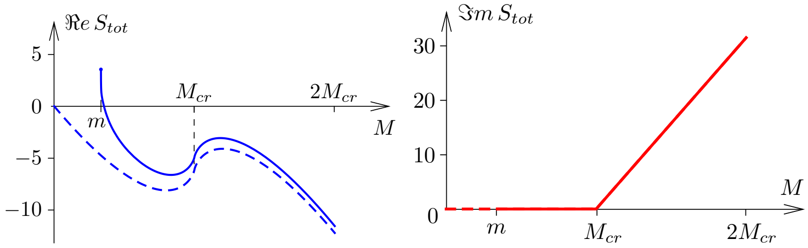

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transition probability $\mathcal{P}_{fi} = |\mathcal{A}_{fi}|^2 \approx \exp\left(-2\Im m\, S_{tot}\right) = \exp\left(-S_{BH}\right)$

where $\boxed{S_{BH} = \frac{2\pi}{\lambda}(M - M_{cr})}$ - entropy in model with stiff boundary.

ArXiv: 2006.03606 M.F., D. Levkov, S. Sibiryakov, 2020

Field S-matrix for gravitational scattering

Coherent state formalism. $\hat{a}_k|a\rangle = a_k|a\rangle$, where $\hat{a}_k$ - annihilation operator.
S-matrix elements

$$\langle b|S|a\rangle = \int \mathcal{D}\{f_{out}, f_{in}, f, \phi, g\}\ \langle b|f_{out}\rangle e^{iS[\Phi]}\langle f_{in}|a\rangle$$

can be computed semiclassically.

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Multiparticle scattering $A_{2\to\text{many}}$ may be unsuppressed (T/θ boundary problem)

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Ref. B. Farkhtdinov's after next talk

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But we failed: CGHS model has exact solutions, but S-matrix functional diverges at $E > E_{thr.}$.
Next attempt: sinh-CGHS - not as solvable as CGHS /w ϕ_0 but numerically tractable at least.

Test of unitarity

Check explicitly $1 = \mathcal{S}^\dagger \mathcal{S}$ or in coherent states basis

$$e^{\int dk a_k^* b_k} = \int \mathcal{D}c_k^* \mathcal{D}c_k \; e^{-\int dk c_k^* c_k} \langle a|\mathcal{S}^\dagger|c\rangle \langle c|\mathcal{S}|b\rangle$$

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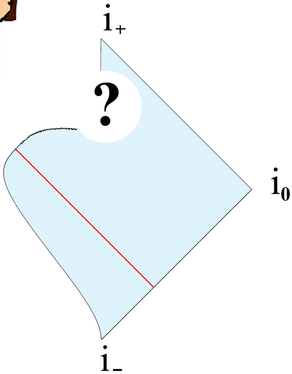
4. 4-derivative scalar theory $\mathcal{L} = \phi\Box^2\phi + g(\partial\phi)^4$ - non-unitary

ArXiv:2212.10599 [hep-th], A. Tseytlin, 2023

Conclusion

- We introduced linear dilaton models for studying gravitational S-matrix:
  - CGHS with dynamical boundary;
  - sinh-CGHS with regular black holes.
- We studied thermodynamics properties and remnant scenario in regular model.
- We proposed a semiclassical path integral method for calculating S-matrix elements and calculated scattering amplitude for point-like particle which is consistent with unitarity.
- New model awaits!



Շնորհակալություն

ε-regularization for dilaton gravity

We choose explicitly

$$T_{\text{int}} = \int d^2x\sqrt{-g}\,\frac{f(\phi)}{\lambda^2}(\lambda^2 - (\partial_\mu\phi)^2)^2$$

where $f(\phi(r))$ has support on $r \gg r_0$

The metric has form $ds^2 = -e^{\nu(r)}dt^2 + e^{\zeta(r)}dr^2$ and $\phi = -\lambda r$, complexified field eqiations, e.g.

$$\partial_r\left(1 - e^{-\zeta}\right) + 2\lambda\left(1 - e^{-\zeta}\right) + \frac{i\varepsilon\lambda}{2}f(-\lambda r)e^{-2\lambda r}\left(1 - e^{-\zeta}\right)^2 = 0\ ,$$

have solution

$$1 - e^{-\zeta(r)} = \frac{M}{2\lambda}e^{-2\lambda r}\left(1 - \frac{i\varepsilon M}{4\lambda}\int_{-\infty}^{\phi(r)} d\phi\, f(\phi)\right)^{-1}$$

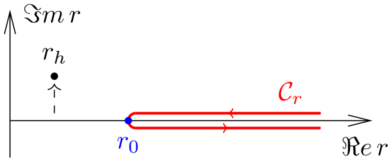
We see that inserting iT_{int} is equivalent to imaginary shift $M \mapsto M + i\varepsilon$.

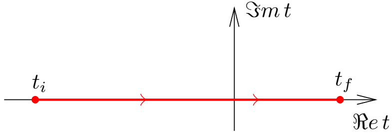
How to deform integration contour

$$t(r) = \int dr \frac{\sqrt{f(r) - V_{eff}(r)}}{f(r)\dot{r}_*(r)} \, ,$$

$$\dot{r}_*(r) = \mp\sqrt{-V_{eff}(r)}$$

$M < M_{\rm cr}$





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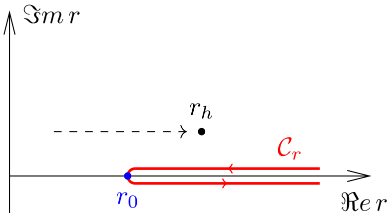
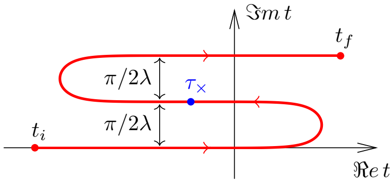




$$\Im m(t_f - t_i) = 2\pi \mathrm{Res} \frac{1}{f(r{=}r_h)} = \frac{\pi}{\lambda}$$

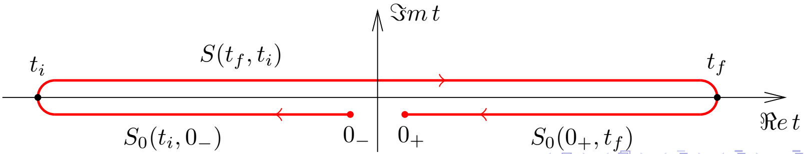
Calculation of total action S_{tot}

Semiclassical scattering amplitude

$$\mathcal{A}_{fi} = \langle \Psi_f | \hat{U} | \Psi_i \rangle = \int \mathcal{D}\Phi \, \Psi_f^*[\Phi] \Psi_i[\Phi] e^{iS'[\Phi]} \simeq F \cdot e^{iS_{\text{tot}}}$$

where $S_{tot} = S(t_f, t_i) + S_0(t_i, 0_-) + S_0(0_+, t_f) - i \ln \Psi_i - i \ln \Psi_f^*$

1. $S(t_f, t_i)$ - interacting action
   - $S_{CGHS}[g, \phi]$ - dilaton field and metric
   - $S_m = -m \int ds$ - point particle
   - $S_{GH} = 2\kappa \int d\sigma \, e^{-2\phi}(K - K_0)$ - Gibbons-Hawking term
2. S_0 - free evolution $\hat{S} = \hat{U}_0 \hat{U} \hat{U}_0 \Big|_{-\infty}^{+\infty}$
3. $\Psi_{i,f} \approx e^{ipr}$ - particle wave functions (in- and out-states)



Calculation of total action S_{tot}

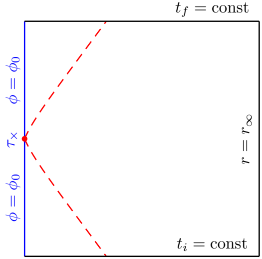
Gravitational part

- CGHS action

$$S_{\mathrm{CGHS}} = 2 \int d^2x \sqrt{-g}\, \Box e^{-2\phi}$$

- Gibbons-Hawking action

$$S_{GH} = 2\kappa \int d\sigma\, e^{-2\phi}(K - K_0)$$



- $K_0 = 2\lambda$, $\kappa = 1$ at $r \to +\infty$
- $K_0 = 0$, $\kappa = -1$ at $t \to \pm\infty$

Field equations of motion $\Rightarrow$ $S_{gr} = 2\kappa \oint d\sigma\, e^{-2\phi_0} K$

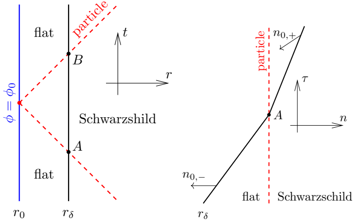
Calculation of total action S_{tot}

- Boundary $\phi = \phi_0$

$$S_{\phi_0} = 2e^{-2\phi_0} \int\limits_{\phi=\phi_0} d\tau\, K$$

$$(n^\tau, n^n) = (-\mathrm{sh}\psi(\tau), -\mathrm{ch}\psi(\tau))$$

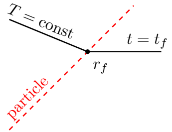
$$\int\limits_{\phi=\phi_0} d\tau\, K = \psi_+ - \psi_-$$



$$K = 2\delta(\tau - \tau_0)\left(\mathrm{arsh}\sqrt{-V_{\mathrm{eff}}(r_0)} - \mathrm{arsh}\sqrt{-V_{\mathrm{eff}}(r_0)/f(r_0)}\right)$$

- Cauchy surfaces $t = t_{f,i}$

$$S_{t_f} = -2\int d\sigma\, e^{-2\phi} K$$



$$S_{t_f} = S_{t_i} \simeq \frac{p}{2\lambda}\,, \qquad p = \sqrt{M^2 - m^2}$$

Calculation of total action S_{tot}

- Point particle action $S_m = -m \left[\int_{r_0}^{r_i} + \int_{r_0}^{r_f} \right] \frac{dr}{\sqrt{-V_{\rm eff}(r)}}$

$$S_m = \frac{m^2}{\lambda p} \ln \left[\frac{1}{2} + \frac{Mm^2}{8M_{\rm cr}p^2} + \frac{p_0}{2p} \right] - \frac{m^2(r_i + r_f - 2r_0)}{p}$$

- Contributions from in- and out- states

$$\Psi_{f,i} = \exp(\mp ipr_{\mp})$$

- Free point particle action $S_{m,0}$

$$S_0(t_i, 0_-) = p(r_- - r_i) - Mt_i \ , \qquad S_0(0_+, t_f) = p(r_+ - r_f) + Mt_f$$

$$t_f - t_i = \frac{M(r_i + r_f - 2r_0)}{p} + \dots$$

Calculation of total action S_{tot}

The result

$$S_{\rm tot} = -\frac{M - M_{\rm cr}}{\lambda} \ln\left(1 - \frac{M + i\varepsilon}{M_{\rm cr}}\right) + \frac{p}{\lambda}\left(1 - \ln\frac{M_{\rm cr}}{2\lambda}\right) +$$

$$-\frac{p}{\lambda}\ln\left(\frac{1}{2} + \frac{Mm^2}{8M_{\rm cr}p^2} + \frac{p_0}{2p}\right) + \frac{2M_{\rm cr}}{\lambda}\ln\left(\frac{4M_{\rm cr}(p_0 + M) + m^2}{4M_{\rm cr}(p_0 + M) - m^2}\right) +$$

$$+\frac{M}{\lambda}\ln\left(\frac{4M^3 - 3m^2M + (4M^2 - m^2)p_0}{(p + M)^3} + \frac{m^2(4M^2 + m^2)}{4M_{\rm cr}(p + M)^3)}\right) ,$$

where $p_0 = \sqrt{(M + m^2/4M_{\rm cr})^2 - m^2}$.

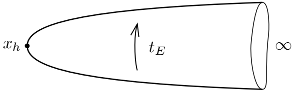
- The part of action which survives in the limit $m \to 0$ has color.
- Imaginary part of whole action

$$\Im m\, S_{\rm tot} = \frac{\pi}{\lambda}(M - M_{\rm cr})\theta(M - M_{\rm cr})$$

contributes to suppression exponent for tunnelling probability.

Euclidean entropy calculation (naive approach)

Euclidean black hole exterior



ignores $\phi = \phi_0$.

- Wick rotate $t \mapsto t_E = it \Rightarrow$ $ds_E^2 = f(r)dt_E^2 + \frac{dr^2}{f(r)}$
- The curvature $R = 4\pi(1-\beta T_H)\frac{\delta^2(x-x_{hor})}{\sqrt{g}} + 2\lambda M e^{-2\lambda r}$ is regular at $x = x_{hor}$ if $\beta = 2\pi/\lambda$. Therefore, $S_E[\Phi_s] = M\beta - M\beta_H$.

Gibbons–Hawking partition functional

$$\boxed{\mathcal{Z}(\beta) := \int_{\Phi[t_E]=\Phi[t_E+\beta]} \mathcal{D}\Phi \, e^{-S_E[\Phi]}} .$$

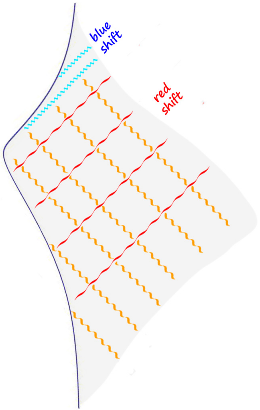
Free energy $F(\beta) := -\frac{1}{\beta}\ln \mathcal{Z}(\beta) \simeq \frac{1}{\beta}S_E(\beta)$

$$\Rightarrow \text{entropy } \Sigma_{BH} = \beta^2 \frac{\partial F(\beta)}{\partial \beta} = \beta\frac{\partial S_E(\beta)}{\partial \beta} - S_E(\beta) \Rightarrow \boxed{\tilde{\Sigma}_{BH} = \frac{M}{T_H} = \frac{2\pi}{\lambda}M}.$$

Parikh and Wilczek: $\mathcal{P}_{fi} \simeq e^{-\Sigma_{BH}} \Rightarrow \mathcal{P}_{fi}(M)$ has discontinuity at $M = M_{cr}$ (it's unphysical).

Euclidean entropy calculation

Argument against $\tilde{\Sigma}_{BH}$.



Consider critical collapse of thermal gas

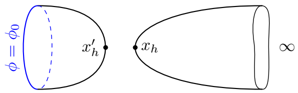
1. т.к. $\mathcal{L}_{\text{grav.}} \sim e^{2\lambda r} \Rightarrow$ decays at distance$\sim \lambda^{-1}$
2. $\Sigma_{\text{gas}} \leq 2M_{\text{cr}}/T_{\text{gas}}, \; T_{\text{gas}} = \sqrt{6\rho_{\text{gas}}/\pi}$;
3. $\rho_{\text{gas}} \simeq M_{\text{cr}}\lambda$;
4. $\Sigma_{\text{gas}} \lesssim e^{-\phi_0} \ll \tilde{\Sigma}_{BH}(M_{cr}) = 4\pi e^{-2\phi_0}$

Critical black hole entropy is parametrically small. Does not match with naive answer.

Euclidean entropy calculation

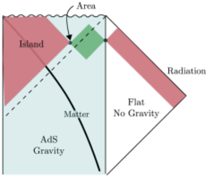
Corrected.

$(-,-)$ $(+,+)$



M' M

Analogy w/ replica wormholes?



- boundary $\phi = \phi_0$ should persist in path integral
- since no regular saddle with boundary

$$\mathcal{D}\Lambda\, e^{-\int d\tau\, \Lambda(\tau)(\phi(x_*(\tau))-\phi_0)}$$

$$S_E(M', M) = M\beta - M\beta_H + M'\beta_H \,,$$

$$S_E(M) = \min_{M'} S_E(M', M) \quad \Rightarrow \quad M' = M_{cr} \,.$$

Corrected answer is consistent with point-particle scattering amplitude

$$\boxed{\Sigma_{BH} = \frac{M}{T_H} = \frac{2\pi}{\lambda}(M - M_{\rm cr})}$$

Black bounce in sinh-CGHS

Coordinate extension:

$$g(r) = \frac{\left(1+\frac{M_{\rm ext}}{M}\right)\tanh(\lambda r) - \frac{2\pi T_H}{\lambda}}{\left(1+\frac{M_{\rm ext}}{M}\right)\tanh(\lambda r) + \frac{2\pi T_H}{\lambda}} e^{4\pi T_H r}$$

$$T = \sqrt{g(r)}\sinh(2\pi T_H t)$$

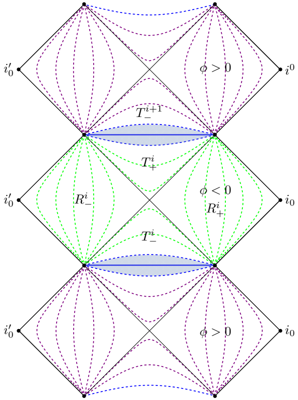
$$R = \sqrt{g(r)}\cosh(2\pi T_H t)$$

Metric takes a form

$$ds^2 = \frac{f(r)}{4\pi^2 {T_H}^2 g(r)}\left(-dT^2 + dR^2\right)$$

Maps $(V_i, U_i) = (T_i + R_i, T_i - R_i)$ are identified by

$$V_{i+1} = -\kappa/V_i\ , \qquad U_{i+1} = -1/\kappa U_i$$



(a)

Collapsing wave packet

Example: conformal matter

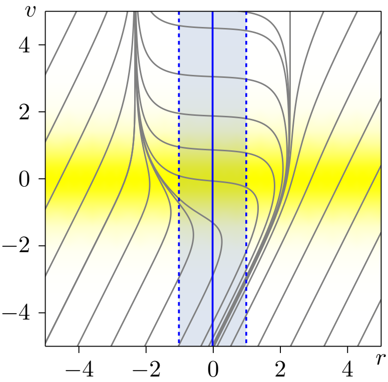
$$T_{\mathrm{m}\,\mu\nu} = \nabla_\mu f \nabla_\nu f - \frac{1}{2} g_{\mu\nu} (\nabla f)^2 \,,$$

Vaidya ansatz $ds^2 = -F(v,r)dv^2 + 2dvdr$ with incident wavepacket $f(v)$ has solution

$$F(v,r) = \left(1 - \frac{\mathcal{M}(v)}{4\lambda \cosh(2\lambda r)}\right) \,,$$

with Bondi mass

$$\mathcal{M}(v) = \int_{-\infty}^{v} dv' (\partial_v f(v'))^2 \,.$$



Vaidya solution with coordinates $(r,\ v)$.

On mass inflation in sinh-CGHS

CGHS regime $M \gg M_{ext}$,

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 vu + g(v) + h(u) \,,$$

$$g(v) = \frac{1}{2}\int_0^v dv' \int_{v'}^{+\infty} dv'' (\partial_v f(v''))^2 \,,$$

$$h(u) = -\frac{1}{2}\int_{-\infty}^u du' \int_{-\infty}^{u'} du'' (\partial_u f(u''))^2 \,,$$

For wavepacket tail profile at late times $f(v) \simeq f_0 \cdot (\lambda v)^{-\alpha}$ $v \to +\infty$

$$g(v) \simeq \frac{M}{2\lambda} - \frac{g_\infty}{(\lambda v)^{2\alpha}} \,, \qquad \alpha > 0 \,,$$

After crossing the core

$$f(v) \mapsto f_0 \cdot (-\lambda v)^{\alpha} \,,$$

Ricci scalar near Cauchy horizon

$$R \simeq 4\lambda^2 e^{2\phi} \left(\frac{M}{2\lambda} + (2\alpha+1) g_\infty (-\lambda v)^{2\alpha} + \frac{\mathcal{E}_{\rm out}(u)}{2\lambda} + \right.$$
$$\left. + \frac{2\alpha+1}{2\alpha-1} \frac{2\alpha g_\infty}{\lambda} (-\lambda v)^{2\alpha-1} \partial_u h(u) \right)$$

is finite if $\alpha > 1/2$.

Role of negative energy densities

Consider S-matrix elements (coherent states $\Rightarrow \hat{a}_k|a\rangle = a_k|a\rangle$)

$$\langle b_k|\hat{S}|a_k\rangle = \int dc_k^* dc_k \, \langle b_k|\hat{S}_{\rm reg}|c_k\rangle\langle c_k|a_k\rangle \approx e^{iS[c_k]}e^{-\Gamma[c_k]}$$

with $\hat{S}_{reg}$ defined on subspace of topologically trivial spacetimes. Saddle point equation

$$i\frac{\delta S}{\delta c_k} = \frac{\delta \Gamma}{\delta c_k}$$

Role of states w/ negative energy density

- Typical semiclassical state Ψ - localized wavepacket into remnant
- For any typical Ψ one can find non-typical Ψ':

$$\langle \Psi'|\hat{T}_{\mu\nu}(x)|\Psi^*\rangle < 0$$

Fulling, Davies (1976)

gr-qc/9711030 Roman, Ford

- Non-typical Ψ' cause remnant decay.

Common QFT counterpart: tunnelling through sphaleron. Neat example:

0903.3916 [quant-ph] Levkov, Panin