Effective potential in the inflationary cosmology

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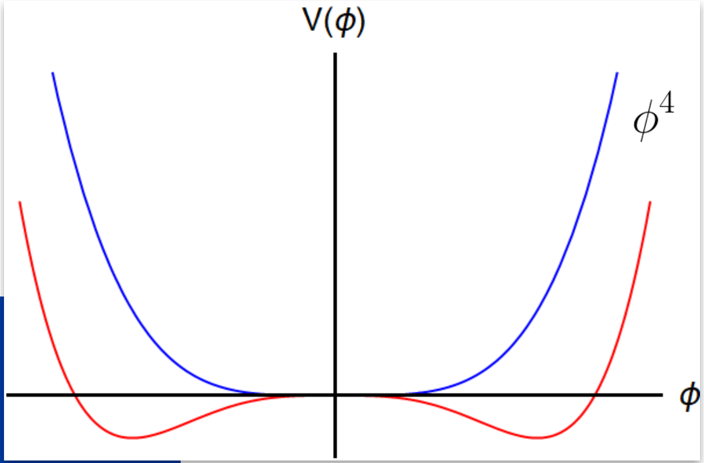
Content

- Effective potentials: Weinberg and Coleman mechanism
- Generalisation to nonrenormalisable theories
- Application to inflationary cosmology

Coleman, Weinberg, 1973

Weinberg-Coleman mechanism

In 1973 E. Weinberg and S. Coleman investigated the mechanism of appearance of an additional minimum in the effective potential after the addition of a one-loop quantum correction.



$$V_{classical}(\phi) \sim \frac{g}{4!}\phi^4$$

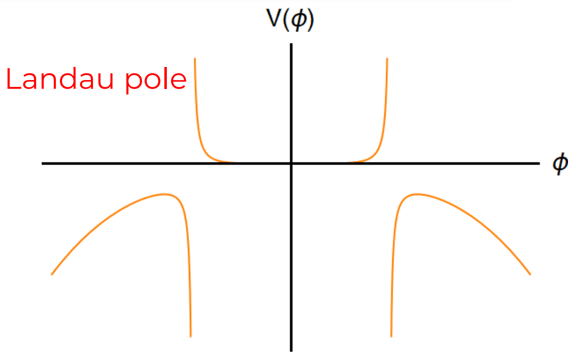
$$V_{eff}(\phi) \sim \frac{g}{4!}\phi^4 \left(1 + \frac{3}{2} g \log(\phi^2/\mu^2)\right)$$

Dimensional transmutation parameter **[m]**

However accounting for **all the corrections** in the effective potential (**RG**) leads to the restoration of the original minimum :

$$f'(y) = \frac{3}{2} f(y)^2$$

$$V_{all-loop}(\phi) \sim \frac{g\phi^4}{1 - \frac{3}{2} g \log(\phi^2/\mu^2)}$$



Summation of LL

renormalizable $\left(g\log(\phi^2/\mu^2)\right)^n$

non-renormalizable $(\Delta V_n(g,\phi)\log(F(g,\phi,\mu)))^n$

However, there are some difficulties

$$f'(y)=f(y)^2$$

Dimensionless combinations

New properties can be expected

D.T., Kazakov et al '18

Bogoliubov-Parasyuk theorem

If we consider a UV **divergent graph** **G** of any local field theory, then after **subtraction** all the divergent subgraphs, the remaining divergence will be local

R-operation:

$$\mathcal{R}G = \prod_{\gamma}(1 - K_{\gamma})G,$$

$d = 4 - 2\epsilon$ not to be confused with the Hubble flow

Final result must not include these term:

$$\frac{\log(\mu)}{\epsilon}$$

include these only term

$$\left(\frac{\Delta V_n(g,\phi)}{\epsilon}\right)^n \quad (\Delta V_n(g,\phi)\log(F(g,\phi,\mu)))^n$$

$$1/\epsilon \to -\log g v_2/\mu^2$$

power coefficient **The same** **power coefficient**

Such a constraint leads to recurrence relation:

$$\mathcal{A}_n^{(n)} = (-1)^{n+1}\frac{\mathcal{A}_1^{(n)}}{n},$$

$$\mathcal{B}_n^{(n)} = (-1)^n\left(\frac{2}{n}\mathcal{B}_2^{(n)} + \frac{n-2}{n}\mathcal{B}_1^{(n)}\right),$$

$$C_n^{(n)} = (-1)^{n+1}\left(\frac{3}{n}C_3^{(n)} + \frac{2(n-3)}{n}C_2^{(n)} + \frac{(n-2)(n-3)}{2n}C_1^{(n)}\right)$$

D.T., Kazakov, Iakhibbaev'23

Effective action

$$Z(J) = \int \mathcal{D}\phi \ \exp\left(i \int d^4x \ \mathcal{L}(\phi, d\phi) + J\phi\right)$$

$$W(J) = -i \log Z(J)$$

Connected correlation function

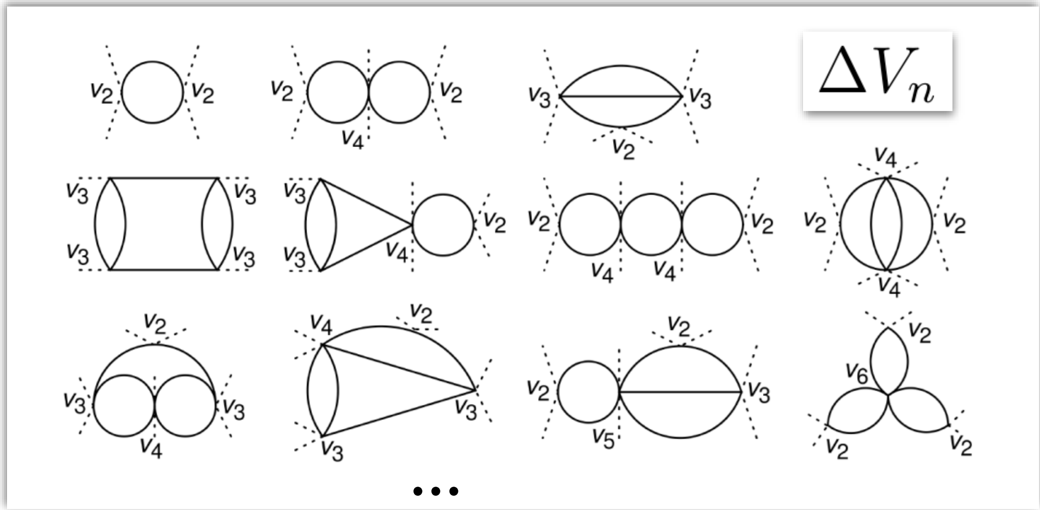
$$\Gamma(\phi) = W(J) - \int d^4x J(x)\phi(x)$$

Feynman's rules

$$v_n = \frac{\partial^n}{\partial \phi^n} V(\phi)$$

acquires

$$m^2(\phi) = g v_2(\phi)$$



D.T., Kazakov, Iakhibbaev, JHEP'23

Generalised equation

$$n\Delta V_n = \frac{1}{2} v_2 D_2 \Delta V_{n-1} + \frac{1}{4}\sum_{k=1}^{n-2} D_2 \Delta V_k D_2 \Delta V_{n-1-k}, \quad n \geq 2 \qquad D_2 = \frac{\mathrm{d}^2}{\mathrm{d}\phi^2}$$

$$d = 4 - 2\epsilon \qquad z = \frac{g}{\epsilon} \qquad \Sigma(z,\phi) = \sum_{n=0}^{\infty} (-z)^n \Delta V_n(\phi)$$

Differential equation:

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2, \quad \Sigma(0,\phi) = V_0(\phi)$$

$$f'(y) = \frac{3}{2} f(y)^2$$

$$\left(g \log(\phi^2/\mu^2)\right)^n$$

$$(\Delta V_n(g,\phi) \log(F(g,\phi,\mu)))^n$$

$$V_{eff}(g,\phi) = \Sigma(z,\phi)\Big|_{z \to -\frac{g}{16\pi^2} \log g v_2/\mu^2}$$

→ additional free parameter

And even leading logarithms do not obey multiplicative behavior, which corresponded to geometric progression and we should expect new properties for the scalar theory

D.T., Kazakov, Iakhibbaev, JCAP'23

Single-field model of slow-roll inflationary scenario

T-type α-attractors

$$\tanh^{2n}(\phi/(\sqrt{6\alpha M_{Pl}^2}))$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$3M_{pl}^2H^2 = \frac{\dot\phi^2}{2} + V,$$
$$-2M_{pl}^2\dot H = \phi^2,$$
$$\ddot\phi + 3H\dot\phi + \frac{\partial V}{\partial\phi} = 0.$$

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2, \quad \Sigma(0,\phi) = V_0(\phi)$$

T^2

$$V = g\tanh^2\left(\frac{\phi\omega}{\sqrt{6}}\right)$$

$x = z\omega^4$ and $y = \tanh^2(\phi\omega/\sqrt{6})$

$$\frac{1}{36}(y-1)^2\left((1-3y)S_y - 2(y-1)yS_{yy}\right)^2 = -S_x$$

$$S(0,y) = y,\ S(x,1) = 1,\ S_x(x,1) = 0$$

T^4

$$V = g\tanh^4\left(\frac{\phi\omega}{\sqrt{6}}\right)$$

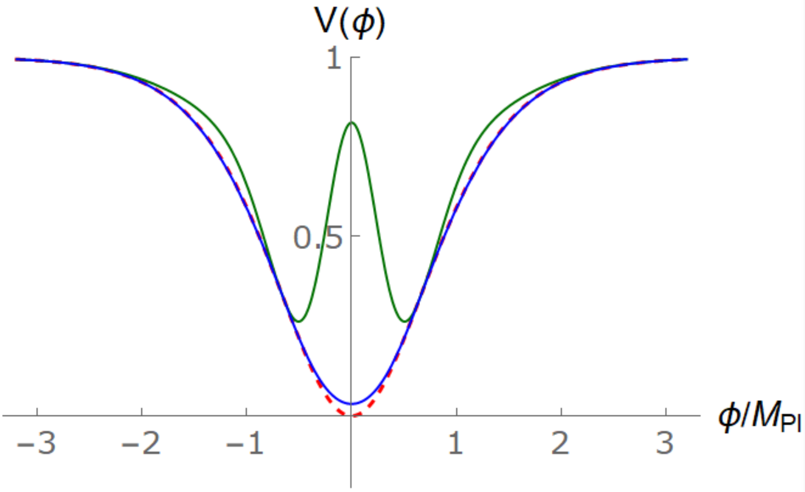
$x = z\omega^4,\ y = \tanh^4(\phi\omega/\sqrt{6})$

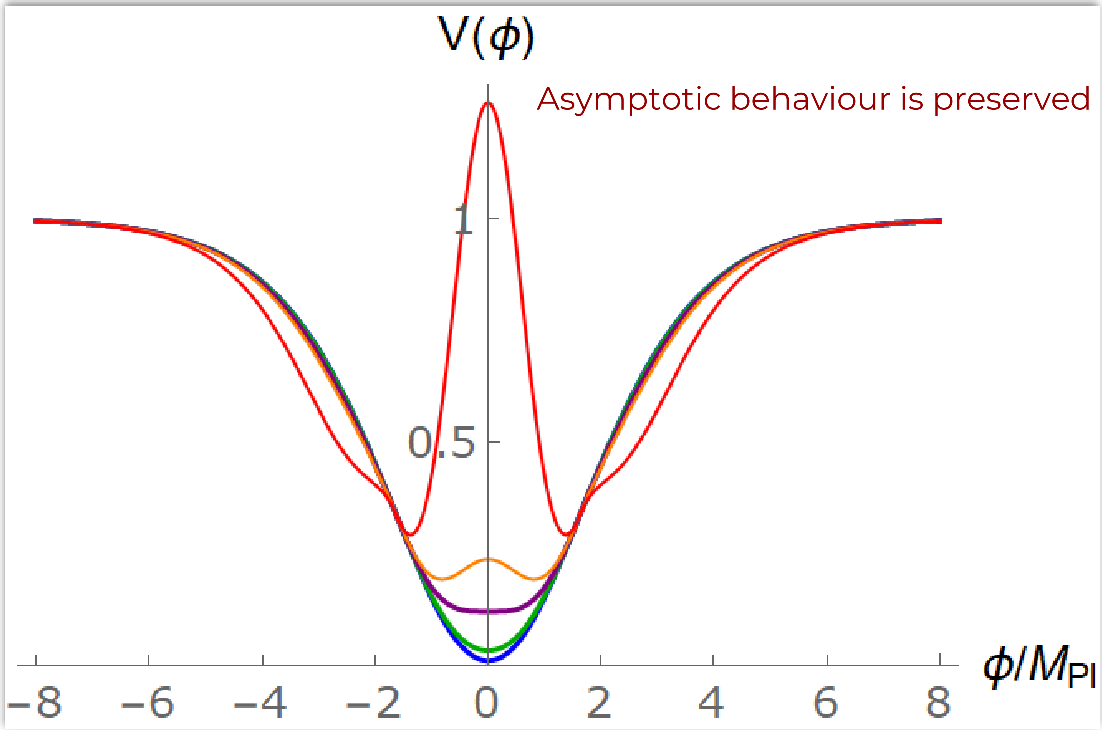
$$\frac{1}{9}\left(y^{1/2}-1\right)^2 y\left(\left(5y^{1/2}-3\right)S_y + 4y\left(y^{1/2}-1\right)S_{yy}\right)^2 = -S_x$$

$$S(0,y) = y,\ S(x,1) = 1,\ S_x(x,1) = 0$$

T^2

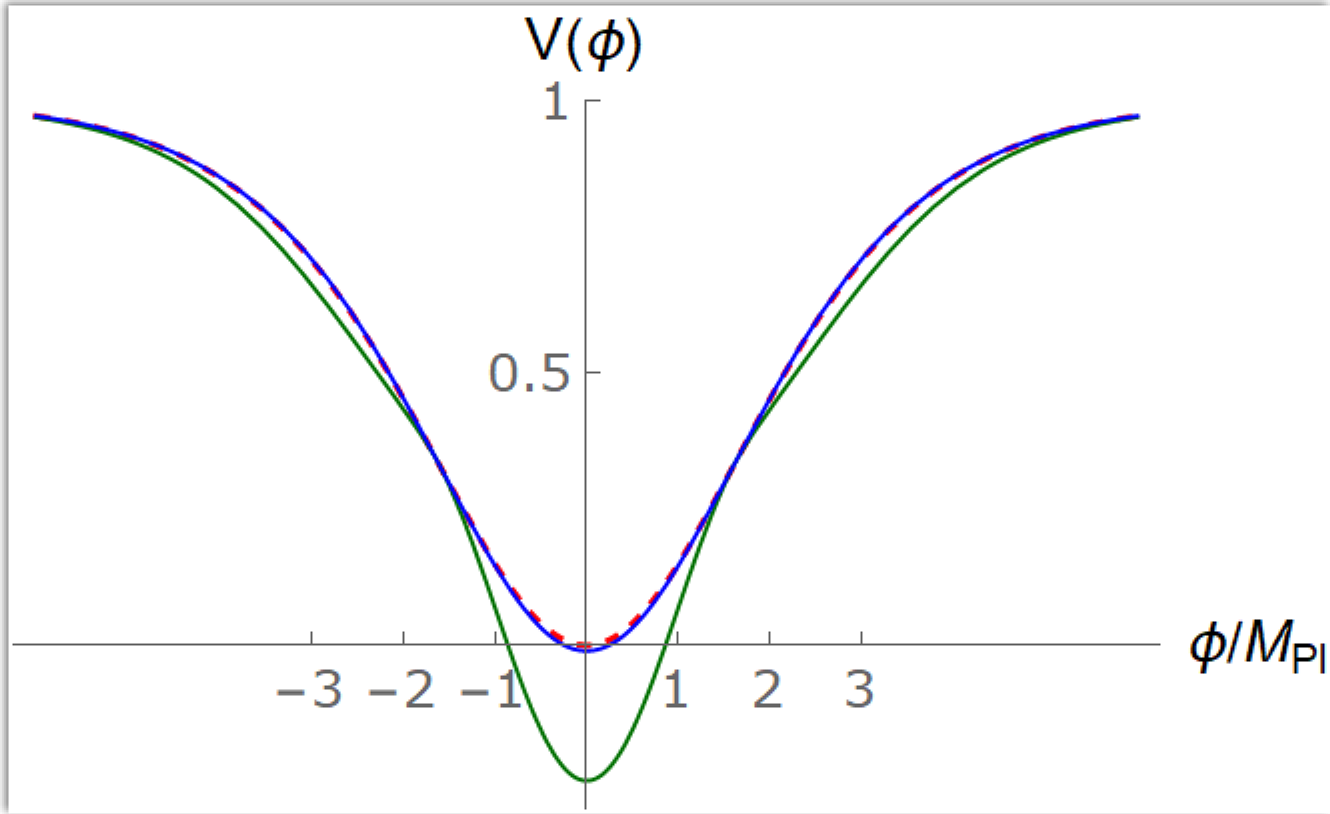
$\mu^2 \sim 10^{-2}$





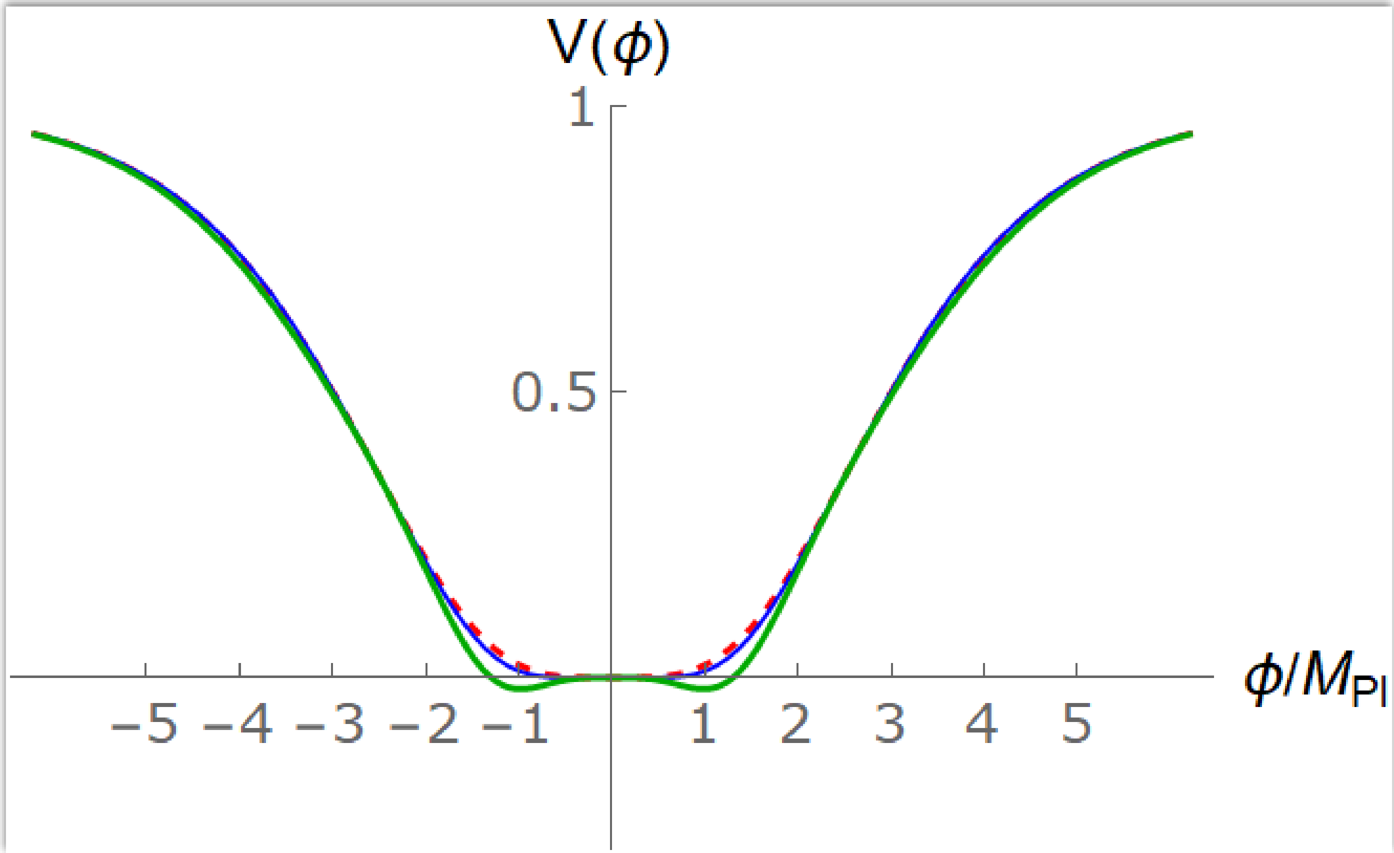
Blue line — $\mu^2 \sim 0.6$, green $\mu^2 \sim 10^{-2}$, purple $\mu^2 \sim 10^{-10}$, orange $\mu^2 \sim 10^{-16}$, red 10^{-36}

T^2



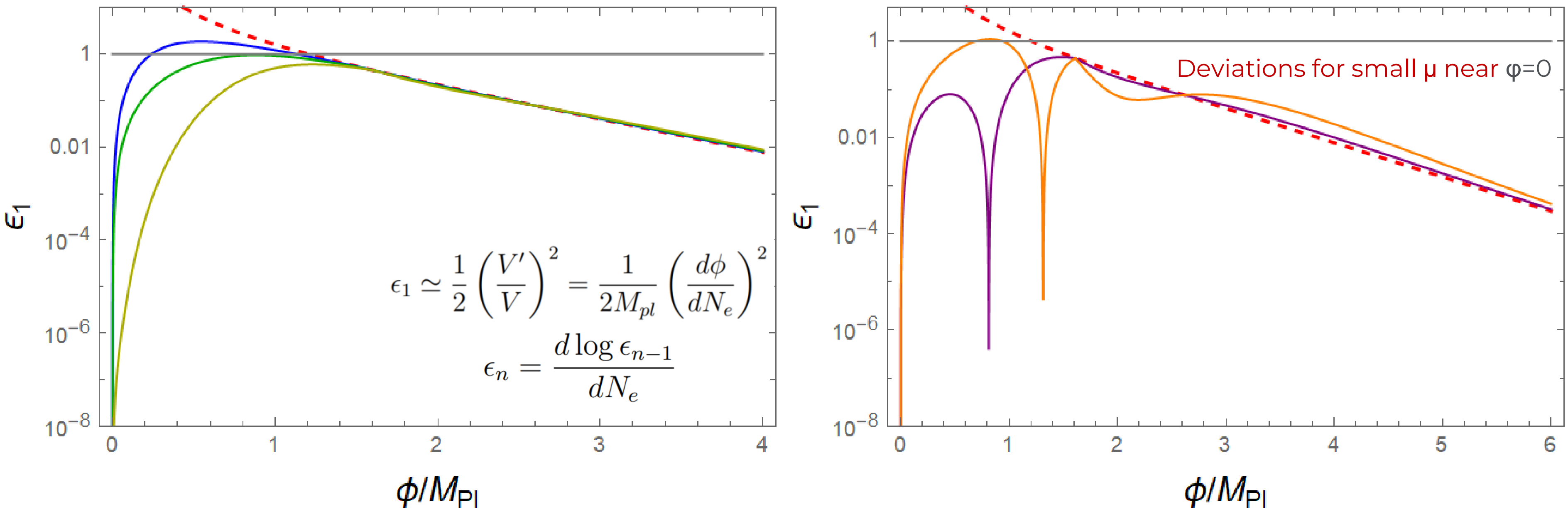
$\mu^2 \sim 10$

T^4



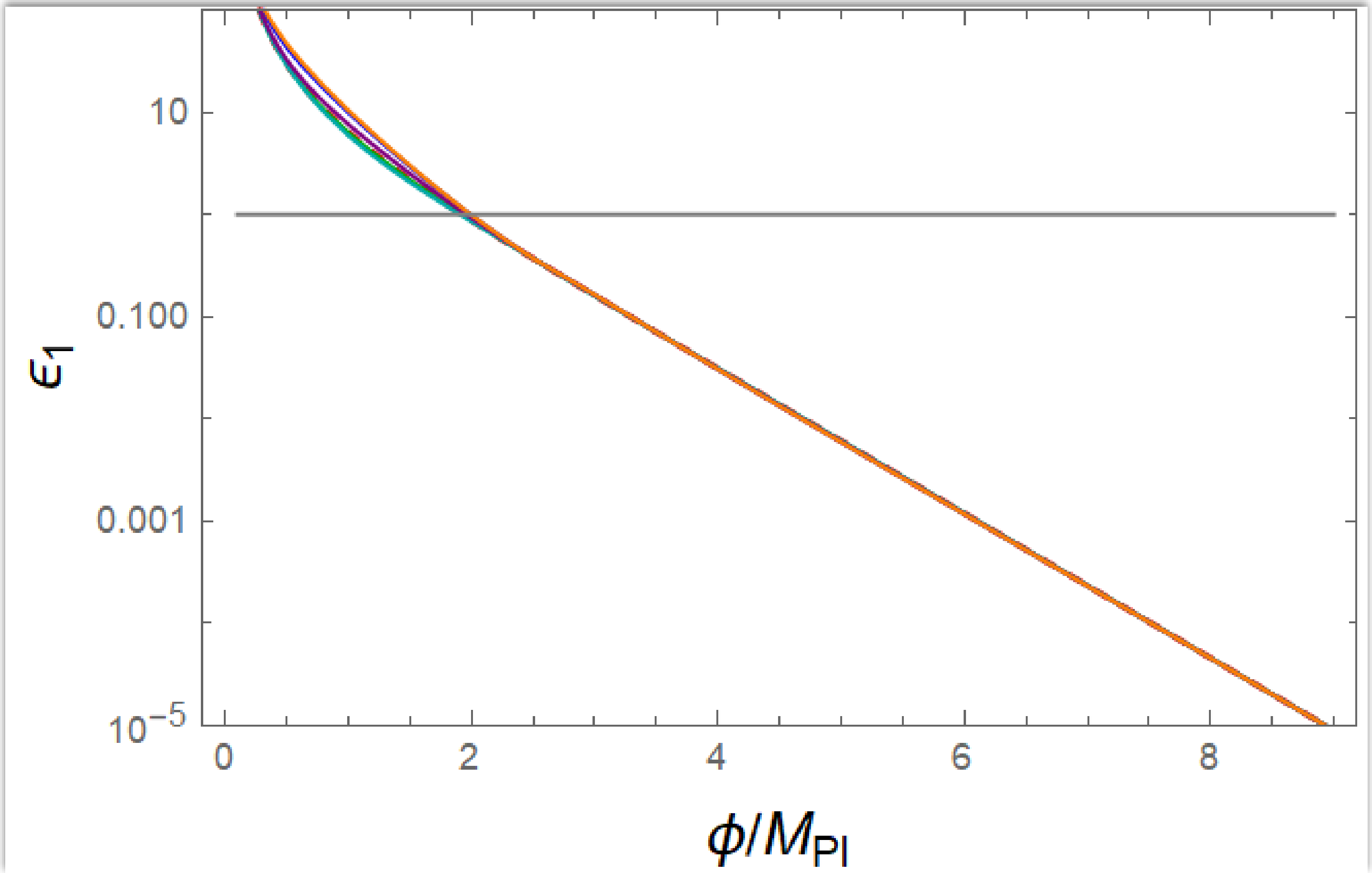
$\mu^2 \sim 10^{-80}$

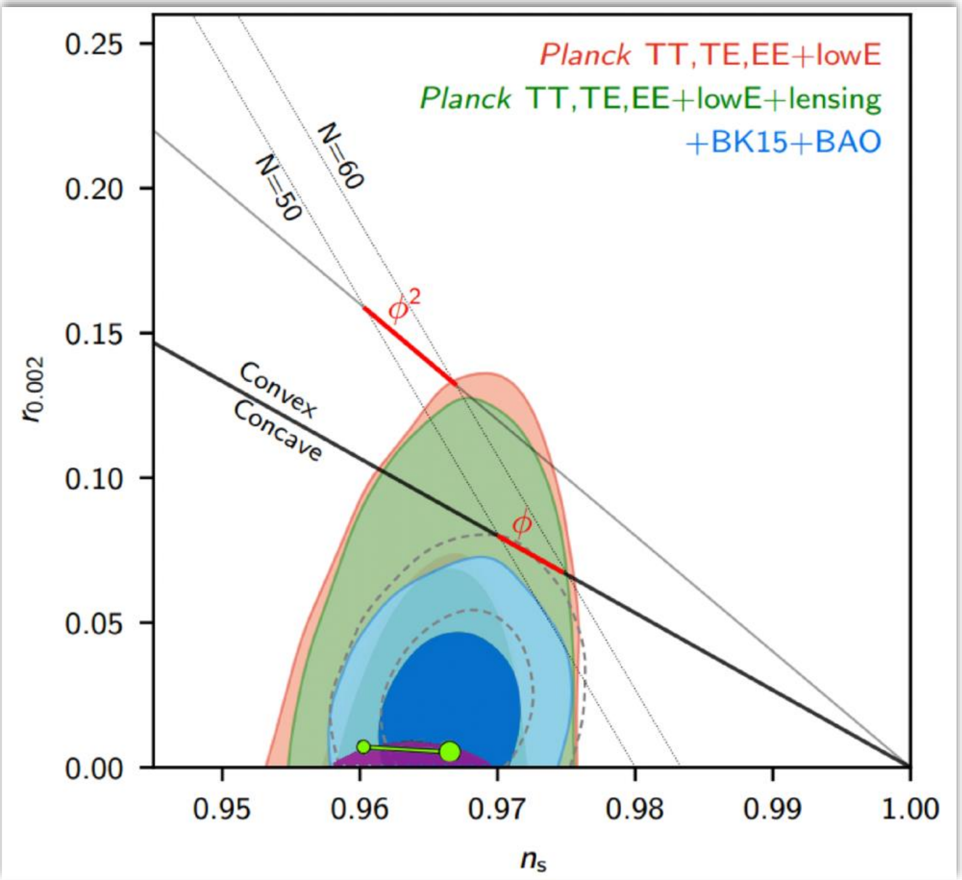
T^2



Plots for the Hubble flow parameter ϵ_1 In Fig. the blue line — $\mu^2 \sim 10^{-2}$, green line — $\mu^2 \sim 10^{-8}$, yellow line — $\mu^2 \sim 10^{-10}$, at the purple line $\mu^2 \sim 10^{-16}$, orange one $\mu^2 \sim 10^{-56}$. The red dashed line corresponds to the classical potential and the grey line indicates the end of inflation

T^4





$$n_s = 1 - 2\epsilon_1 + \epsilon_2$$

$$r = 16\epsilon_1$$

$$r < 0.036,\ n_s = 0.9649 \pm 0.0042$$

Comparison in the n_s versus r plot. The dark purple region corresponds to the full effective potentials for T^2 in the limits $10^2 < \mu^2 < 10^{-3}$ and T^4 in the limits $10^{10} < \mu^2 < 10^{-14}$ in the interval from 50 to 60 e-folds. The bright green line corresponds to the classical T^2-model.

Thanks for attention

Conclusions

We have obtained a universal generalised RG equation for an arbitrary scalar potential

We applied our formalism on T-models. This is good example of nonrenormalisable scalar theory, in which the spontaneous symmetry breaking may be preserved after all-loop summation

Effective potential is consistent with the cosmological observational data for a wide range of the parameter μ.