Some aspects of Tsallis Holographic dark energy

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Outline

Introduction

Accelerating expansion of the universe

Discovery of the accelerated expansion of the Universe in 1998.

Dark energy is introduced to explain the accelerated expansion.

A standard cosmological model, the ΛCDM model, is proposed, where the cosmological constant Λ acts as dark energy.

Problems with Standard Model:

- Smallness of the cosmological constant
- The coincidence problem

Holographic principle: The holographic principle states that all physical quantities within the universe, including the dark energy density, can be described by setting some quantities at the boundary of the universe.

Based on the dimensional analysis (*arXiv:hep-th/9803132*), we have

$$\rho_{de} = C_1 M_p^4 + C_2 M_p^2 L^{-2} + C_3 L^{-4} + \ldots$$

The energy within a Schwarzschild radius L is $L^3 \Lambda^4$

$$L^3 \Lambda^4 < M_p^2 L^{-2}, \qquad \rho_{de} \sim \Lambda^4 < M_p^2 L^{-2}$$

Applying HP to The DE Probleml

A model of holographic dark energy was proposed:

$$\rho_{de} = \frac{3C^2}{L^2},$$

where C is a constant and L can be chosen as:

$$L = \frac{1}{H}, \qquad L = a\int_0^t \frac{dt}{a}, \qquad L = a\int_t^\infty \frac{dt}{a}, \qquad L \sim R = 6\left(\dot{H} + 2H^2\right)$$

Applying HP to The DE Probleml

In addition to the dimensional analysis mentioned above, there are also a number of other theoretical motivations leading to the form of HDE:

- Entanglement Entropy from Quantum Information Theory (*arXiv:gr-qc/9504049*)
- Holographic Gas as Dark Energy (*arXiv:0811.3332*)
- Casimir Energy in de Sitter Space (*arXiv:0910.3375*)
- Dark Energy from Entropic Force (*arXiv:1001.4466*)
- HDE from Action Principle (*arXiv:1210.0966*)

Tsallis generalized the Boltzmann–Gibbs entropy for a black hole with area A (*J Stat Phys 52, 479–487 (1988)*):

$$S_\gamma = \mu A^\gamma$$

Tsallis holographic dark energy model (*arXiv:1802.07722*):

$$\rho_{de} = \frac{3C^2}{L^{4-2\gamma}},$$

Cosmological Observations

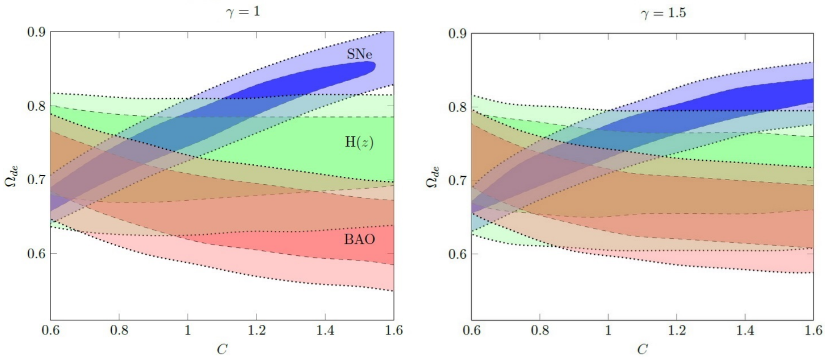
Cosmological Observations

We used observational data:

- Type Ia Supernova
- Baryon Acoustic Oscillation
- Dependence of the Hubble parameter on redshift

Int. J. Mod. Phys. D Vol. 29, no. 01, 1950176 (2020).

Cosmological Observations



1σ (dashed lines) and 2σ (dotted lines) allowed areas on plane $C - \Omega_{de}$

Cosmological Observations

Dark energy model	χ^2_{SN}	χ^2_{A}	χ^2_{H}	$\sum \chi^2$
ΛCDM	346.72	17.0	0.036	**363.756**
$\gamma = 0.9, C = 0.7, \Omega_{de} = 0.7.$	347.42	17.36	0.089	**364.870**
$\gamma = 1, C = 0.7, \Omega_{de} = 0.7.$	347.34	17.21	0.108	**364.653**
$\gamma = 1.2, C = 0.7, \Omega_{de} = 0.7.$	347.27	16.97	0.160	**364.391**
$\gamma = 1.5, C = 0.8, \Omega_{de} = 0.72.$	346.91	16.23	0.614	**363.757**
$\gamma = 1.9, C = 0.9, \Omega_{de} = 0.72.$	347.15	16.49	1.010	**364.642**

χ^2 values for Tsalis holographic dark energy model.

Dynamical analysis

Dynamical analysis

The cosmological equations for this metric can be written in the following form:

$$H^2 = \frac{1}{3}(\rho_m + \rho_{de} + \rho_r),$$

$$\dot{H} = -\frac{1}{2}(\rho_m + \rho_{de} + 4\rho_r/3 + p_{de}).$$

Dynamical analysis

It is convenient to introduce the following parameters

$$x = \frac{\rho_m}{3H^2}, \quad y = \frac{\rho_{de}}{3H^2}, \quad z = \ln H, \quad \Omega_r = \frac{\rho_r}{3H^2}.$$

And

$$\frac{\dot{\rho_{de}}}{3H^3} = (2\gamma - 4)y\,(1 - \delta)\,, \quad \delta = \left(e^{2(1-\gamma)z}\frac{C^2}{y}\right)^{\frac{1}{2\gamma-4}}.$$

Int. J. Mod. Phys. D., 2023. arXiv:2305.10573

Dynamical analysis

We obtain a system of dynamic equations for x, y, z:

$$\frac{dx}{d\eta} = x - x^2 - xy(4 + (2\gamma - 4)(1 - \delta)),$$

$$\frac{dy}{d\eta} = -xy + y(1 - y)(4 + (2\gamma - 4)(1 - \delta)),$$

$$\frac{dz}{d\eta} = -\frac{1}{2}\left(4 - x - 4y - (2\gamma - 4)y(1 - \delta)\right),$$

where $\eta = \ln a$

Dynamical analysis

Point	x	y	Ω_r	Existence, behavior of H
1	0	0	1	$H \to \infty$ (repeller)
2	0	1	0	$H \to 0$ or $H \to \infty$ (attractor)
3	0	1	0	$H = C^{\frac{1}{\gamma-1}}$ (saddle)
4	1	0	0	$H \to 0$ (saddle)
5	1	0	0	$H \to \infty$ (repeller)

Critical points for the system of equations describing the evolution of the Universe filled with matter, matter and holographic dark energy at $\gamma \neq 1$.

Scalar perturbations

Scalar perturbations

The perturbed metric is:

$$ds^2 = -\left[1 + 2\Phi(r,t)\right] dt^2 + a^2(t)\left[1 - 2\Phi(r,t)\right] dx^2$$

With the scalar type fluctuations, the future event horizon becomes (*arXiv:0801.1407*)

$$L_h(0,t) = \int_0^{l_h(t)} a(t)\left[1 - \Phi(r,t)\right] dr$$

where $l_h(t)$ is the coordinate distance to the future event horizon

The coordinate distance $l_h(t)$ can be written as

$$l_h(0,t) \equiv l_{h0} + \delta l_h, \quad \delta l_h = \int_t^{\infty} \frac{2\Phi(l_{h0}(t'), t')}{a(t')} dt'$$

Thus the fluctuation of the future event horizon can be written as

$$\delta L_h(0,t) \equiv L_h(0,t) - L_{h0} = a(t) \left[\int_t^{\infty} \frac{2\Phi(l_{h0}(t'), t')}{a(t')} dt' - \int_0^{l_{h0}} \Phi(r,t) dr \right]$$

and the THDE energy density has fluctuation

$$\delta \rho_{de} = \frac{2\gamma - 4}{2} \rho_{de} \frac{\delta L_h}{L_h}$$

Scalar perturbations

Inserting this equation into the 00-component of the perturbated Einstein equation, one obtains

$$\frac{\Delta^2}{a^2}\Phi - 3H\dot{\Phi} - 3H^2\Phi = \frac{1}{2M_p^2}(\delta\rho_{de} + \delta\rho_m)$$

For simplicity, we neglect the matter density perturbation, so $\delta\rho_m = 0$. To solve this equation, we expand Φ using its eigenfunction. We suggest

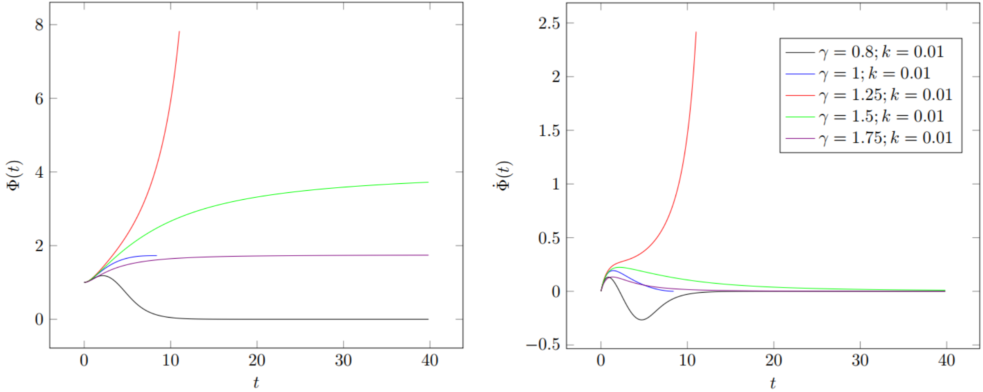
$$\Phi(r,t) = \sum_k \Phi_k(t)\frac{\sin(kr)}{r}$$

where we have dropped the $\cos(kr)/r$ terms, which lead to a singularity at $r = 0$.

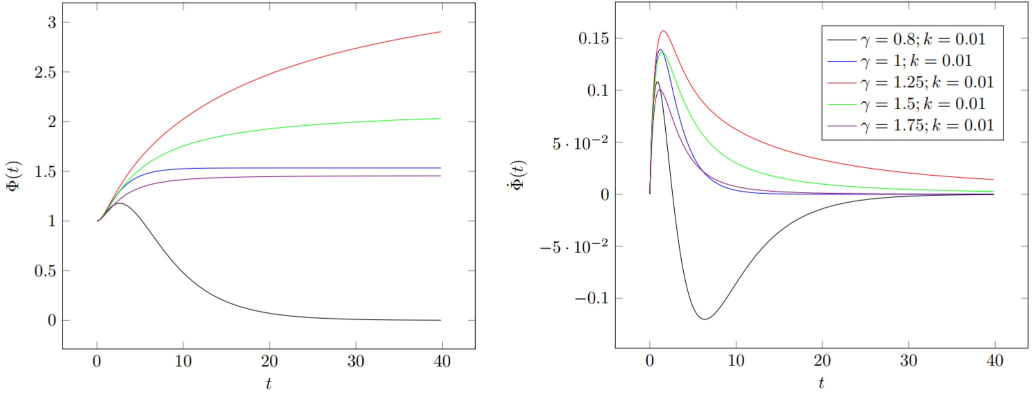
One way to deal with this equation is to take derivative with respect to t. This integral equation becomes a differential equation

$$\ddot{\Phi}_k + \frac{1}{3H}\left(\frac{k^2}{a^2} + 3\dot{H} + 3H^2 + 3(4-2\gamma)H^2 - \frac{3H(5-2\gamma)}{L}\right.$$
$$\left. + \frac{2\gamma-4}{2}\frac{a\rho_{de}}{L}\int_0^{l_{h0}(t)}\frac{\sin(kr)}{kr}dr\right)\dot{\Phi}_k + \frac{1}{3H}\left(6\dot{H}H + 3(4-2\gamma)H^3 + (4-2\gamma)H\frac{k^2}{a^2}\right.$$
$$\left. - \frac{3H^2(5-2\gamma)}{L} - (5-2\gamma)\frac{k^2}{a^2L} + \frac{2\gamma-4}{2}\frac{\rho_{de}}{kL}\frac{\sin(kl_{h0}(t))}{l_{h0}(t)}\right)\Phi_k = 0$$

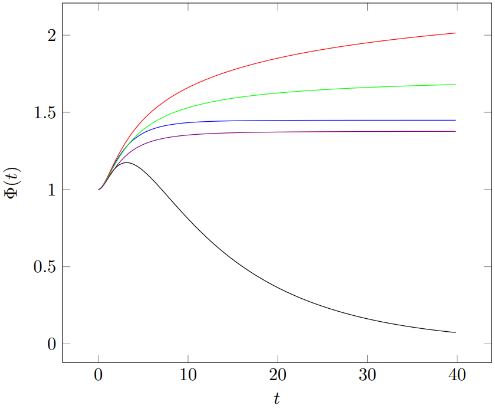
Scalar perturbations with C = 0.8 and $\Omega = 0.72$

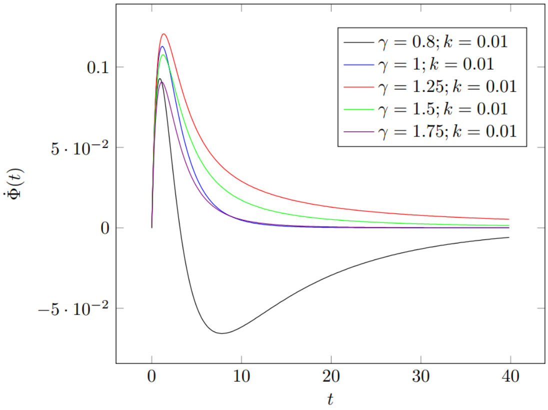


Scalar perturbations with $C = 1$ and $\Omega = 0.72$

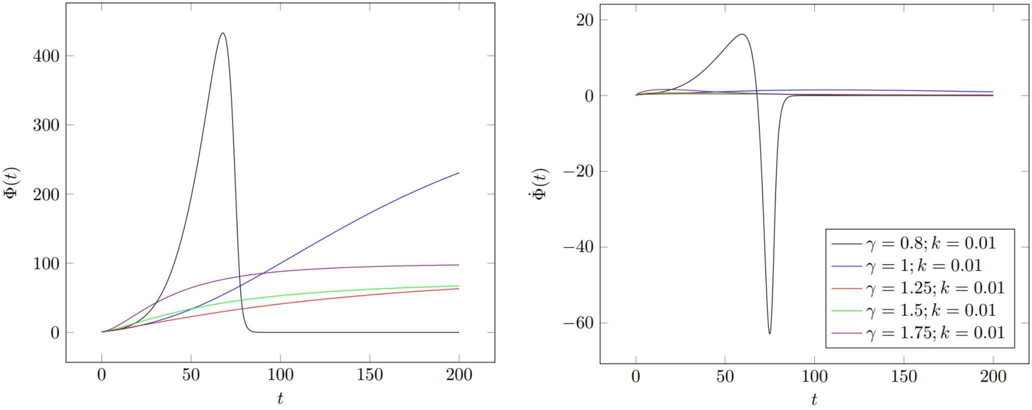


Scalar perturbations with $C = 1.2$ and $\Omega = 0.72$

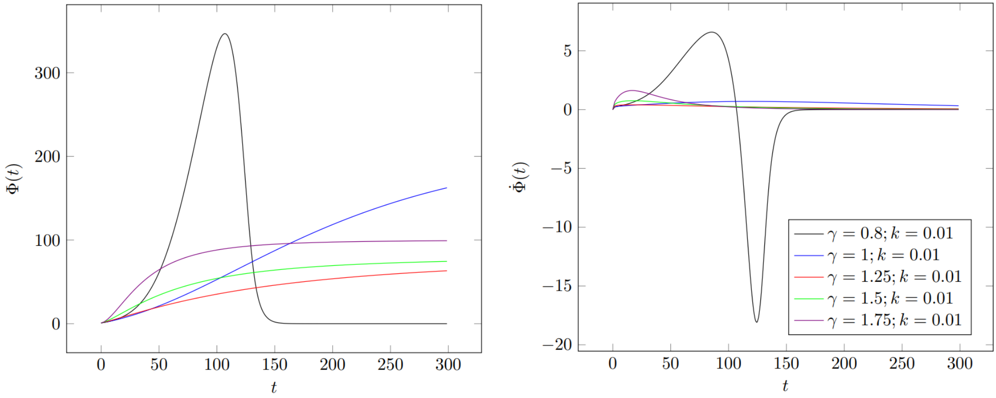




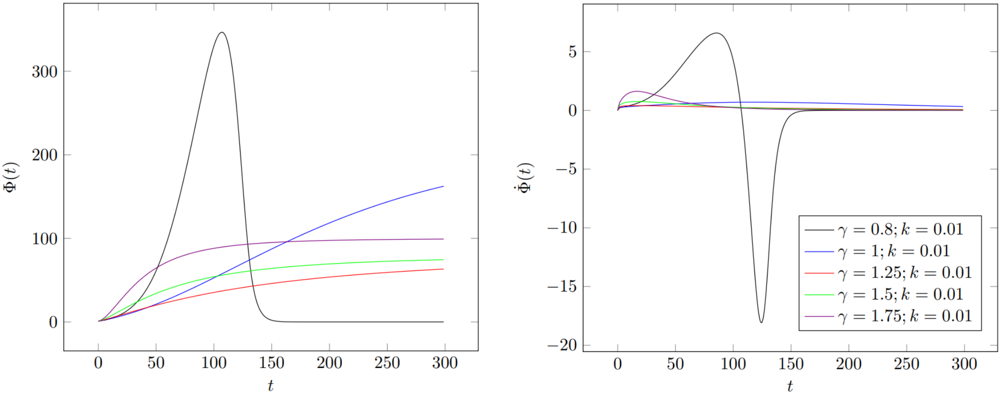
Scalar perturbations with C = 0.8 and $\Omega = 0.01$



Scalar perturbations with C = 1 and $\Omega = 0.01$



Scalar perturbations with C = 1.2 and $\Omega = 0.01$



Conclusions

- THDE at various parameter values is in satisfactory agreement with observational data.
- The nature of the critical points for the cosmological system of equations was studied. Compared to conventional HDE, new points appear.
- An analysis of the growth of possible fluctuations showed that for a sufficiently wide range of parameters there is no unlimited growth of perturbations in the dark energy density.

Thank you for attention