Effects of the creation of particles in the field of waves of high intensity

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- **Particle creation by a dynamic classical field. Example: the production of matter particles by the inflaton field ostillation at the end of inflation (reheating stage) [1]**
- **Particle production enhancement effect - parametric resonance**
- **Creation of massive particles by an intense wave of a massless field with resonant amplification [2], small mass m«ω**
- **The purpose of the work is to consider the case of arbitrary masses**

[1] Rubakov V.A. Gorbunov D.S. Introduction to the theory of the early universe. Cosmological perturbations. Inflationary theory. URSS, 2009.

[2] A.Arza. Phys.Rev.D 105 (2022) arXiv:2009.03870

$$\mathcal{H} = g\phi\chi^2$$

Two-scalar model

Equations of motion

$$(\Box + m_\phi^2)\phi = -g\chi^2$$

$$(\Box + m_\chi^2)\chi = -2g\phi\chi$$

Monochromatic plane wave

$$\phi(\vec{x},t) = \frac{\sqrt{2\rho_\phi}}{\omega_p}\cos(\vec{p}\cdot\vec{x} - \omega_p t)$$

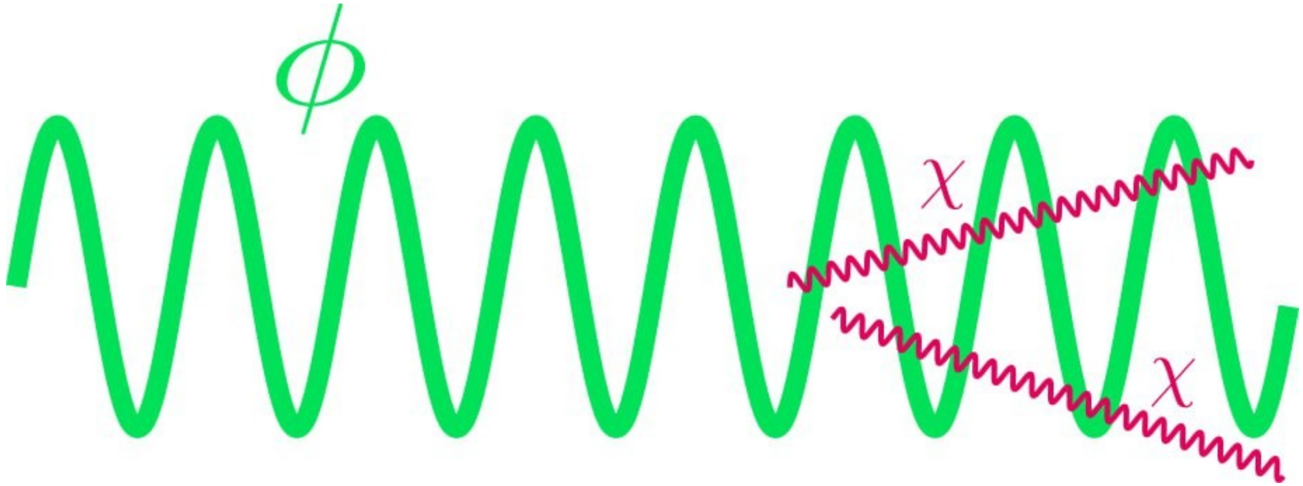
$$\Omega_{\vec{k}} = \sqrt{k^2 + m_\chi^2}$$

$$\omega_{\vec{p}} = \sqrt{p^2 + m_\phi^2}$$

Quantum field

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\Omega_{\vec{k}}}}\left(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{x}}\right)$$

$$\mathcal{H} = g\phi\chi^2$$



Denote

$$A_{\vec{k}} = \chi_{\vec{k}} + \chi^{\dagger}_{-\vec{k}}$$

$$(\partial_t^2 + \Omega_{\vec{k}}^2)A_{\vec{k}} = -\omega_{\vec{p}}^2\alpha\left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}A_{\vec{k}-\vec{p}}e^{-i\omega_{\vec{p}}t} + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}A_{\vec{k}+\vec{p}}e^{i\omega_{\vec{p}}t}\right)$$

where $\alpha \equiv \frac{g\sqrt{2\rho_\phi}}{\omega_{\vec{p}}^3}$

Suppose $\chi_{\vec{k}} = a_{\vec{k}}(t)e^{-i\Omega_{\vec{k}}t}$

$$\underline{e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}})} + e^{i\Omega_{-\vec{k}}t}(\ddot{a}^{\dagger}_{-\vec{k}} + 2i\Omega_{-\vec{k}}\dot{a}^{\dagger}_{-\vec{k}}) =$$

$$= -\omega_{\vec{p}}^2\alpha\left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}\left(a_{\vec{k}+\vec{p}}e^{-i(\Omega_{\vec{k}+\vec{p}}-\omega_{\vec{p}})t} + \underline{a^{\dagger}_{-\vec{k}-\vec{p}}e^{i(\Omega_{-\vec{k}-\vec{p}}+\omega_{\vec{p}})t}\right)} + \right.$$

$$\left. + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}\left(a_{\vec{k}-\vec{p}}e^{-i(\Omega_{\vec{k}-\vec{p}}+\omega_{\vec{p}})t} + \underline{a^{\dagger}_{-\vec{k}+\vec{p}}e^{i(\Omega_{-\vec{k}+\vec{p}}-\omega_{\vec{p}})t}\right)\right)}$$

The amplitude $a^{\dagger}_{-\vec{k}+\vec{p}}$ - leading, $a^{\dagger}_{-\vec{k}-\vec{p}}$ - subleading

The final equation

- **Approximation (A.Arza, PRD 2022): $\ddot{a}$ term neglected →**

$$m_\chi \ll \omega_p$$

$$\dot{a}_{\vec{k}} = -i\sigma_{\vec{k}}\, a^\dagger_{\vec{p}-\vec{k}}\, e^{i\epsilon_{\vec{k}}\, t}$$

$$\sigma_{\vec{k}} = g\sqrt{\frac{\rho_\phi/2}{\omega^2_{\vec{p}}\,\Omega_{\vec{p}-\vec{k}}\,\Omega_{\vec{k}}}}$$

- **Without approximation**

$$e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}}) = \sigma_{p-k}a^\dagger_{-\vec{k}+\vec{p}}e^{i(\Omega_{-\vec{k}+\vec{p}}-\omega_{\vec{p}})t}$$

$$\sigma_{p-k} = -\omega_p^2\alpha\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}$$

Solution

- **In the approximation**

$$\epsilon_{p-k} = \Omega_k + \Omega_{p-k} - \omega_p = \epsilon_k$$

$$a_k(t) = e^{i\epsilon_k t/2}\left[a_k(0)\left(\cosh(s_k t) - i\frac{\epsilon_k}{2s_k}\sinh(s_k t)\right) - i\frac{\sigma_k}{s_k}a^\dagger_{p-k}(0)\sinh(s_k t)\right]$$

$$s_{\vec{k}} = \sqrt{\sigma^2_{\vec{k}} - \epsilon^2_{\vec{k}}/4}$$

- **Without approximation**

$$a_k(t) = e^{i\epsilon_{p-k}t/2}\Big[a_k(0)(\cosh(st) - i\frac{\frac{\epsilon^2_{p-k}}{4} - s^2 - \Omega_k\epsilon_{p-k}}{s(\epsilon_{p-k} - 2\Omega_k)}\sinh(st)) -$$

$$-i\frac{\sigma_{p-k}}{s(\epsilon_{p-k} - 2\Omega_k)}a^\dagger_{p-k}(0)\sinh(st)\Big]$$

$$s_{p-k} = \sqrt{-\frac{\epsilon^2_{p-k}}{4} - 2\Omega^2_k + \epsilon_{p-k}\Omega_k \pm \sqrt{\Omega^2_{p-k}\epsilon^2_{p-k} + 4\Omega^4_k + \sigma^2_{p-k} - 4\epsilon_{p-k}\Omega^3_k}}$$

Conditions of instability

$$s^2 > 0$$

Dimensionless quantities:

$\vec{\kappa} = \vec{k}/\omega_p \qquad \vec{v} = \vec{p}/\omega_p \qquad \beta_k = \sqrt{\kappa^2 + \mu^2} \qquad \mu = m_\chi/\omega_p \qquad \eta_{p-k} = s_{p-k}/\omega_p$

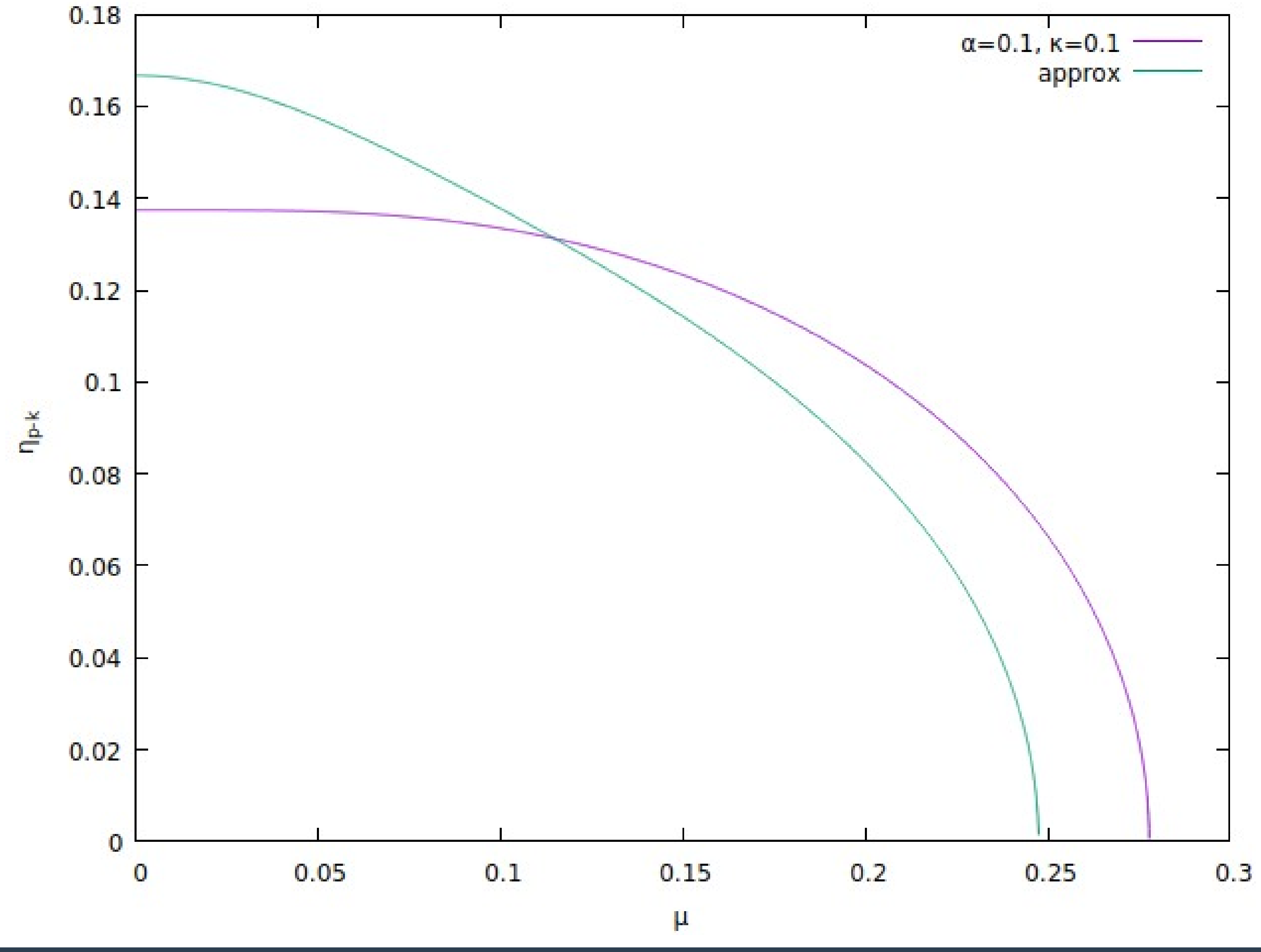
- **In the approximation**

$$\eta_{\vec{\kappa}} = \sqrt{\frac{\alpha^2}{4\beta_{\vec{v}-\vec{\kappa}}\beta_{\vec{\kappa}}} - \frac{1}{4}(\beta_{\vec{\kappa}} + \beta_{\vec{v}-\vec{\kappa}} - 1)^2}$$

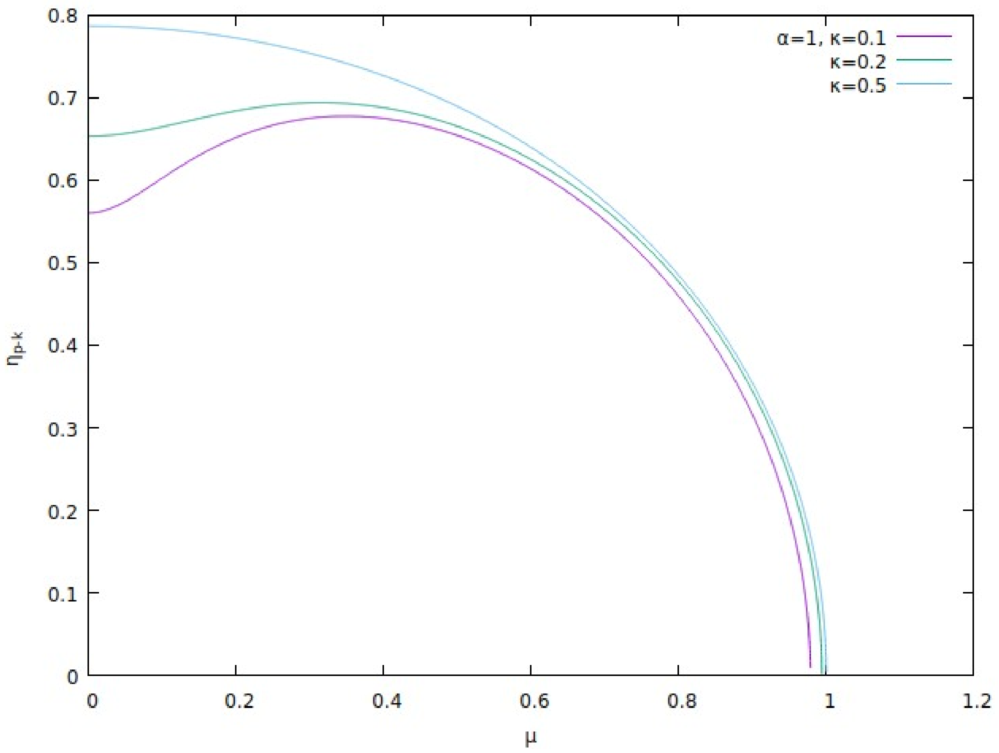
- **Without approximation**

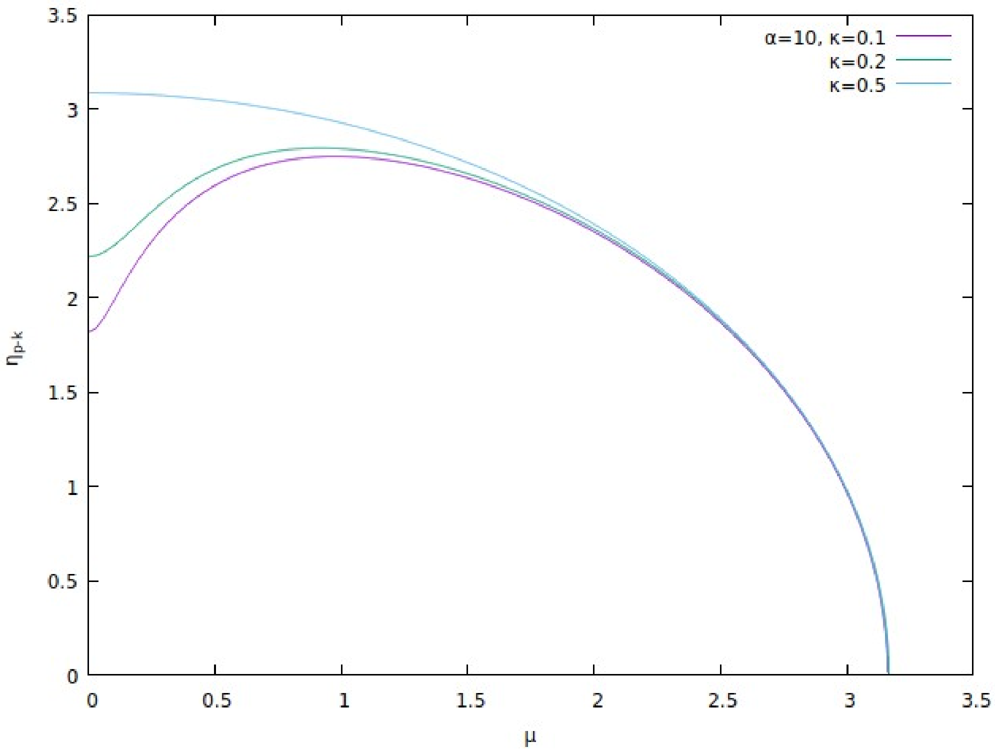
$$\eta_{p-k} = \Bigg(-\frac{1}{4}(\beta_k + \beta_{v-k} - 1)^2 - 2\beta_k^2 + (\beta_k + \beta_{v-k} - 1)\beta_k + \\ + \sqrt{\beta_k^2(\beta_k + \beta_{v-k} - 1)^2 + 4\beta_k^4 - 4(\beta_k + \beta_{v-k} - 1)\beta_k^3 + \frac{\beta_k \alpha^2}{\beta_{v-k}}}\Bigg)^{\frac{1}{2}}$$

Dependence of $\eta_{p\text{-}k}$ on μ at α=0.1 and κ=0.1 in the approximation and without it



Dependences of $\eta_{p\text{-}k}$ on μ for various α



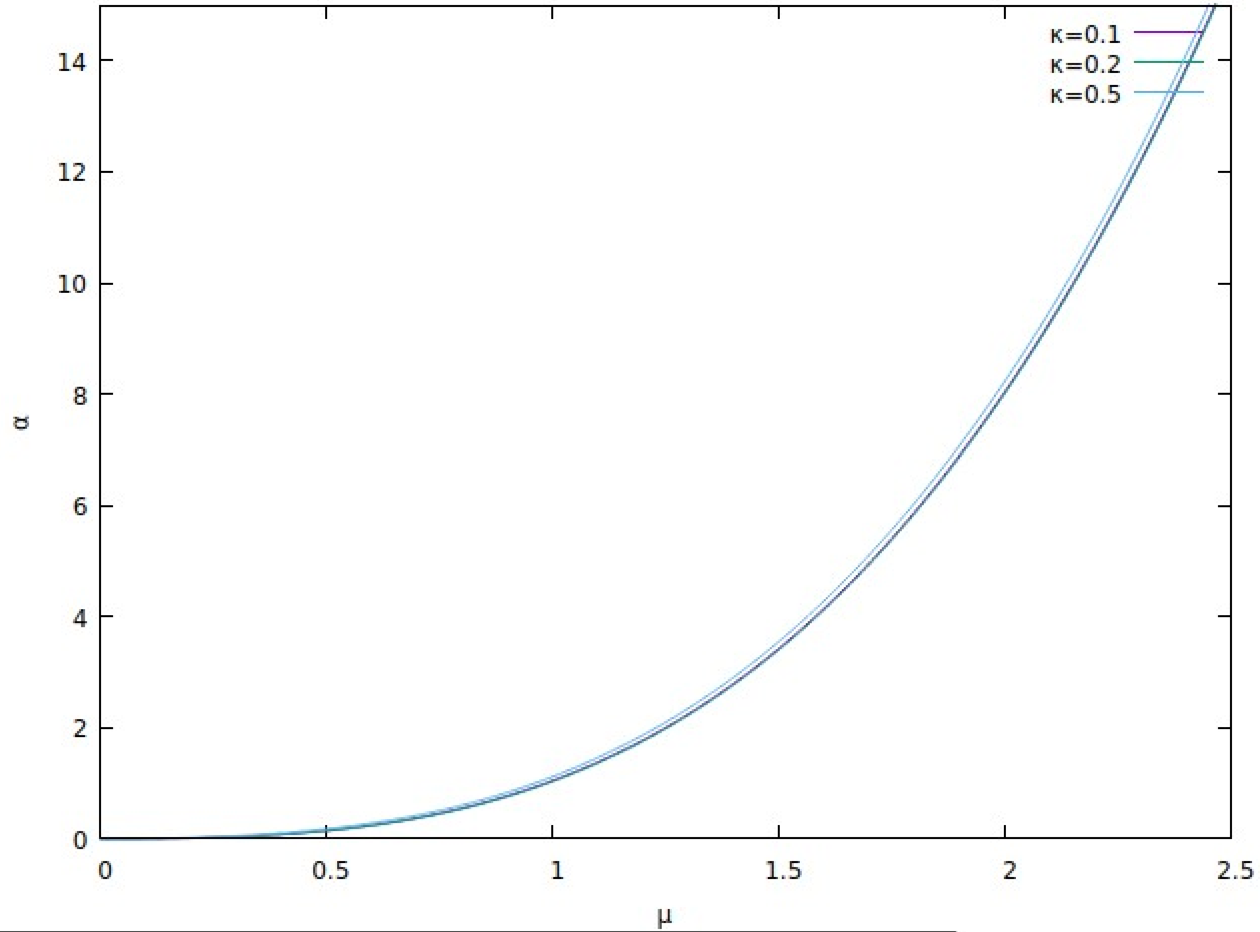


$\alpha=1$

$\alpha=10$

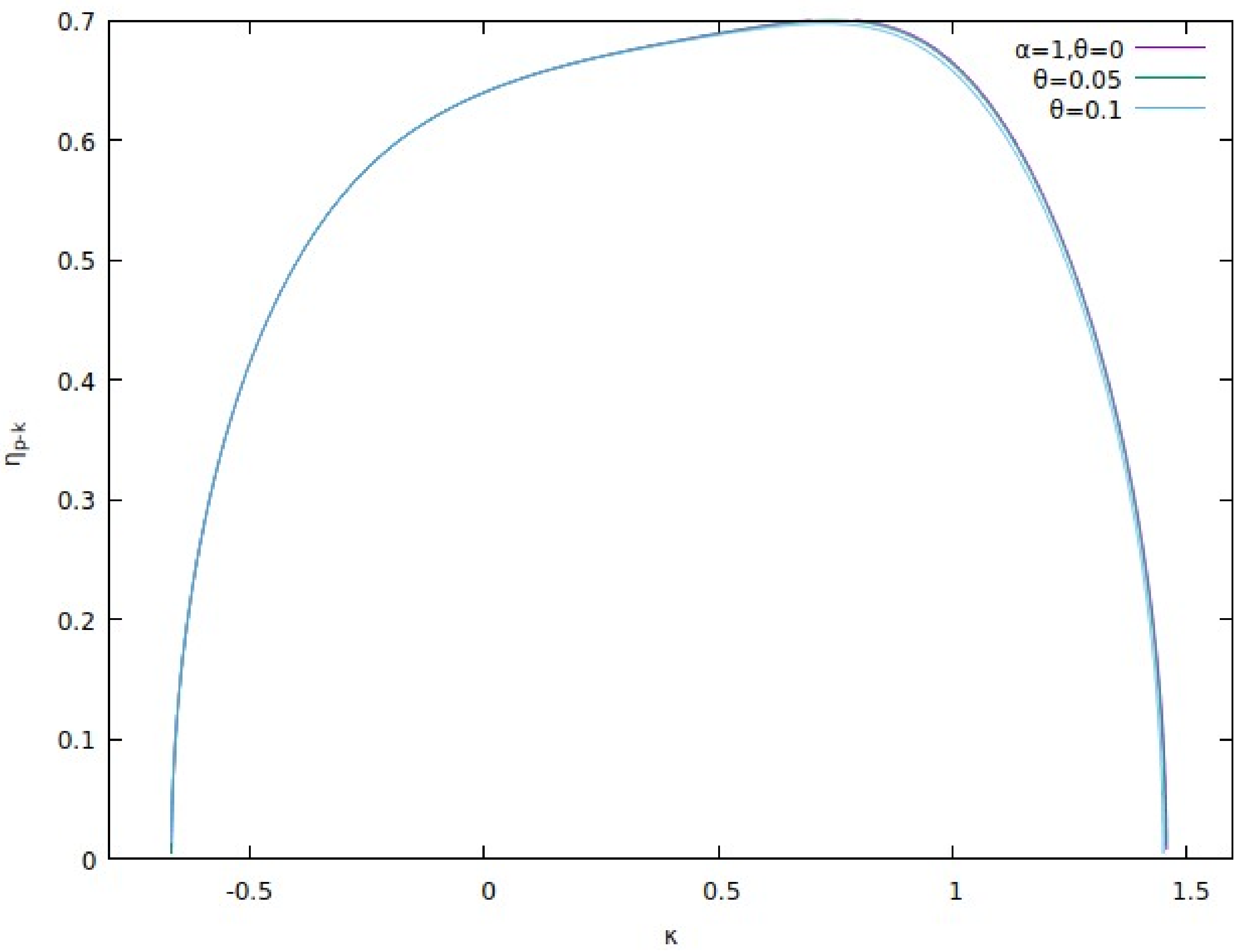
The boundary of stability

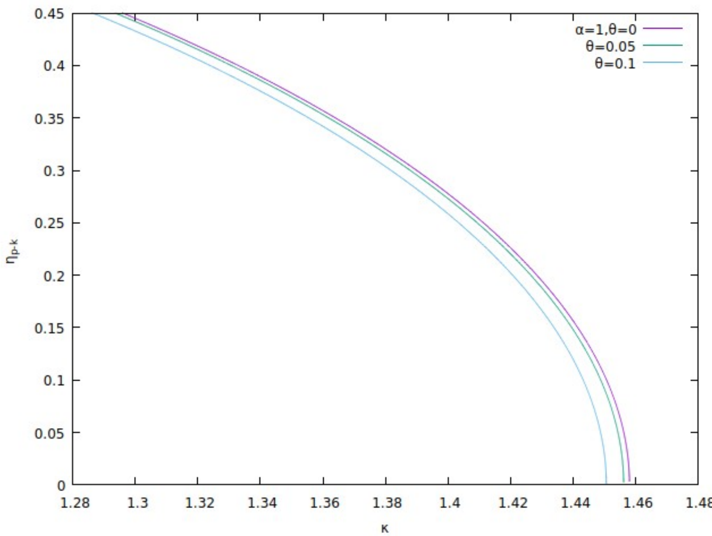
$$\eta^2_{p-k} = 0$$



$$\alpha(\mu) = 4\sqrt{\frac{\beta_{v-k}}{\beta_k}(\beta_k + \beta_{v-k} - 1)^2(-3\beta_k + \beta_{v-k} - 1)^2}$$

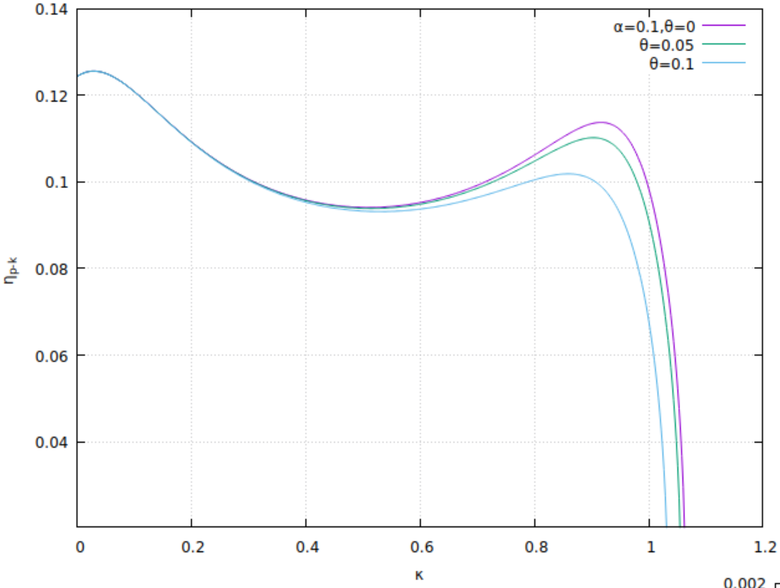
Dependence of $\eta_{p\text{-}k}$ on κ for α=1

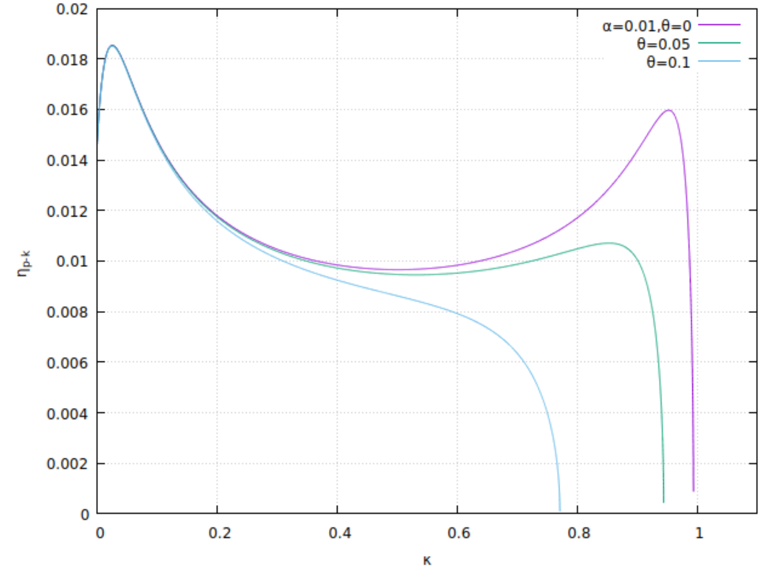




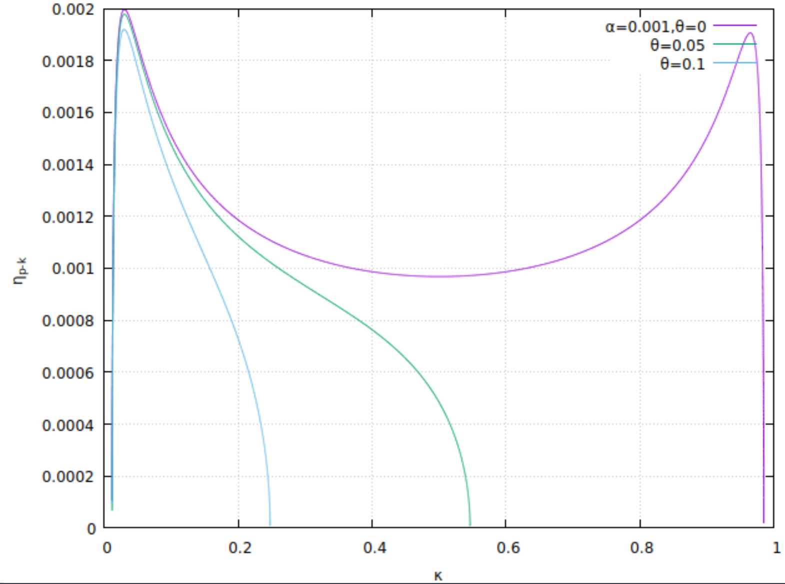
$$\mu = \sqrt{\alpha}/2$$

Dependence of $\eta_{p\text{-}k}$ on κ for various α and fixed θ





α=0.001



α=0.1

α=0.01

Decay rate

The decay rate for the process $\phi_i \to \phi_j + \phi_k$ [3]

$$\Gamma = \int \frac{d^3p_j}{(2\pi)^3} \int \frac{d^3p_k}{(2\pi)^3} \frac{d|S_{fi}|^2}{dt}$$

Scattering matrix for this process

$$S_{fi} = \int_0^T dt \int d^3x \langle f|H_I|i\rangle,$$ **where** $H_I = g\phi\chi^2$

[3] N.Herring, B.Pardo, D.Boyanovsky arXiv:1808.02539

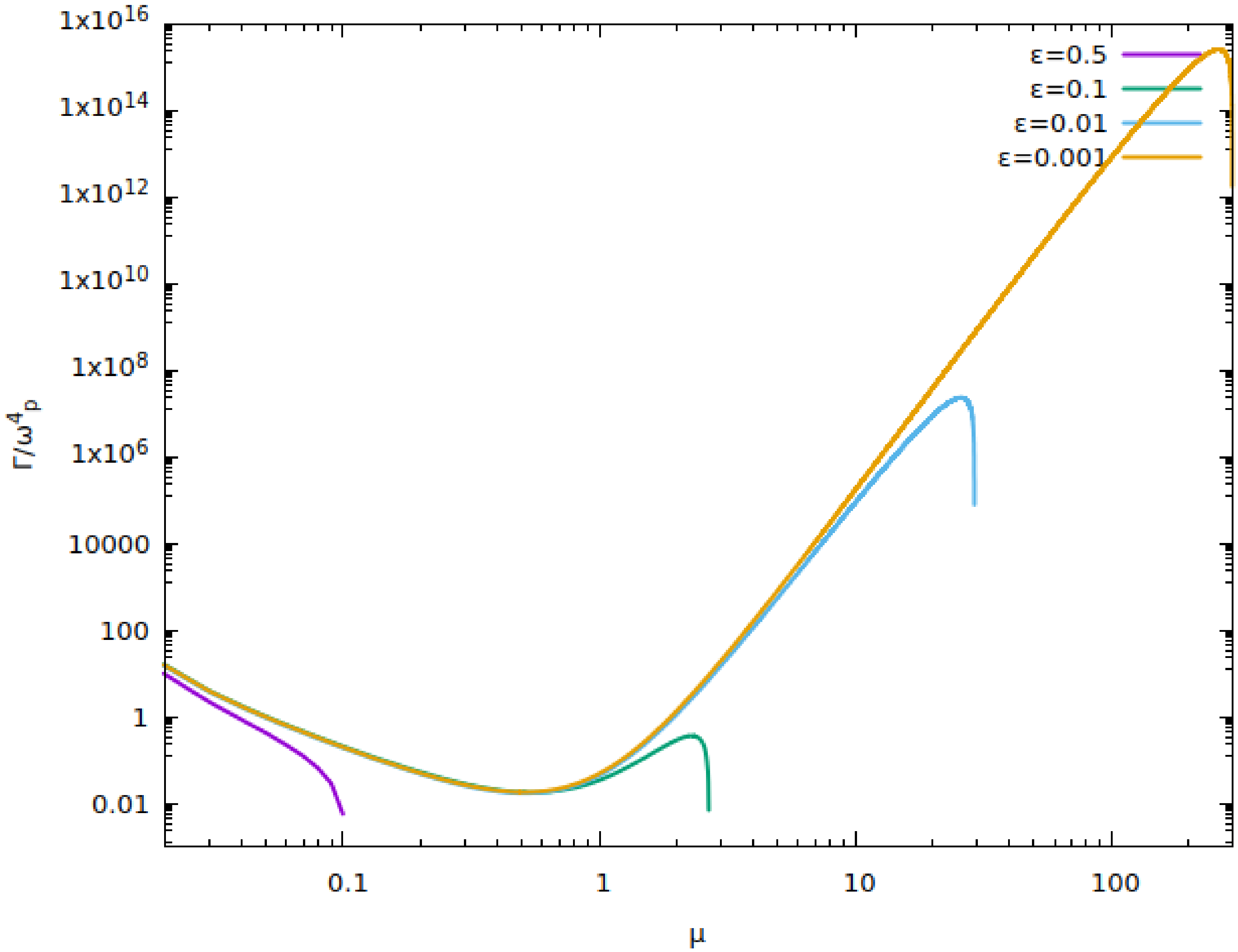
The final equation for the decay rate

$$\Gamma = \frac{\omega_{\vec{p}}^4 \alpha}{16(2\pi)^5} \int_0^{\pi} \int_{\kappa_-}^{\kappa_+} \frac{\sinh(\theta)\kappa^2 d\kappa d\theta}{\beta_{\vec{k}} \beta_{\vec{v}-\vec{\kappa}}} \Big(16 C_1^2 \eta^2 (\sinh(2) - \\ -4\eta_{\vec{v}-\vec{\kappa}} \sinh(1)) + \frac{(1 - C_2^2 + C_1^2)^2}{4\eta_{\vec{v}-\vec{\kappa}}} + \\ +\eta_{\vec{v}-\vec{\kappa}} (1 - (C_2^2 - C_1^2)^2 (\cosh(1) + (1))) + \\ +2\eta_{\vec{v}-\vec{\kappa}}^3 \sinh(2)(1 + C_2^2 - C_1^2) \Big)$$

where

$$C_1 = \frac{\frac{\epsilon_{\vec{v}-\vec{\kappa}}^2}{4} - \eta_{\vec{v}-\vec{\kappa}}^2 - \beta_{\vec{\kappa}} \epsilon_{\vec{v}-\vec{\kappa}}}{\eta_{\vec{v}-\vec{\kappa}}(\epsilon_{\vec{v}-\vec{\kappa}} - 2\beta_{\vec{\kappa}})} \qquad C_2 = \frac{\sigma_{\vec{v}-\vec{\kappa}}}{\eta_{\vec{v}-\vec{\kappa}}(\epsilon_{\vec{v}-\vec{\kappa}} - 2\beta_{\vec{\kappa}})}$$

Dependence of the decay rate on the μ for $\kappa_-=0,1$ and $\kappa_+=0,9$



Standing wave

$$\phi(\xi,t) = 8\frac{\sqrt{2\rho_\phi}}{\omega}\cos(p_x\xi)\cos(p_y\xi)\cos(p_z\xi)\cos(\omega t)$$

The equation

$$e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}}) = \sigma_{\vec{p}_j-\vec{k}}a^\dagger_{\vec{p}_j-\vec{k}}e^{i(\Omega_{\vec{p}_j-\vec{k}}-\omega)t}$$

$$j = 1, .., 4$$

Solution

$$a_{\vec{k}}(t) = e^{i\epsilon_{\vec{p_j}-\vec{k}}t/2}\Big[a_{\vec{k}}(0)(\cosh(s_{\vec{p_j}-\vec{k}}t) - i\frac{\frac{\epsilon^2_{\vec{p_j}-\vec{k}}}{4} - s^2_{\vec{p_j}-\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{p_j}-\vec{k}}}{s_{\vec{p_j}-\vec{k}}(\epsilon_{\vec{p_j}-\vec{k}} - 2\Omega_{\vec{k}})}\sinh(s_{\vec{p_j}-\vec{k}}t)) -$$

$$-i\frac{\sigma_{\vec{p_j}-\vec{k}}}{s_{\vec{p_j}-\vec{k}}(\epsilon_{\vec{p_j}-\vec{k}} - 2\Omega_{\vec{k}})}a^\dagger_{\vec{p_j}-\vec{k}}(0)\sinh(s_{\vec{p_j}-\vec{k}}t)\Big]$$

$$\vec{\xi} = (\xi_x, \xi_y, \xi_z), \vec{p_1} = (p_x, p_y, p_z), \vec{p_2} = (p_x, -p_y, p_z)$$

$$\vec{p_3} = (-p_x, -p_y, p_z), \vec{p_4} = (-p_x, -p_y, p_z)$$

Conclusions

- **The decay of φ→χχ occurs at the amplitude of the field φ above the threshold, not only in the case of small masses($m_\chi \ll \omega_p$) - the case of A.Arza, but also at an arbitrary mass**
- **In the case of large masses ($m_\chi \gg \omega_p$), the required threshold amplitude of the field φ is powerfully greater in comparison with the case of small amplitudes**

Thanks for your attention!

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