Stochastic Relaxation of the Electroweak Scale

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QUANTUM UNIVERSE

Outline of this talk

- Review of the **relaxion mechanism**
- Role of quantum **fluctuations** during relaxation
  - **Stochastic** dynamics
  - Can the relaxion explain **dark matter**?
  - Can the relaxion be a **QCD axion**?

- Review of the **relaxion mechanism**

- Role of quantum **fluctuations** during relaxation
  - **Stochastic** dynamics
  - Can the relaxion explain **dark matter?**
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Scalar fields in cosmology

Wide range of applications

- Strong CP problem, dark matter, dark energy, inflation, …

Weinberg, PRL 40, 223.

Peccei, Quinn, PRL 38, 1440.

Well-motivated example: **axion** (dynamical solution to the **strong CP problem**)

- The standard model Lagrangian allows for a term

$$\mathcal{L}_{\bar{\Theta}} = -\bar{\Theta}\frac{g_s^2}{32\pi^2}F_{a\mu\nu}\tilde{F}_a^{\mu\nu}$$

- Experiments constraint $\bar{\theta} < 10^{-10}$, (no CP violation in the strong sector)
- The axion field φ couples to the QCD anomalous term

$$\mathcal{L} = -\frac{\varphi}{f}\frac{g_s^2}{32\pi^2}F_{a\mu\nu}\tilde{F}_a^{\mu\nu}$$

- The effective low-energy potential is minimized at the CP conserving value $\varphi = 0$.

Vafa, Witten, Nucl.Phys. B234 (1984)

$$U(\varphi) = \Lambda_b^4[1 - \cos(\varphi/f)]$$

In this work I focus on the **relaxion** and the **Higgs mass hierarchy problem**.

The (GKR) relaxion mechanism

Graham et. al., 1504.07551

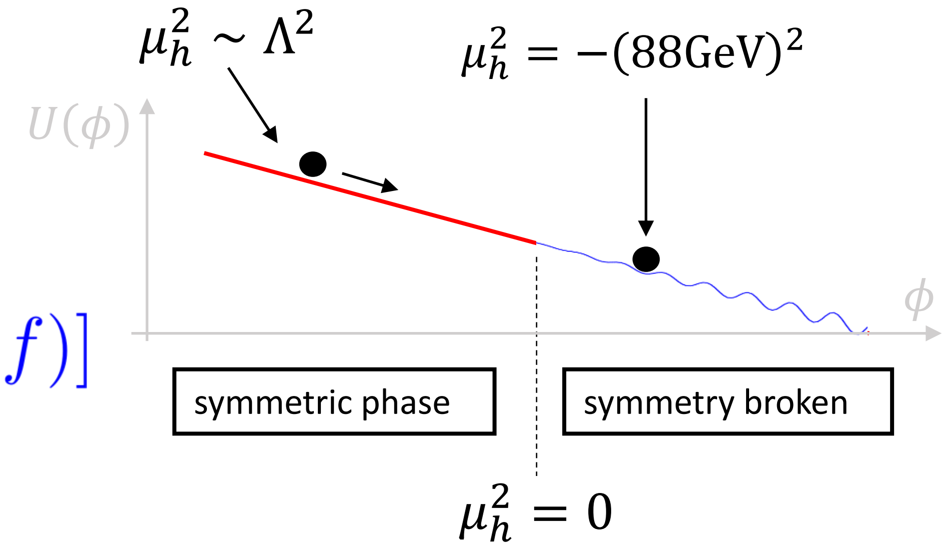
- **Dynamical Higgs mass,** controlled by vev of ϕ

$$\mu_h^2 \to \mu_h^2(\phi) = \Lambda^2 - g\Lambda\phi$$

  - **Rolling potential** for ϕ

$$U(\phi) = -g\Lambda^3\phi + \Lambda_b^4(v_h)[1 - \cos(\phi/f)]$$

- Higgs-vev-dependent **relaxion barriers.**



Stopping mechanism

Slow-roll dynamics **during inflation,**

$$\dot{\phi}_{SR} = \frac{U'}{3H_I}$$

⬇

The relaxion stops near the first minimum: $\Lambda_b^4 \sim g\Lambda^3 f$.

The relaxion-Higgs potential

$$U(h,\phi) = \frac{1}{2}[\Lambda^2 - g\Lambda\phi]h^2 + \frac{\lambda_h}{4!}h^4 - g\Lambda^3\phi + \Lambda_b^4(v_h)[1 - \cos(\phi/f)]$$

The QCD and nonQCD models

The **QCD** relaxion model

- Higgs-dependent barriers from the **QCD anomaly,**

$$\Lambda_b^4(v_h) \approx \Lambda_{QCD}^3 m_u$$

- Problem: the relaxion no longer solves the **strong CP problem!**

$$\bar{\theta} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right) \sim \mathcal{O}(1)$$

The **nonQCD** relaxion model

- Higgs-dependent barriers from a **hidden gauge group**

Parameter region: the nonQCD model

1) Vacuum energy

The **change of relaxion energy** much less compared to the **energy scale of inflation**

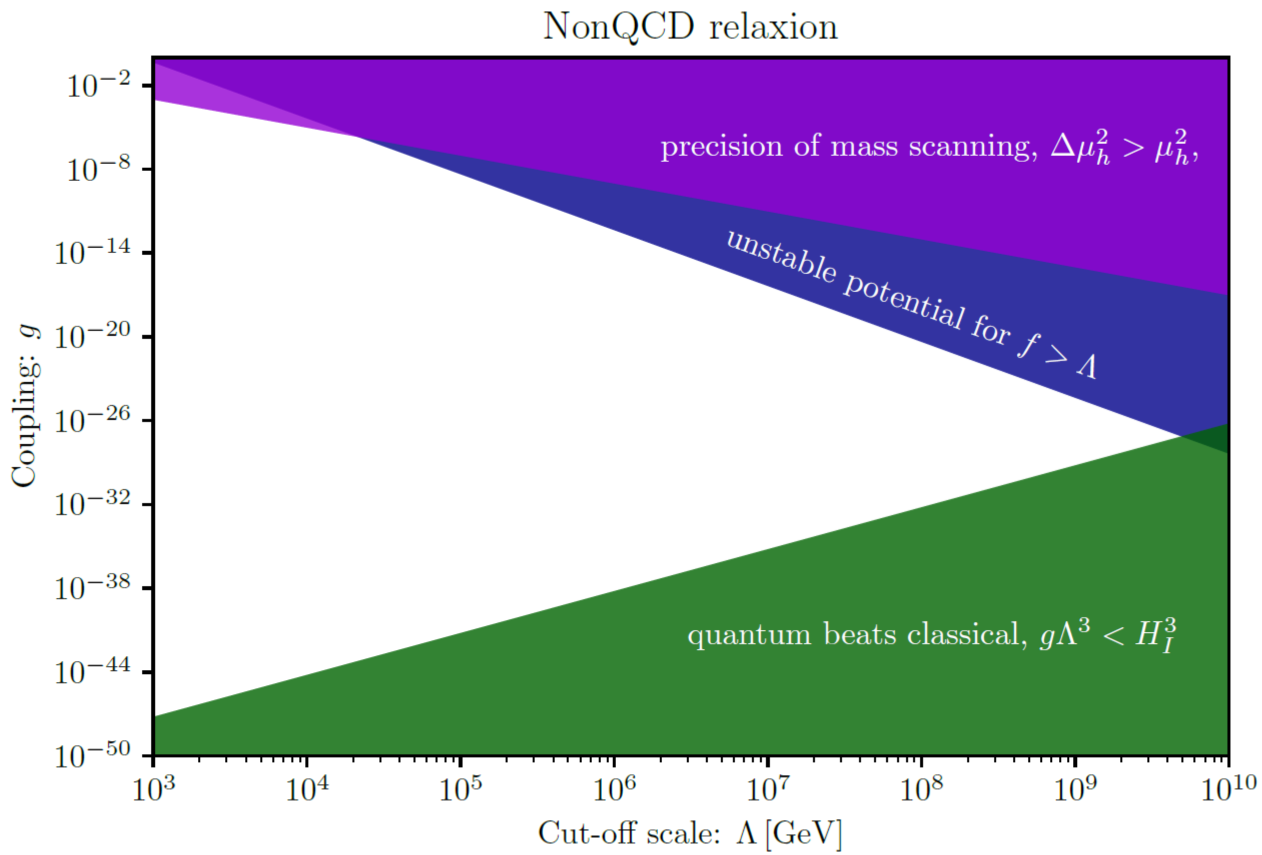
$$\Delta U \sim \Lambda^4 < H_I^2 M_{Pl}^2$$

2) Classical beats quantum

The **slow-roll** ($\dot{\phi} = g\Lambda^3/3H_I$) per unit Hubble time dominates over the random walk ($\Delta\phi \sim H_I$)

$$H_I < (g\Lambda^3)^{1/3}$$

1) + 2) ➡ $$\frac{\Lambda^2}{M_{Pl}} < H_I < g^{1/3}\Lambda$$



$$\Lambda < 4 \times 10^9 \mathrm{GeV} \left(\frac{\Lambda_b}{\sqrt{4\pi v_h}}\right)^{4/7}$$

- Review of the **relaxion mechanism**

- Role of quantum **fluctuations** during relaxation
  - **Stochastic** dynamics
  - Can the relaxion explain **dark matter?**
  - Can the relaxion be a **QCD axion?**

The stochastic formalism

- The relaxion vev receives random kicks from superhorizon fluctuations

- The dynamics can be described in terms of a **Fokker-Planck equation**

$$\frac{d\rho}{dt} = \boxed{\frac{1}{3H_I}\frac{\partial(\rho\,\partial_\phi U)}{\partial\phi}} + \boxed{\frac{H_I^3}{8\pi^2}\frac{\partial^2\rho}{\partial\phi^2}}$$

$\rho(\phi, t)$ - probability distribution

Drift term

Diffusion term

e.g. 9407016

Diffusion introduces new effects, such as

- the ***broadening*** of the distribution, $\sigma(t) = \sqrt{\frac{H_I^3}{4\pi^2}t}$

Nelson et. al., 1708.00010

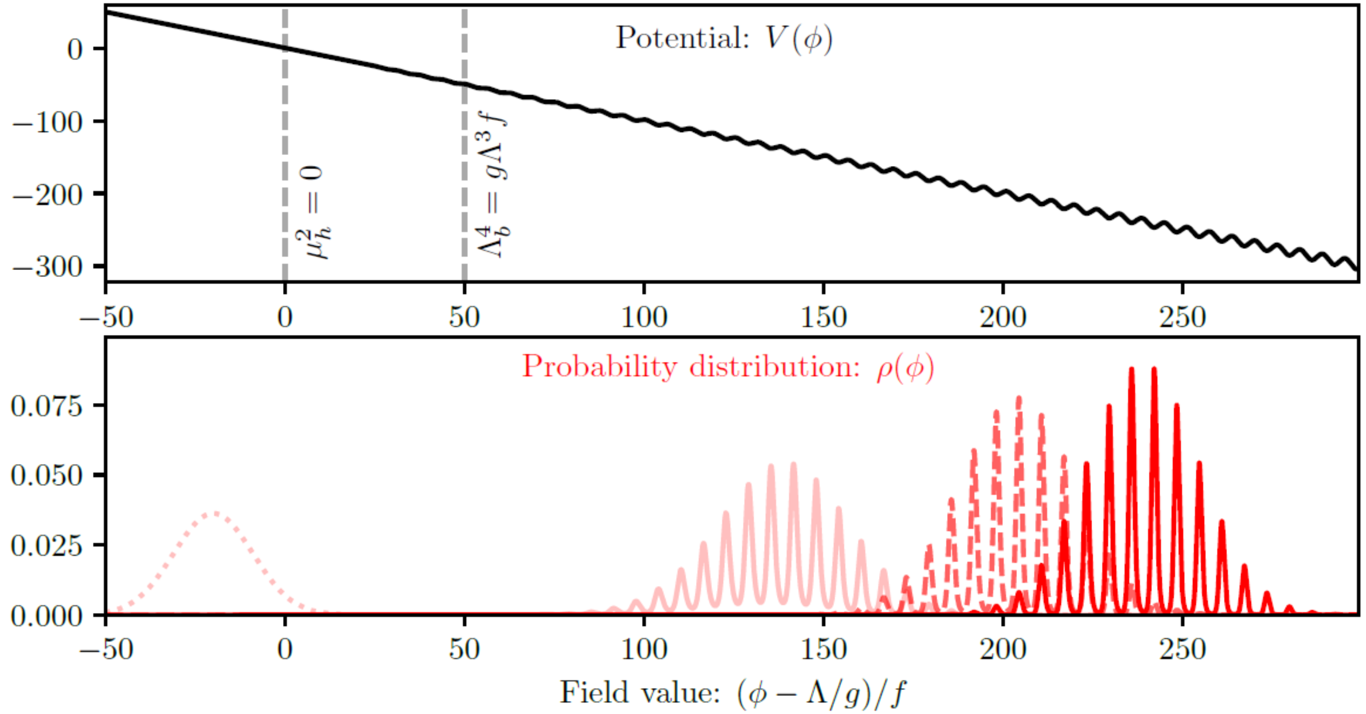
- ***probability fluxes*** between neighboring local minima,

$$k \approx \frac{\sqrt{U_0''|U_b''|}}{6\pi H_I}e^{-B}, \qquad B = \frac{8\pi^2\Delta U_b}{3H_I^4}$$

Hawking-Moss instanton

PLB 110 (1982) 35.

Illustration of the dynamics & stopping



The relaxion slows down after

$$B = \frac{8\pi^2 \Delta U_b}{3H_I^4} \sim 1$$

The new stopping condition

$$\Lambda_b^4 \sim \max\left(g\Lambda^3 f, H_I^4\right).$$

- Review of the **relaxion mechanism**

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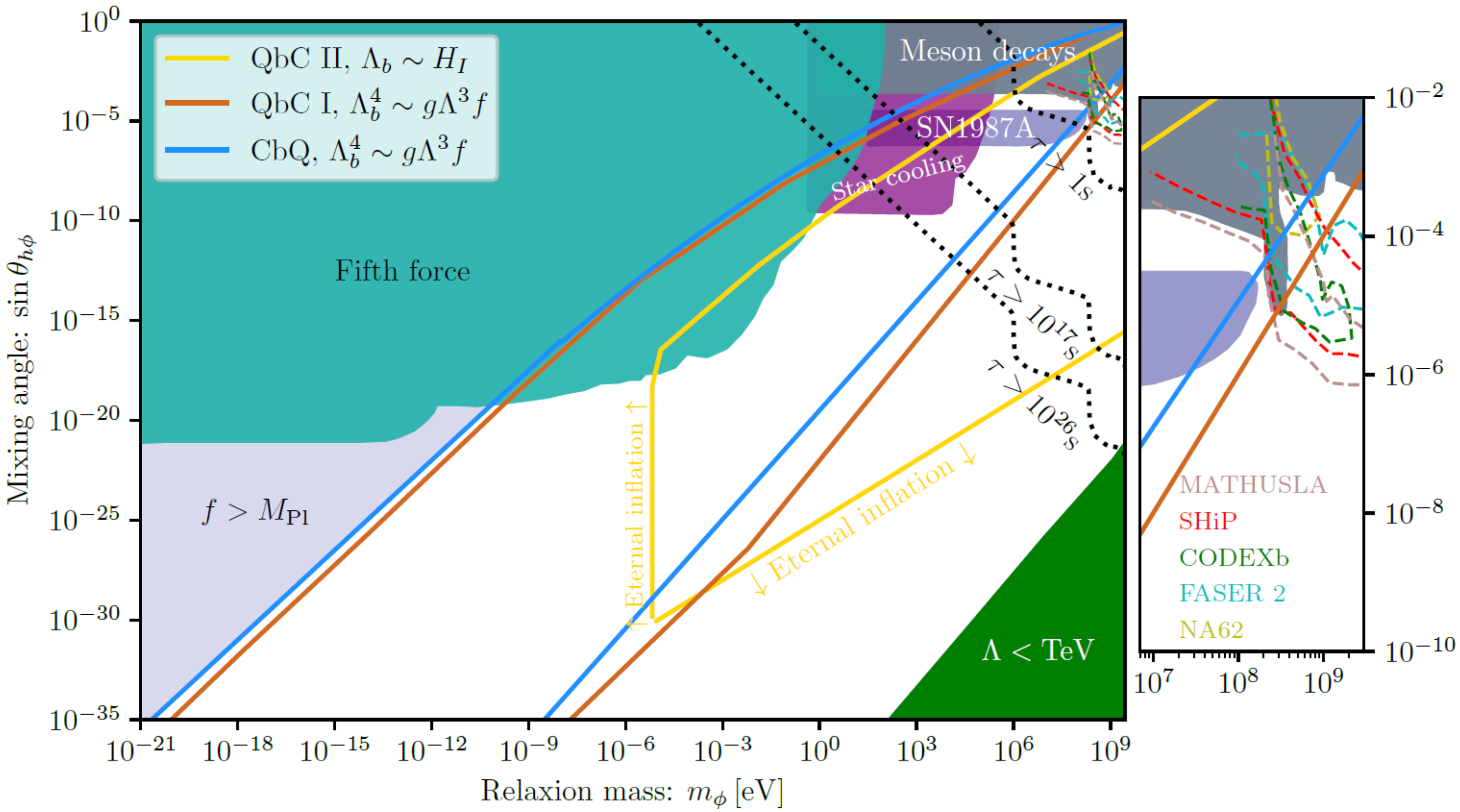
Relaxion-Higgs mixing

In most of the parameter space the relaxion is light and long-lived, interacts via its Higgs mixing

$$\sin(2\theta_{h\phi}) = -\frac{2m_{h\phi}^2}{\sqrt{(2m_{h\phi}^2)^2 + (m_h^2 - m_\phi^2)^2}}.$$

$$m_{h\phi}^2 = \frac{\partial^2 U}{\partial h \partial \phi}$$



Flacke et. al., 1610.02025

Hardy et al., 1611.05852

Balaji et al., 2205.01699

Relaxion dark matter window

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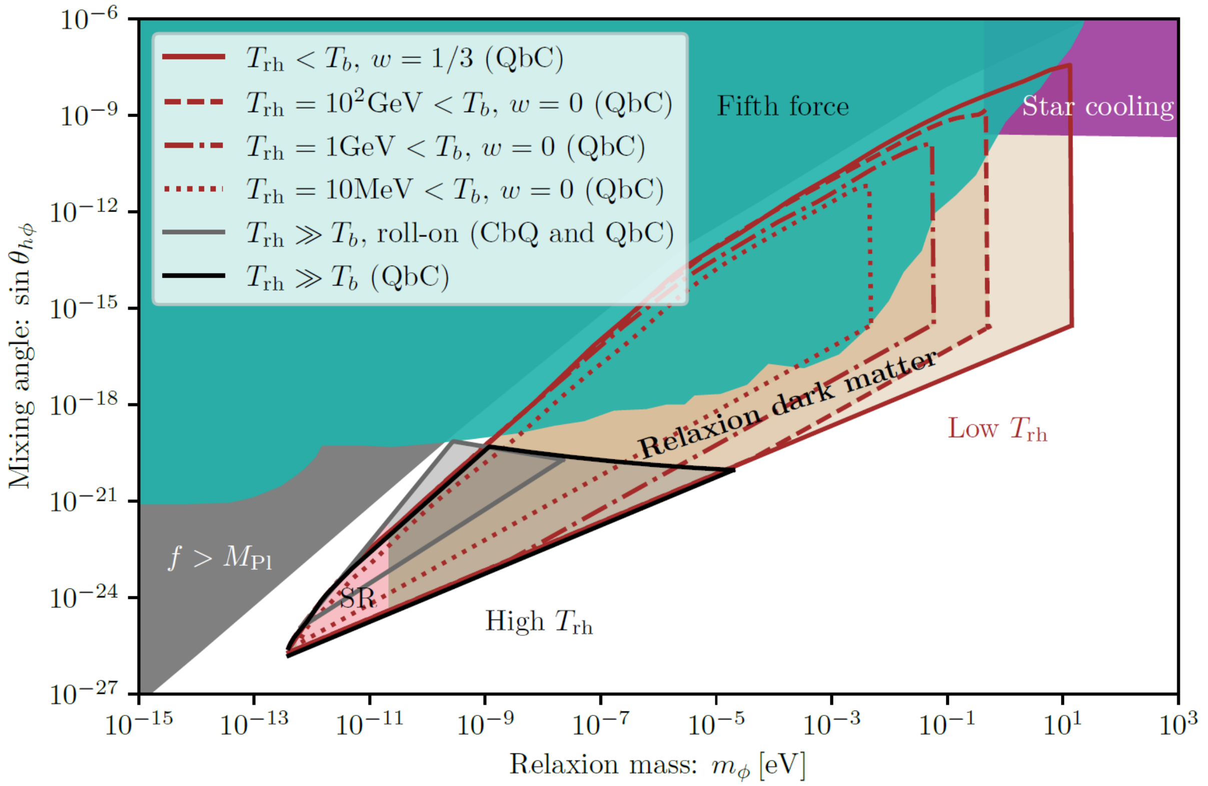

Can the relaxion account for the dark matter?

Yes, via its misalignment from the minimum

Brown: low reheating temperature, stochastic misalignment

$$\rho(\phi) \propto \exp\left(-\frac{8\pi^2 V(\phi)}{3H_I^4}\right)$$

Graham et al 1805.07362
Takahashi et al 1805.08763

Grey: high reheating temperature, misalignment from roll-on after reheating Banerjee et. al., 1810.01889

Black: high reheating temperature, stochastic misalignment

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Implications for the QCD relaxion model

The local minima of the relaxion potential are at

$$0 = V'(\phi_0) = -g\Lambda^3 + \frac{\Lambda_b^4(\phi_0)}{f} \sin\left(\frac{\phi_0}{f}\right)$$

resulting in a nonvanishing θ-angle,

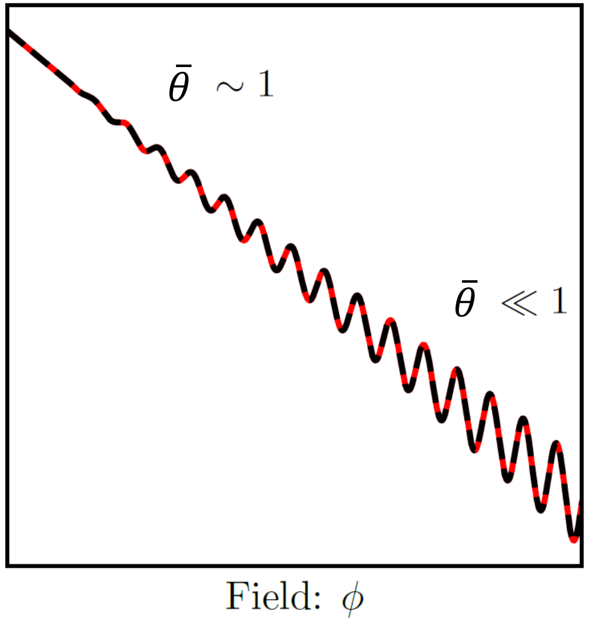
$$\bar{\theta} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right)$$

Strong CP problem **can be solved in the QbC regime** if

$$H_I \sim \Lambda_b \approx 75\text{MeV}$$

and

$$g\Lambda^3 f < 10^{-10}\Lambda_b^4$$



Summary

- New perspective into the relaxation mechanism when considering the effects of fluctuations i.e. the **stochastic dynamics**.

- The mechanism **can address multiple problems at the same time.**

- Future directions: Other applications or observational signatures?

Thanks for your attention!

Backup slides

The parameter space

1) Vacuum energy

The **change of relaxion energy** much less compared to the **energy scale of inflation**

$$\Delta U \sim \Lambda^4 < H_I^2 M_{Pl}^2$$

2) Classical beats quantum (CbQ)

The **slow-roll** ($\dot{\phi} = g\Lambda^3/3H_I$) per unit Hubble time dominates over the random walk ($\Delta\phi \sim H_I$)

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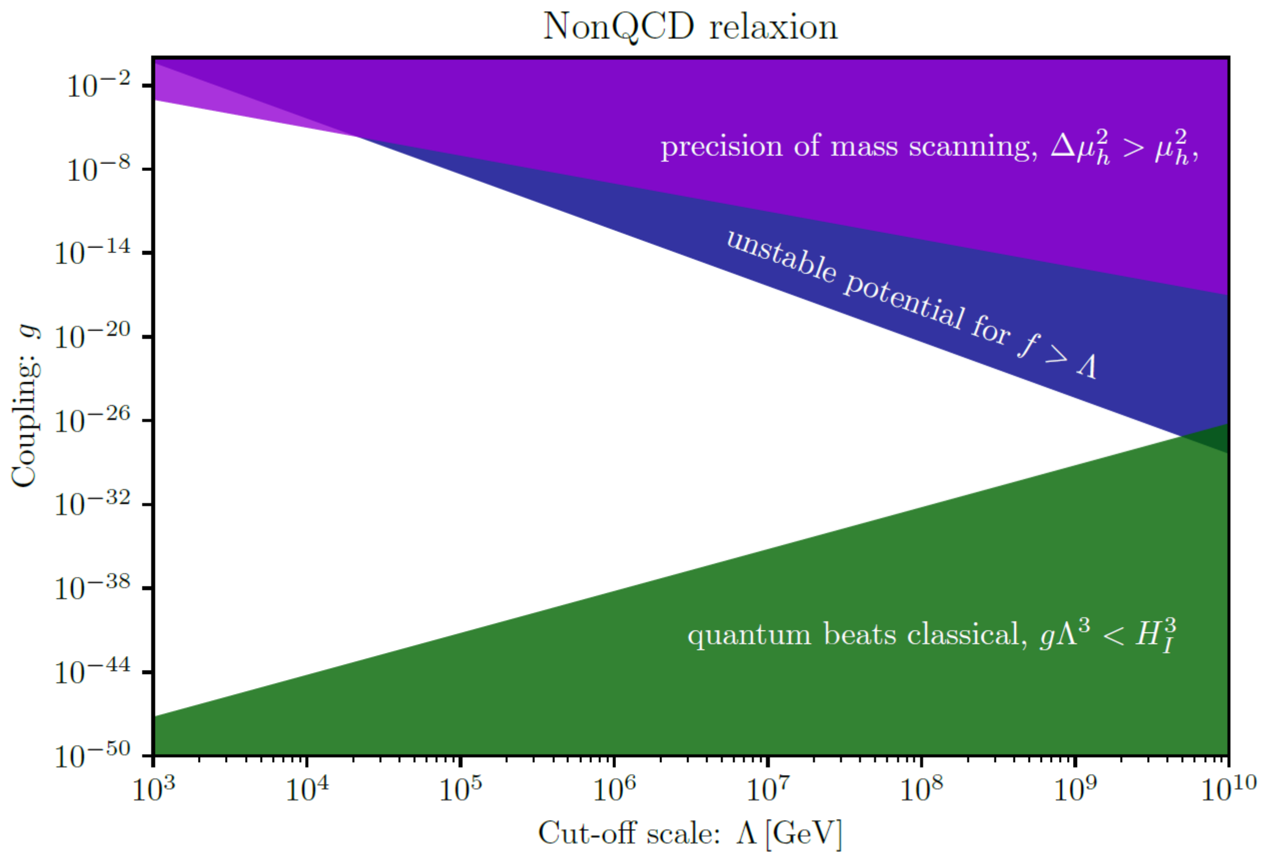


1) + 2) ➡

$$\frac{\Lambda^2}{M_{Pl}} < H_I < g^{1/3}\Lambda$$

$$\Lambda < 4 \times 10^9 \text{GeV} \left(\frac{\Lambda_b}{\sqrt{4\pi} v_h} \right)^{4/7}$$

The QCD model with a change of slope

Graham et. al., 1504.07551

The local minima of the relaxion potential are **not** CP conserving

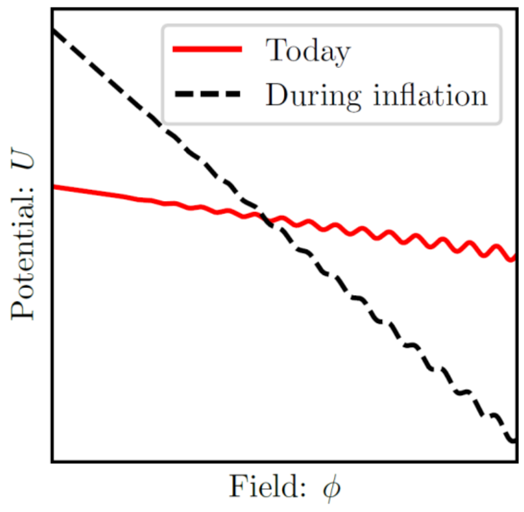
$$0 = U'(\bar{\theta}) = -g\Lambda^3 + \frac{\Lambda_b^4}{f}\sin\bar{\theta} \quad \longrightarrow \quad \bar{\theta} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right)$$

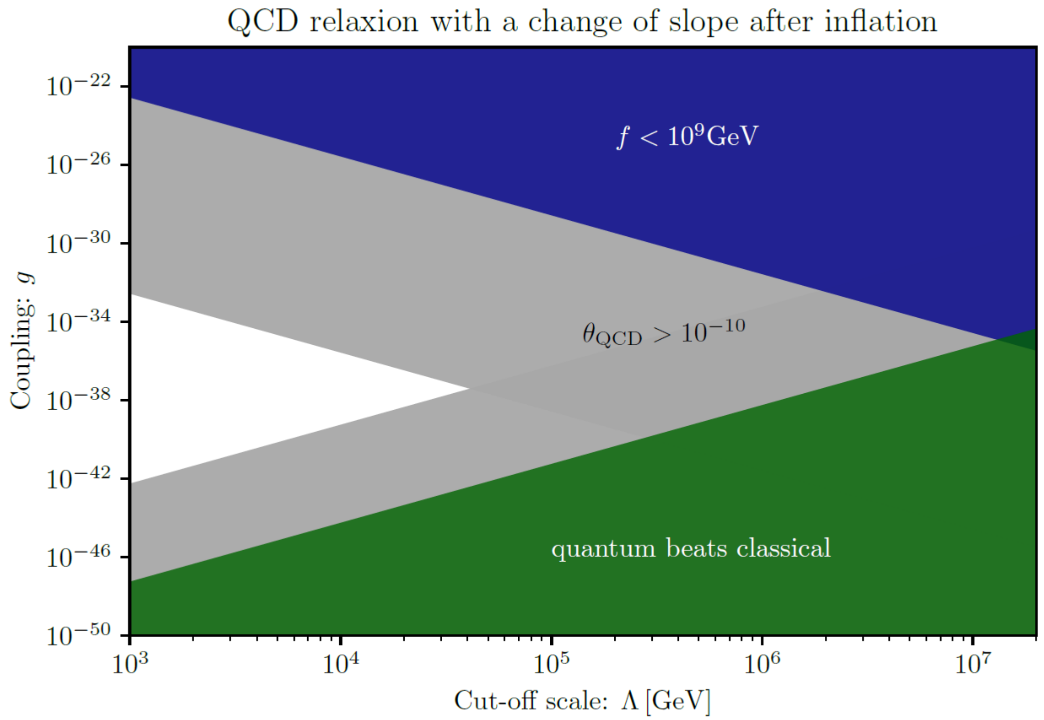
Solution: the slope of the potential drops after inflation,

$$g = \xi g_I, \quad \xi < 10^{-10},$$

to reduce CP violation

$$\bar{\theta} = \xi\bar{\theta}_I < 10^{-10}.$$





$$\Lambda < 3 \times 10^4\text{GeV}\left(\frac{10^9\text{GeV}}{f}\right)^{1/6}\left(\frac{\xi}{10^{-10}}\right)^{1/4}.$$

Eternal inflation and volume weighting

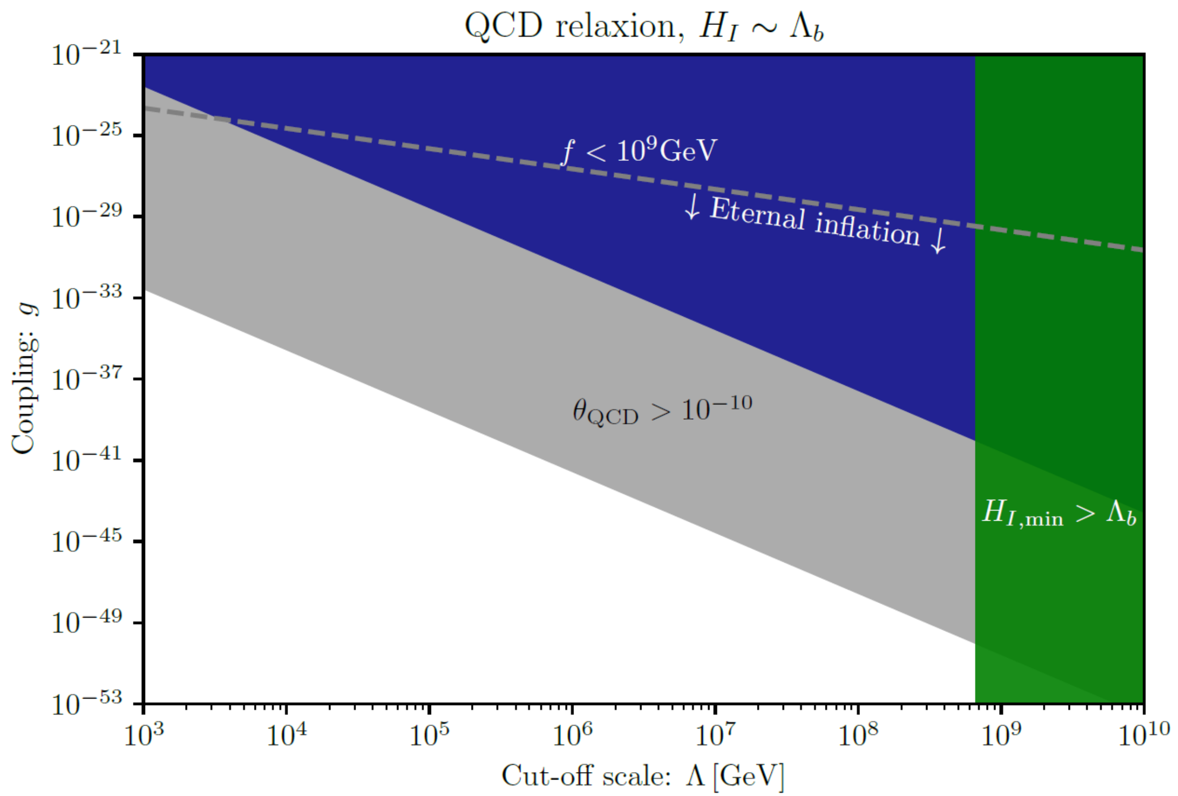
- The minimum number of e-folds of inflation required to relax the Higgs mass from $\mu_h \sim \Lambda$ to $\mu_h = 0$ is given by

$$N_I = H_I t_I > N_{\rm req} = \frac{3H_I^2}{g^2\Lambda^2}$$

- If $N_I > N_c \sim \frac{2\pi^2}{3}\frac{M_{\rm Pl}^2}{H_I^2}$, inflation is **eternal**.

0802.1067

- Eternal inflation has associated measure problems.

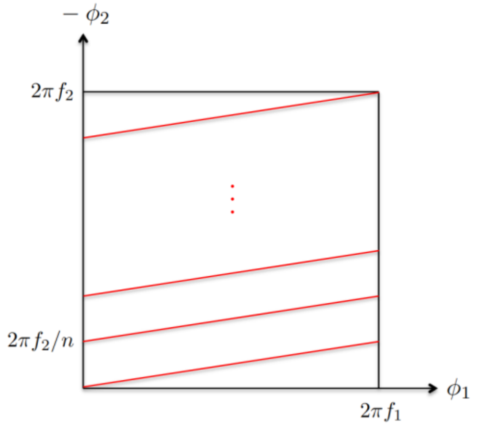


- Possible solution: using scale factor cut-off measure.

Nelson et. al., 1708.00010

Large field range from the clockwork mechanism

Example: 2 axions



$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \left(\tilde{V}_0 + V_0 + \mu_h^2|h|^2 + V_{\rm br} + \ldots\right)$$

$$\begin{aligned}
\tilde{V}_0 &= -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right), \\
V_0 &= -\epsilon f_2^4 \cos\left(\frac{\phi_2}{f_2} + \delta_2\right), \\
\mu_h^2 &= M_h^2 - \epsilon' f_2^2 \cos\left(\frac{\phi_2}{f_2} + \delta_2'\right), \\
V_{\rm br} &= -\Lambda_{\rm br}^4(h) \cos\left(\frac{\phi_1}{f_1} + \delta_1\right),
\end{aligned}$$

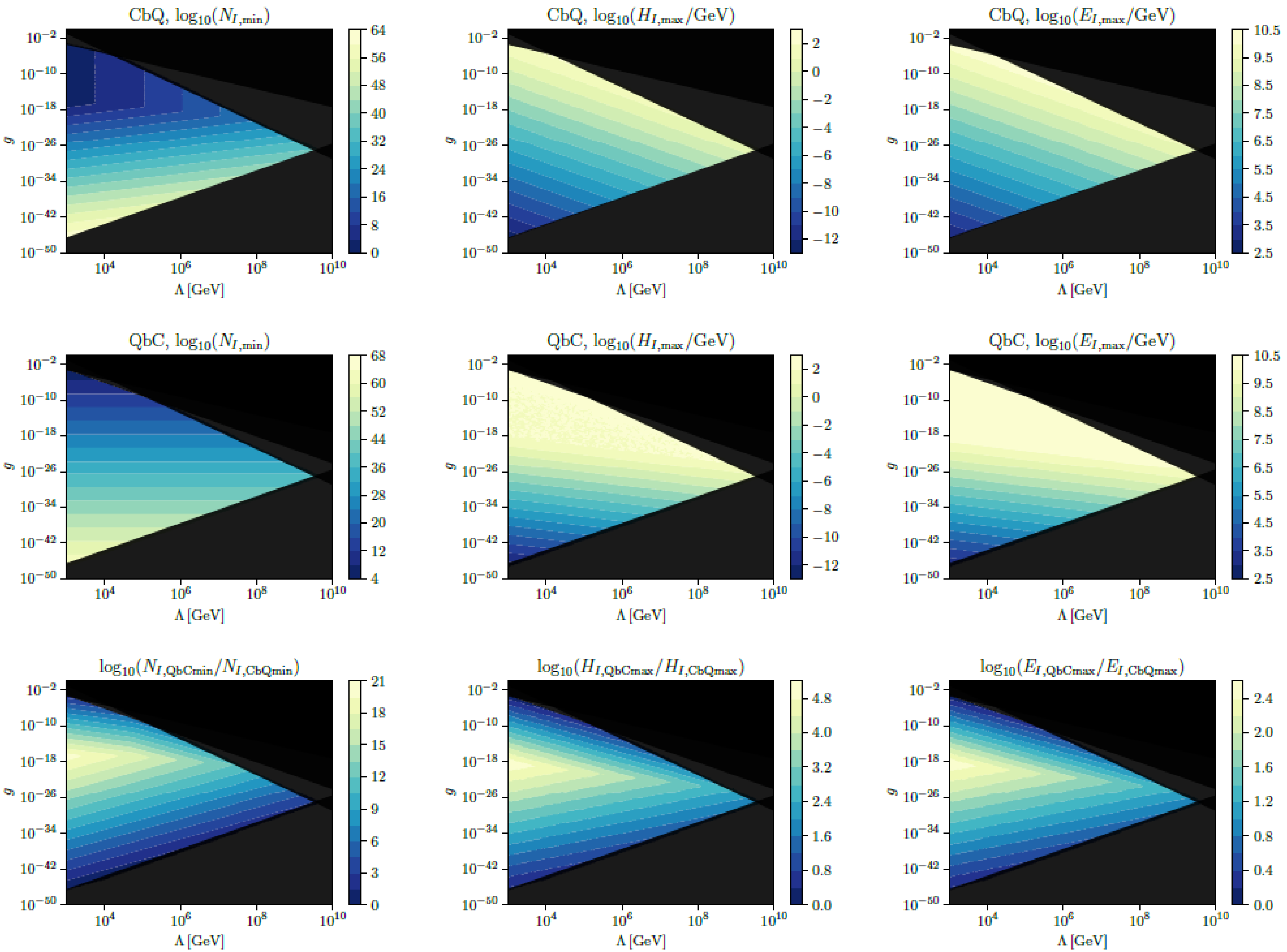
Choi et. al., 1511.00132

Low energy potential along the light direction

$$\begin{aligned}
V_{\rm eff} = &-\epsilon f_2^4 \cos\left(\frac{\phi}{f_{\rm eff}} - \delta_2\right) + \left(M_h^2 - \epsilon' f_2^2 \cos\left(\frac{\phi}{f_{\rm eff}} - \delta_2'\right)\right)|h|^2 \\
&- \Lambda_{\rm br}^4(h) \cos\left(\frac{\phi}{f} + \delta_1\right),
\end{aligned}$$

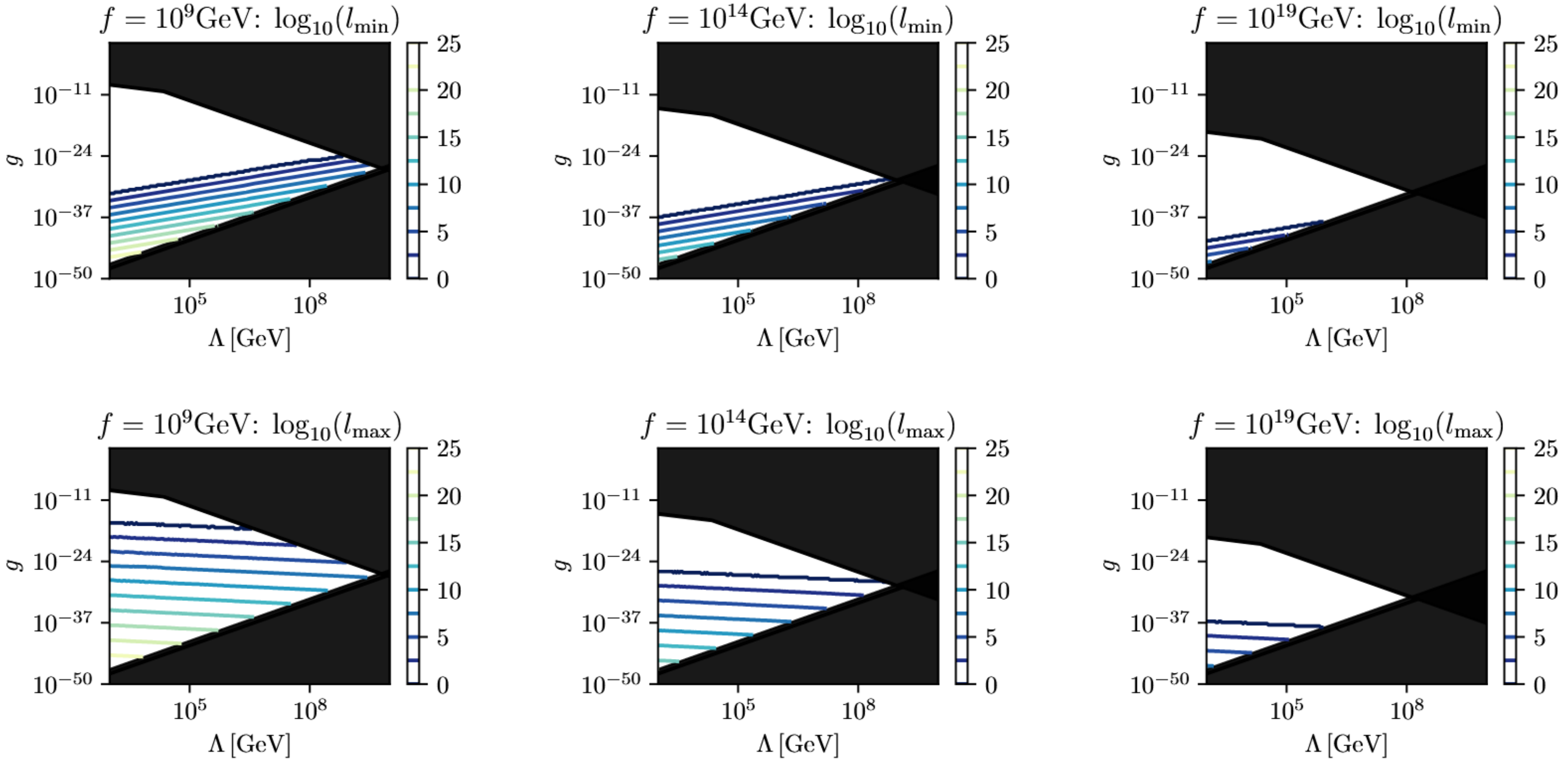
where $f_{\rm eff} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf.$

Required number of e-folds and scales of inflation



Implications for the classical-beats-quantum regime

- The usual stopping condition $\Lambda_b^4 \approx g\Lambda^3 f$ holds.
  - Only the nonQCD model is viable
- The relaxion does not always get trapped in the first minimum



$$l \sim \frac{1}{\delta_1^2}\left(\frac{9H_I^4}{16\pi^2\Lambda_b^4}\right)^{2/3} \approx \frac{(-\mu_h^2)}{g'\Lambda f}\left(\frac{9H_I^4}{16\pi^2\Lambda_b^4}\right)^{2/3}$$

Volume-weighting

- Volume-weighted Fokker-Planck equation

$$\frac{dP}{dt} = \frac{1}{3H_I}\frac{\partial(P\,\partial_\varphi V)}{\partial\phi} + \frac{H_I^3}{8\pi^2}\frac{\partial^2 P}{\partial\phi^2} + \frac{4\pi}{M_{\rm Pl}^2}\frac{V}{H_I}P \qquad P(\phi,t) = e^{3(H(\phi)-H_I)t}\rho(\phi,t)$$

- Does the relaxion climb up during inflation?

No, if $N_I < N_c$

$$\phi_{\rm peak}(t) = \dot{\phi}_{\rm SR}t - \frac{g\Lambda^3 H_I^2 t^2}{M_{\rm Pl}^2\pi}$$

- The fate of "wrong" Hubble patches ($\mu_h \sim \Lambda$) after inflation

The field slow-rolls down to the region with a small Higgs vev.

Gupta, 1805.09316

Axion abundance from stochastic misalignment

- If $H_{rh} > H_{osc}$, the onset of oscillations in the radiation dominated era.

$$\rho_{\phi,0} \approx \rho_{\phi,\mathrm{osc}} \left(\frac{a_{\mathrm{osc}}}{a_0}\right)^3 \approx \frac{m_\phi^2 \phi^2}{2} \left(\frac{T_0}{T_{\mathrm{osc}}}\right)^3 \left(\frac{g_{s,0}}{g_{s,\mathrm{osc}}}\right)$$

- If $H_{rh} < H_{osc}$, the onset of oscillations is before reheating. The fractional energy density today depends on the equation of state before reheating

$$\rho_{\phi,0} \approx \rho_{\phi,\mathrm{osc}} \left(\frac{a_{\mathrm{osc}}}{a_{\mathrm{rh}}}\right)^3 \left(\frac{a_{\mathrm{rh}}}{a_0}\right)^3 \approx \frac{m_\phi^2 \phi^2}{2} \left(\frac{H_{\mathrm{rh}}}{H_{\mathrm{osc}}}\right)^{2/(1+w)} \left(\frac{T_0}{T_{\mathrm{rh}}}\right)^3 \left(\frac{g_{s,0}}{g_{s,\mathrm{rh}}}\right)$$

Combining the two cases:

$$\frac{\langle \Omega_{\phi,0} \rangle}{\Omega_{\mathrm{DM}}} \approx 20 \left(\frac{\mathrm{eV}}{m_\phi}\right)^{3/2} \left(\frac{H_I}{100\mathrm{GeV}}\right)^4 \min\left\{1, \left(\frac{H_{\mathrm{rh}}}{H_{\mathrm{osc}}}\right)\right\}^{\frac{1-3w}{2(1+w)}}$$

The case of high reheating temperature

- The displacement after inflation

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} - g\Lambda^3 + C(T)\frac{\Lambda_b^4}{f}\sin\left(\frac{\phi}{f}\right) = 0$$

Where for simplicity we take $C(T(t)) = \theta(T_b/T(t) - 1)$

- The total displacement of the field

$$\Delta\phi \approx \frac{g\Lambda^3}{4H_b^2}$$

- The field gets re-trapped if $\Delta\phi < \phi_b - \phi_0$
  - Additional constraints on the parameter region.

- DM from roll-on was studied in Banerjee et. al., 1810.01889
- DM from stochastic misalignment

$$10^{-13}\left(\frac{\Lambda}{\text{TeV}}\right)^{\frac{16}{7}}\left(\frac{M_{Pl}}{f}\right)^{\frac{4}{7}} < \frac{m_\phi}{\text{eV}} < 6\times 10^{-6}\left(\frac{g(T_b)}{100}\right)\left(\frac{T_b}{100\text{GeV}}\right)^4\left(\frac{\text{TeV}}{\Lambda}\right)^2$$

The case of the relaxion

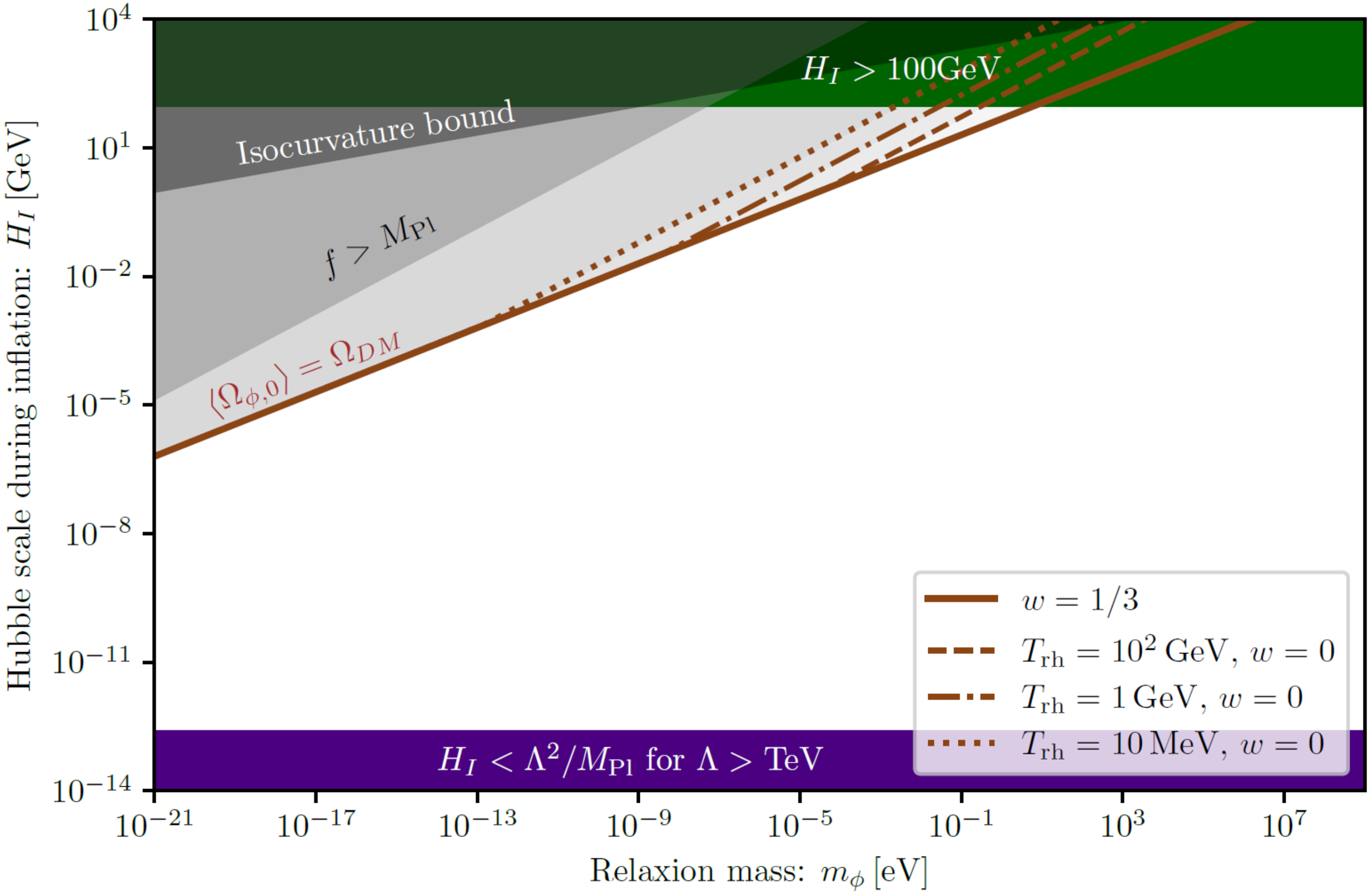
In which local minimum does the relaxion end up?

$$B = \frac{8\pi^2 \Delta V_b^{\rightarrow}}{3H_I^4} \sim 1$$

Stopping condition

The barriers disappear at $T > T_b$
(T_b is at most the weak scale)

- Additional displacement for $T_{rh} \gg T_b$



Bounds on isocurvature fluctuations:

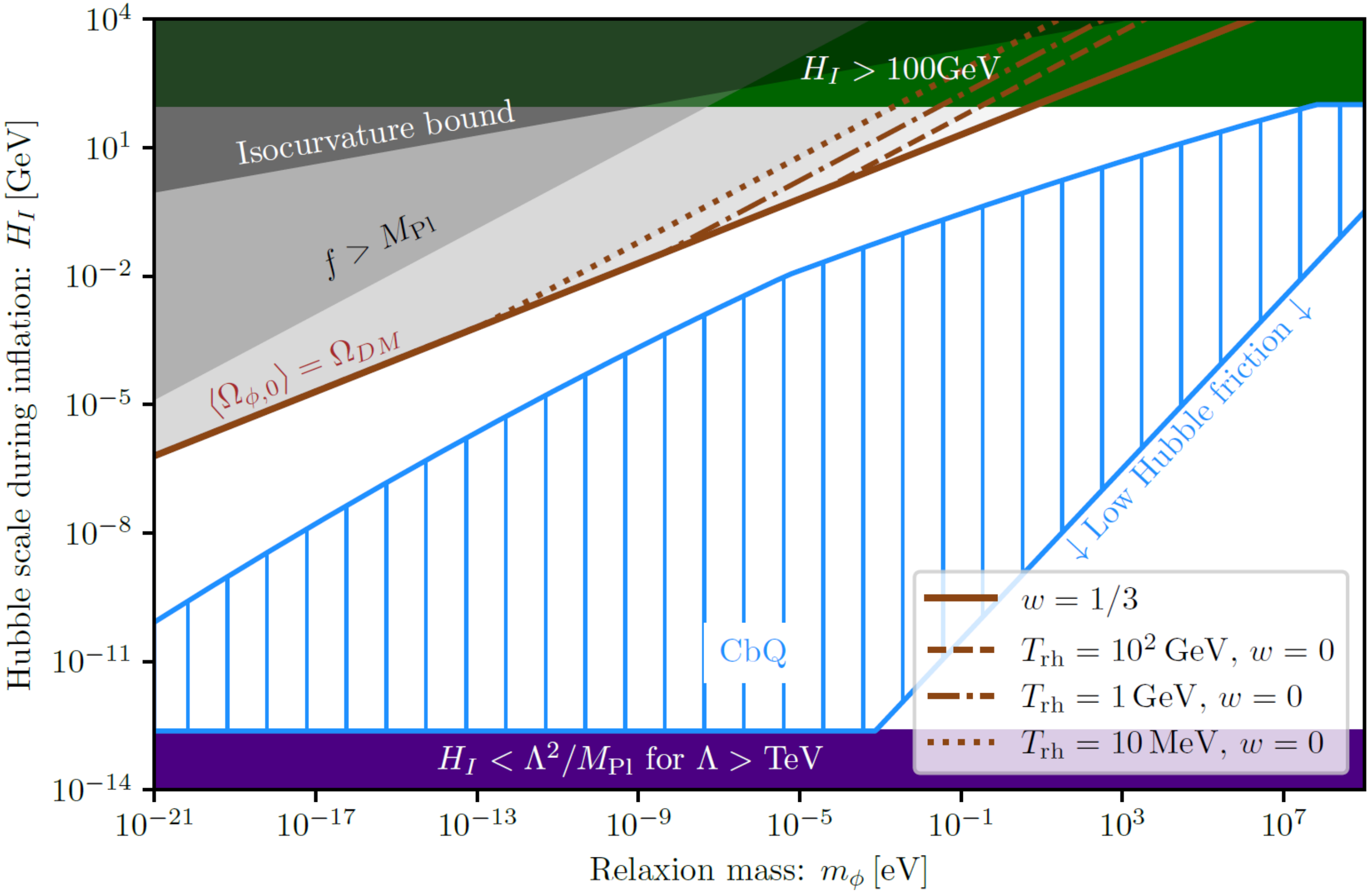
$$\frac{H_I}{\text{GeV}} < 0.3 \times 10^7 \frac{\phi}{10^{11}\text{GeV}} \left(\frac{\Omega_{DM}}{\Omega_{\phi,0}}\right)$$

Relaxion DM window, $T_{rh} < T_b$

The classical beats quantum (CbQ) regime

$$H_I^3 < g\Lambda^3$$

The relaxion is always under-abundant



Relaxion DM window, $T_{rh} < T_b$

The classical beats quantum (CbQ) regime

$$H_I^3 < g\Lambda^3$$

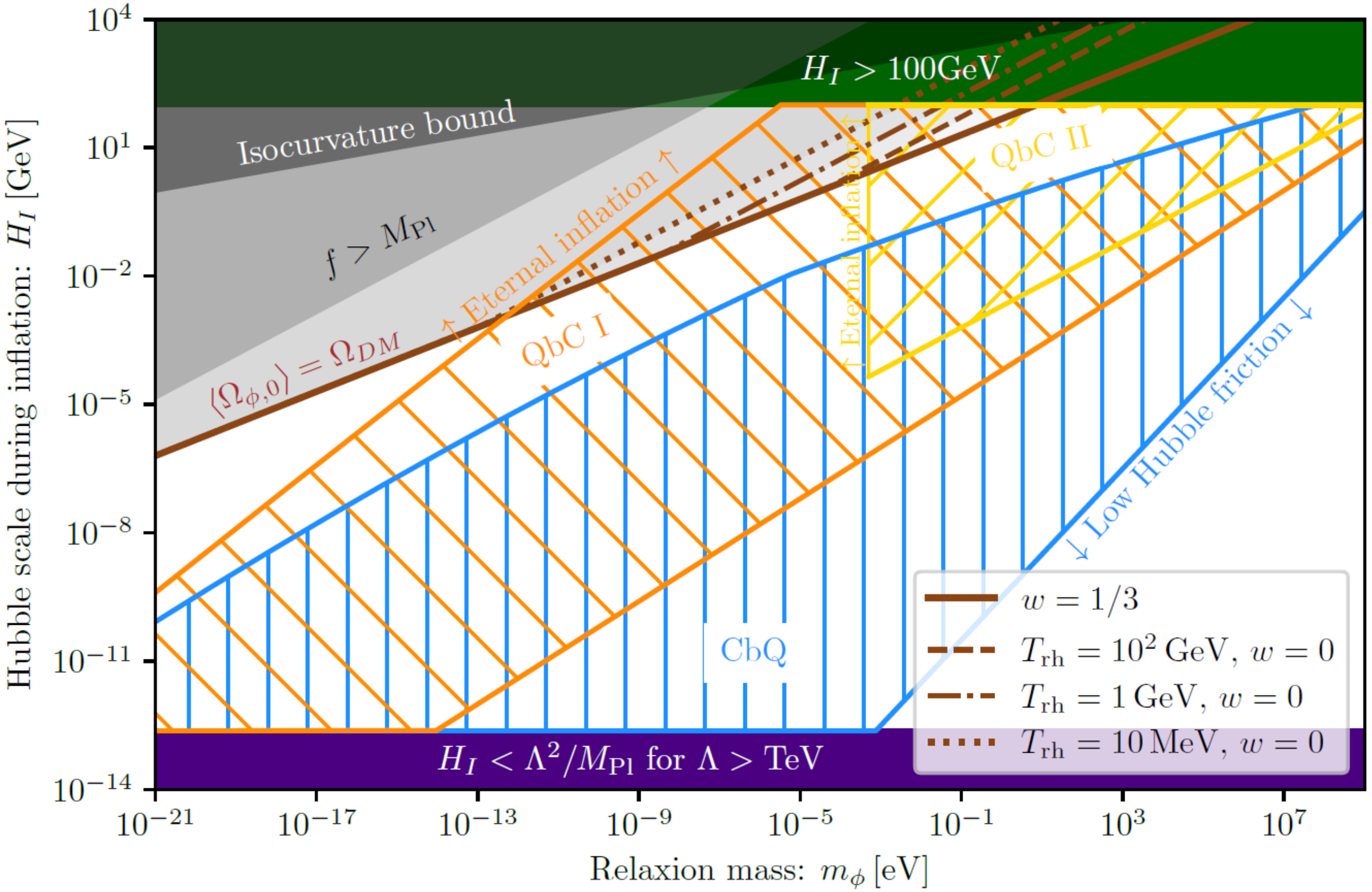
The relaxion is always under-abundant

The quantum beats classical (QbC) regime

$$H_I^3 > g\Lambda^3$$

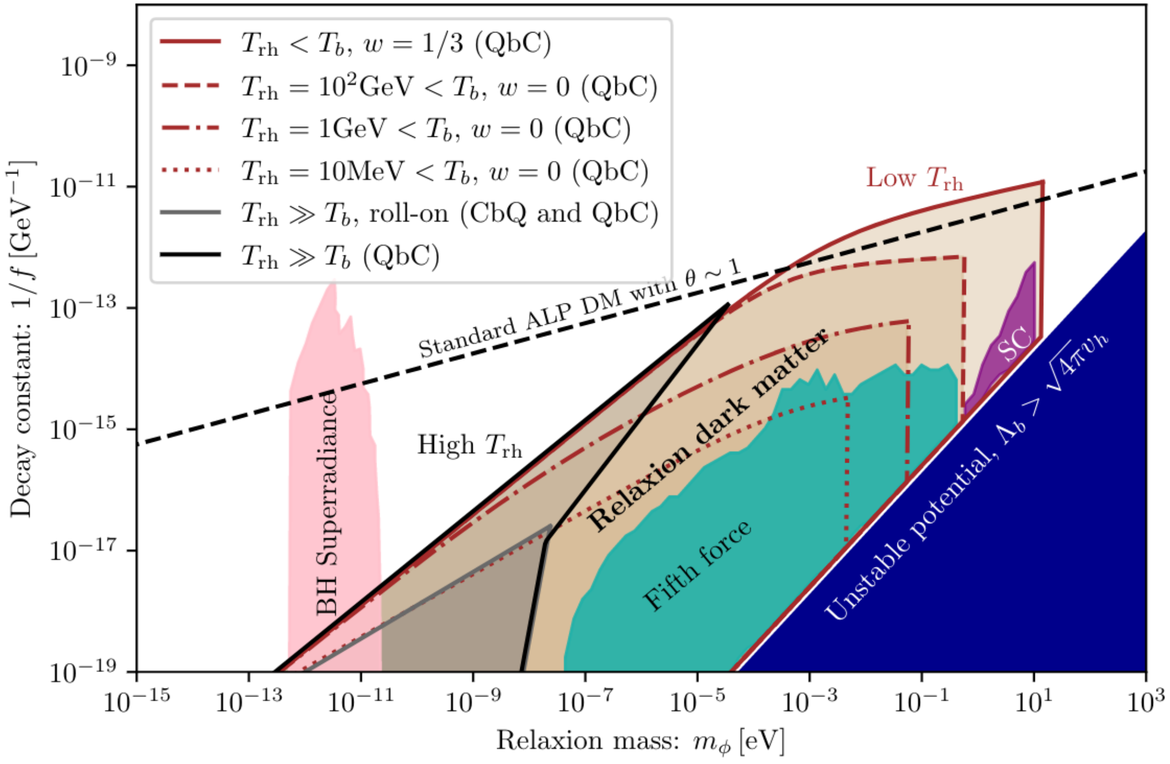
The lower bounds is to avoid eternal inflation

if $N_{\min} > N_c = \frac{2\pi^2}{3}\frac{M_{Pl}^2}{H_I^2}$



$$10^{-13}\left(\frac{\Lambda}{\text{TeV}}\right)^{\frac{16}{7}}\left(\frac{M_{Pl}}{f}\right)^{\frac{4}{7}} < \frac{m_\phi}{\text{eV}} < 0.4\times 10^{4w}\left(\frac{H_I}{100\text{GeV}}\right)^{2(1+w)}\left[\frac{T_{\rm rh}}{100\text{GeV}}\left(\frac{g(T_{\rm rh})}{100}\right)^{\frac{1}{4}}\right]^{\frac{1-3w}{2}}$$

Relaxion DM window (nonQCD model)



Brown: low reheating temperature, stochastic misalignment

Grey: high reheating temperature, misalignment from roll-on after reheating

Banerjee et. al., 1810.01889

Black: high reheating temperature, stochastic misalignment

The stopping condition

- The barriers of the relaxion potential are suppressed due to the rolling term

$$\phi_b - \phi_0 = 2f \times \delta$$

From $V' = 0$ one finds that $\cos\delta = \frac{g\Lambda^3 f}{\Lambda_b^4}$, as well as

$$m_\phi^2 = \frac{\Lambda_b^4}{f^2} \times \sin\delta$$

$$\Delta V_b^{\rightarrow} = 2\Lambda_b^4 \times [\sin\delta - \delta\cos\delta]$$

$$d = \frac{3}{8\pi^2}\frac{H_I^4}{g\Lambda^3 f}$$

- The stopping condition:

$$B = \frac{16\pi^2\Lambda_b^4}{3H_I^4}(\sin\delta - \delta\cos\delta) = \frac{2}{d}[\tan(\delta) - \delta] \sim 1$$

  - $d \ll 1 \;\; \rightarrow \;\; B \sim 1$ already for $\delta \ll 1 \;\; \rightarrow \;\; \Lambda_b^4 \approx g\Lambda^3 f$
  - $d \gg 1 \;\; \rightarrow \;\; B \sim 1$ only for $\delta \approx \frac{\pi}{2} \;\; \rightarrow \;\; \Lambda_b^4 \approx 3H_I^4/16\pi^2$

$$\boxed{\Lambda_b^4 \sim \max\left(g\Lambda^3 f, H_I^4\right).}$$

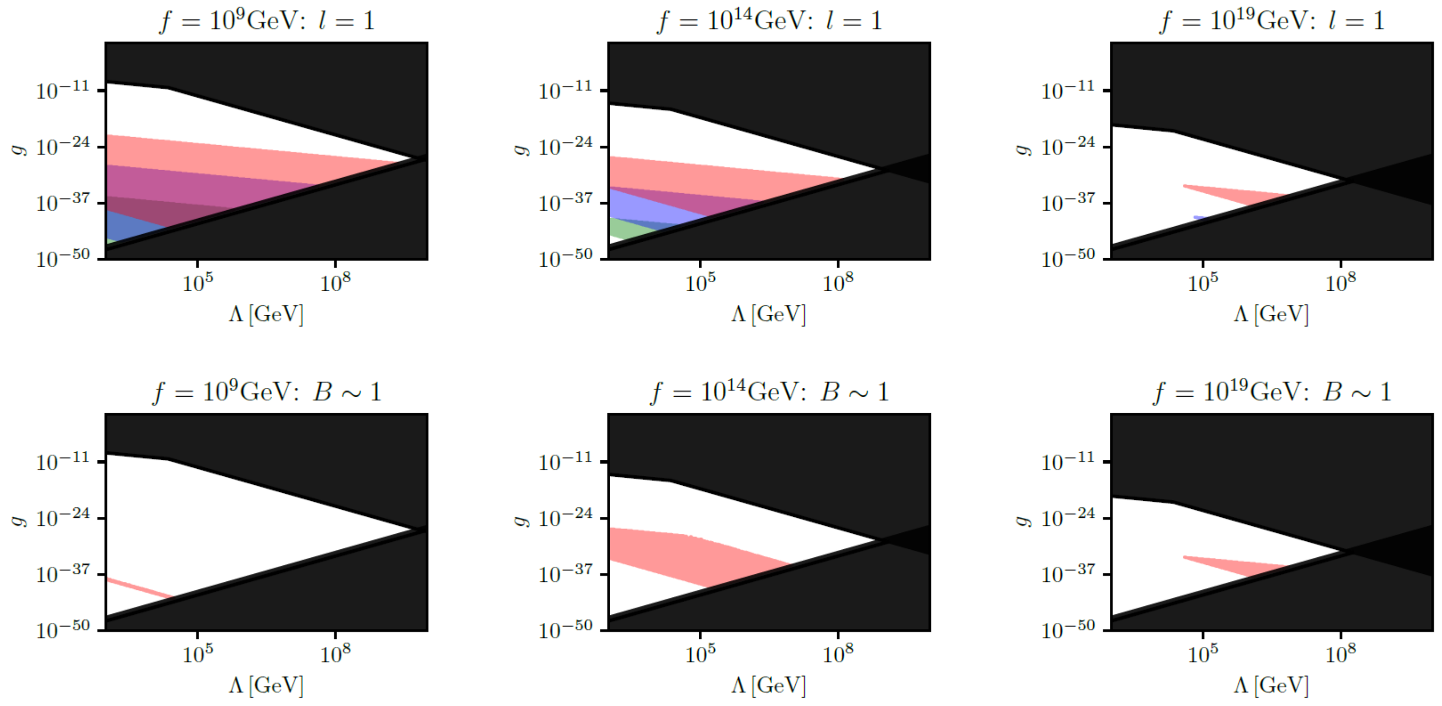
Density-triggered phase transitions in stars

The basic idea

- Large baryon density suppresses the higgs vev/barrier height
- If the minimum disappears and the star is large enough, an expanding bubble can form.

$$n > \frac{\Lambda_b^4 v_h}{\sin\theta_{h\phi} g_{hN} \Lambda^2} \qquad r > \frac{f}{\Lambda_b^2}$$

Budnik et al 2006.14568, Balkin et al 2106.11320



Red giants

White dwarfs

The Sun

Weaker constraints if correct values of l are used!

Stochastic dynamics of the relaxion

Exact solution in the $v_h = 0$ region,

$$\rho(\phi, t) = \frac{1}{\sqrt{2\pi\sigma^2(t)}} \exp\left\{-\frac{(\phi - \dot{\phi}_{SR} t)^2}{2\sigma^2(t)}\right\},$$

with

$$\dot{\phi}_{SR} = \frac{g\Lambda^3}{3H_I} \qquad \sigma(t) = \sqrt{\frac{H_I^3}{4\pi^2} t}$$

Stochastic dynamics of the relaxion

9407016
1708.00010

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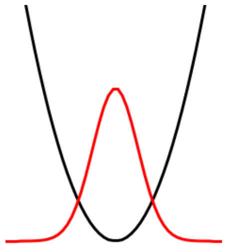
In a bounded potential, $\rho(\phi)$ reaches equilibrium

$$\rho_{\rm eq}(\phi) \propto \exp\left(-\frac{8\pi^2 V(\phi)}{3H_I^4}\right)$$



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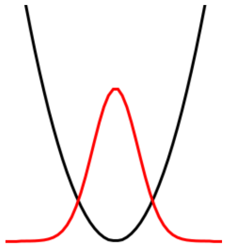
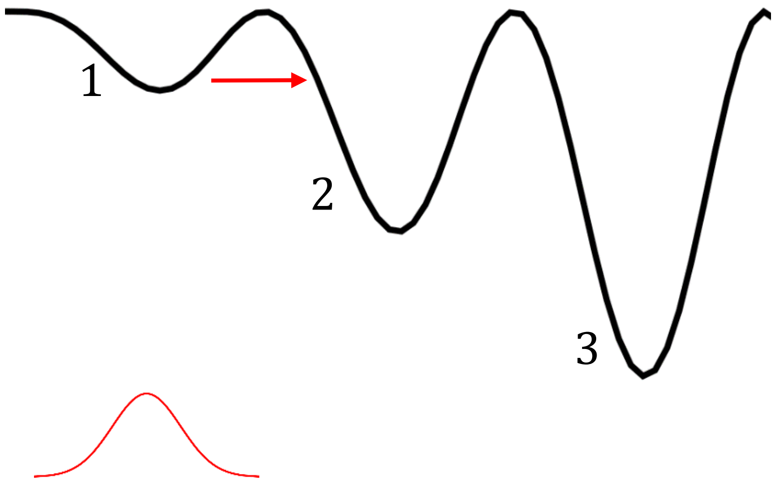
In a bounded potential, $\rho(\phi)$ reaches equilibrium

$$\rho_{\rm eq}(\phi) \propto \exp\left(-\frac{8\pi^2 V(\phi)}{3H_I^4}\right)$$



Diffusion generates a **flux of probability** to a lower minimum



$$\frac{dN_1}{dt} = -k_{12}^{\rightarrow} N_1$$

$$k \approx \frac{\sqrt{V_0''|V_b''|}}{6\pi H_I} e^{-\frac{8\pi^2 \Delta V_b}{3H_I^4}}$$

Hawking-Moss instanton

PLB 110 (1982) 35.

Stochastic dynamics of the relaxion

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Exact solution in the $v_h = 0$ region,

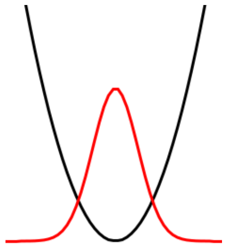
$$\rho(\phi, t) = \frac{1}{\sqrt{2\pi\sigma^2(t)}} \exp\left\{-\frac{(\phi - \dot{\phi}_{SR} t)^2}{2\sigma^2(t)}\right\},$$

with

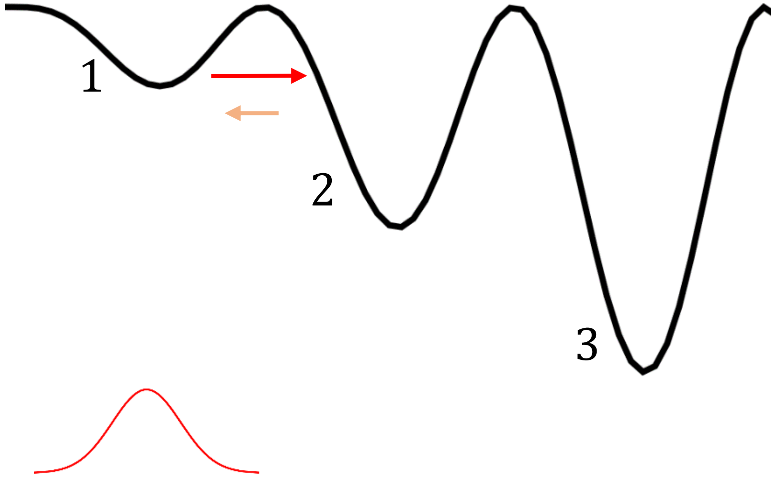
$$\dot{\phi}_{SR} = \frac{g\Lambda^3}{3H_I} \qquad \sigma(t) = \sqrt{\frac{H_I^3}{4\pi^2} t}$$

In a bounded potential, $\rho(\phi)$ reaches equilibrium

$$\rho_{\text{eq}}(\phi) \propto \exp\left(-\frac{8\pi^2 V(\phi)}{3H_I^4}\right)$$



Diffusion generates a **flux of probability** to a lower minimum



$$\frac{dN_1}{dt} = -k_{12}^{\rightarrow} N_1 + k_{21}^{\leftarrow} N_2$$

$$k \approx \frac{\sqrt{V_0'' |V_b''|}}{6\pi H_I} e^{-\frac{8\pi^2 \Delta V_b}{3H_I^4}}$$

Hawking-Moss instanton

PLB 110 (1982) 35.

Stochastic dynamics of the relaxion

Diffusion introduces to new effects, such as

- ***probability fluxes*** between neighboring local minima,

$$k \approx \frac{\sqrt{U_0''|U_b''|}}{6\pi H_I} e^{-B}, \qquad B = \frac{8\pi^2 \Delta U_b}{3H_I^4}$$

Hawking-Moss instanton

PLB 110 (1982) 35.

Dynamics in the region with local minima,

$$\frac{dn_{0,i}}{dt} = -k_{\rightarrow} n_{0,i} - k_{\leftarrow} n_{0,i} + k_{\leftarrow} n_{0,i+1} + k_{\rightarrow} n_{0,i-1}$$

Mean velocity of the field

$$\langle \dot{\phi} \rangle = \int \rho(\phi) \dot{\phi} d\phi \approx \sum_i \dot{n}_{0,i} \phi_{0,i} = 2\pi f (k_{\rightarrow} - k_{\leftarrow})$$

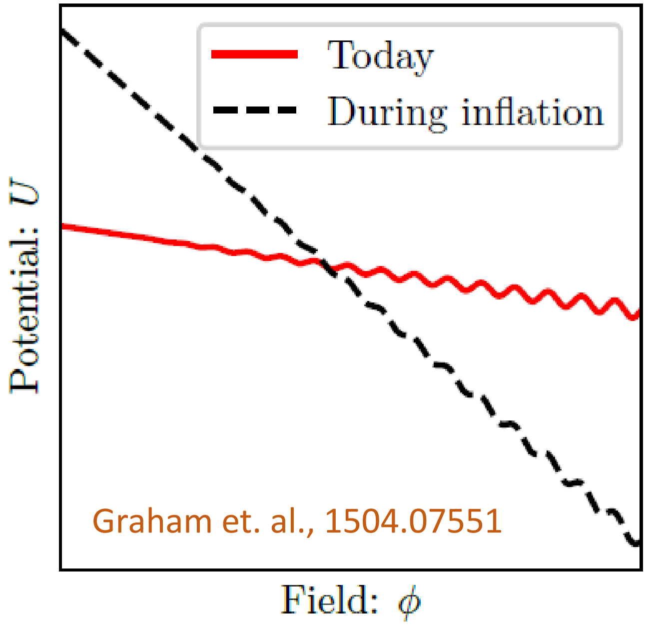
Summary

- New parameter space for the relaxion when considering quantum fluctuations.
- The relaxion in this regime can reproduce the observed dark matter relic abundance.

	With eternal inflation	No eternal inflation	No eternal inflation + dark matter
Classical beats quantum (GKR)	-	$\Lambda < 10^9\mathrm{GeV}$	-
Quantum beats classical	$\Lambda < 10^{11}\mathrm{GeV}$	$\Lambda < 10^9\mathrm{GeV}$	$\Lambda < 10^6\mathrm{GeV}$

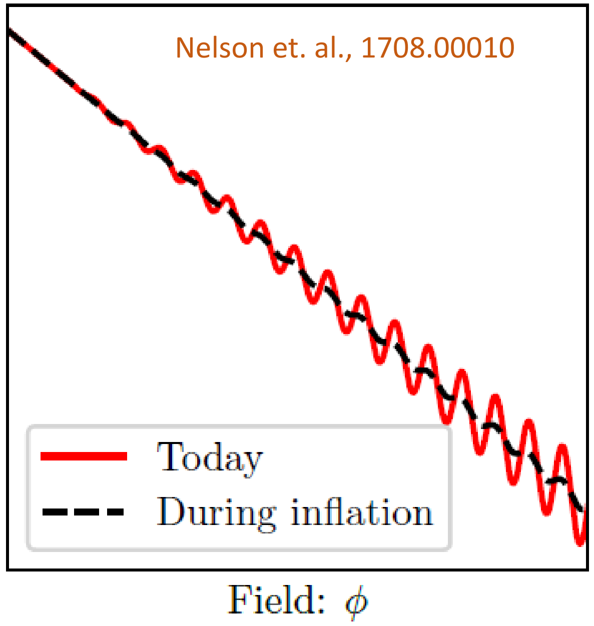
Different approaches to the QCD relaxion

(a)



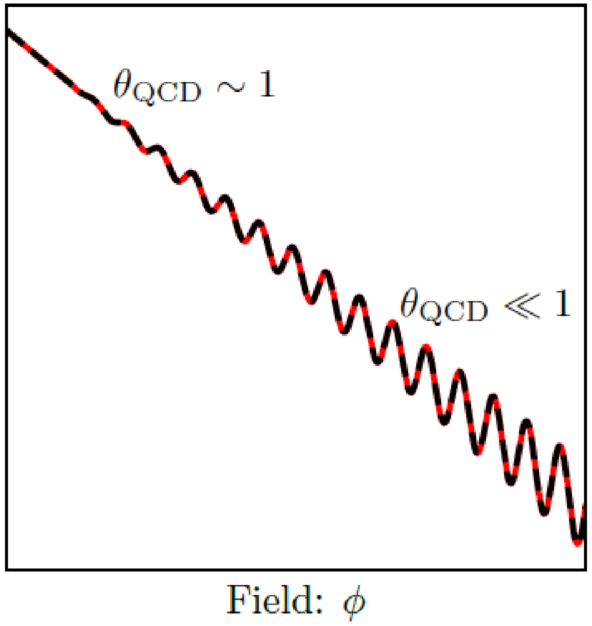
$$10^{-7}\Lambda_b < H_I < 10^{-3}\Lambda_b$$

(b)



$$3\text{GeV} < H_I < 100\text{GeV}$$

(c)



$$H_I \sim \Lambda_b \approx 75\text{MeV}$$

$$\Lambda_b^4(T,h) \approx \frac{\Lambda_b^4(0,h)}{1+(T/\Lambda_{\text{QCD}})^m}$$

Wrong stopping condition used, $\Lambda_b^4(\phi; T_I) \sim g\Lambda^3 f$